

Engineering Mechanics

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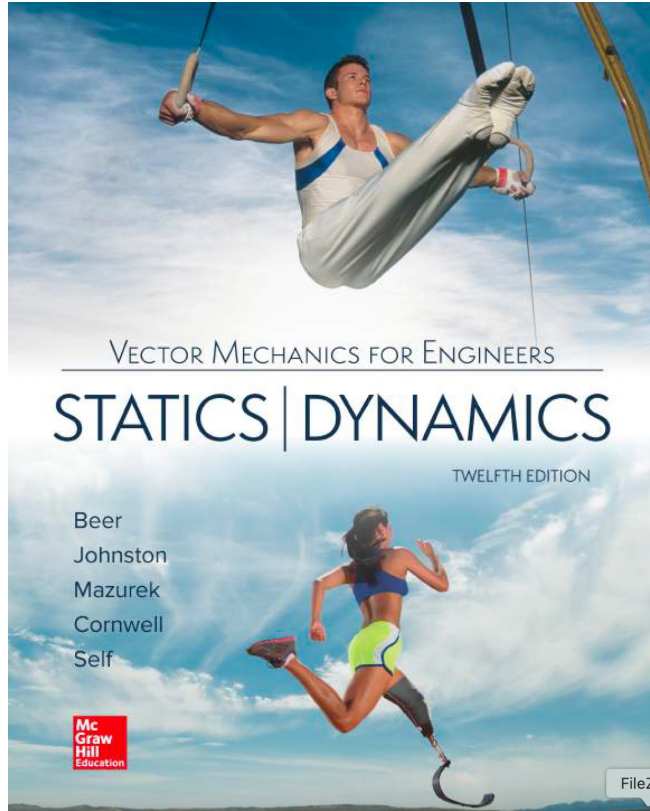
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Cabin: CBMR 304-B

BMEE201L	Engineering Mechanics				L	T	P	C
					2	1	0	3
Pre-requisite	NIL				Syllabus version			
					1.0			
Course Objectives:								
1. To enable students to apply fundamental laws and basic concepts of rigid body mechanics to solve problems of bodies under rest or in motion.								
2. To enable the students to apply conditions of static equilibrium to analyse physical systems.								
3. To compute the properties of areas and bodies.								
Course Outcome:								
Upon successful completion of the course the students will be able to								
1. Compute the resultant and analyse equilibrium (without and with friction) of system of forces acting on particles and rigid bodies in plane and space.								
2. Predict the support-reactions and the internal forces of the members of trusses and frames.								
3. Apply transfer theorems to determine properties of various sections.								
4. Calculate motion parameters of particles and rigid bodies.								
Module:1 Statics of Particles					5 hours			
Fundamental concepts and principles - Resolution of a force -Resultant of forces in a plane- Equilibrium of a particle in a plane; Addition of concurrent forces in space- Equilibrium of a particle in space.								
Module:2 Statics of Rigid Bodies					7 hours			
Equivalent systems of forces- Principle of Transmissibility - Moment of a force about a point and an axis- Couples and force-couple systems- Equilibrium of rigid bodies in two and three dimensions- Types of beams, supports and reactions; Principle of virtual work – System of connected rigid bodies.								
Module:3 Analysis of Structures					5 hours			
Analysis of plane trusses - Method of joints and method of sections- Frames								
Module:4 Friction					5 hours			
The laws of dry friction – Coefficients of Friction- Angles of Friction- Types of Friction Problems - Wedges and Ladder friction- Belt friction.								
Module:5 Properties of Surfaces and Solids					7 hours			
First moments of areas and lines- Centroids of composite areas and lines- - Theorems of Pappus-Guldinus- Second moment of area- Parallel axis theorem- Rectangular and Polar Moments of inertia of composite areas- Radius of Gyration- Product of Inertia- Principal Axes and Principal Moments of Inertia- Mass moments of inertia of thin plates.								
Module:6 Dynamics of Particles					8 hours			
Kinematics of Particles: Displacement, Velocity and Acceleration – Rectilinear motion – Curvilinear motion – Tangential and Normal components – Radial and Transverse components.								
Kinetics of Particles: Newton's Second Law- Energy and Momentum Methods-Principle of Work and Energy-Principle of Impulse and Momentum- Direct Central Impact								
Module:7 Dynamics of Rigid Bodies					8 hours			
Kinematics of rigid bodies: Translation and fixed-axis rotation- General plane motion: velocity- Instantaneous centre of rotation- General plane motion: acceleration.								
Kinetics of rigid bodies:Equations of motion -Angular momentum- Plane motion of a rigid body- Principle of work and energy for rigid bodies- Principle of impulse and momentum for rigid bodies.								
Total Lecture hours:					45 hours			

Textbook for this course



Beer, Johnston, Cornwell, David Mazurek,
and Sanghi, Vector Mechanics for
Engineers: Statics and Dynamics, 12th
Edition, McGraw-Companies

Module 7

Kinematics of Rigid bodies

Module Overview

- Translation
- Fixed Rotation

Introduction

The various types of rigid-body motion can be conveniently grouped as follows:

1. **Translation**. A motion is said to be a translation if any straight line inside the body maintains the same orientation during the motion. In a translation, all of the particles forming the body move along parallel paths. If these paths are straight lines, the motion is called rectilinear translation; if the paths are curved lines, the motion is called curvilinear translation.
2. **Rotation about fixed axes**: In this motion, the particles forming the rigid body move in parallel planes along circles centered on the same fixed axis.

Translational motion

- When a rigid body is in translation, all the points of the body have the same velocity and the same acceleration at any given instant

$$\mathbf{v}_B = \mathbf{v}_A$$

$$\mathbf{a}_B = \mathbf{a}_A$$

Rotation about fixed axis

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\boldsymbol{\omega} = \omega \mathbf{k} = \dot{\theta} \mathbf{k}$$

The vector is directed along the axis of rotation. It is called **the angular velocity** of the body and is equal in magnitude to the rate of change $\dot{\theta}$ of the angular coordinate

We can determine the acceleration $\mathbf{a}$ of particle P. Differentiating it and recalling the rule for the differentiation of a vector product

$$\begin{aligned}\mathbf{a} &= \frac{d\mathbf{v}}{dt} = \frac{d}{dt}(\boldsymbol{\omega} \times \mathbf{r}) \\ &= \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} + \boldsymbol{\omega} \times \frac{d\mathbf{r}}{dt} \\ &= \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} + \boldsymbol{\omega} \times \mathbf{v}\end{aligned}$$

The vector $d\boldsymbol{\omega}/dt$ is denoted by $\boldsymbol{\alpha}$ and is called the angular acceleration of the body

$$\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

Equation defining Rotation of rigid body about fixed axis

The motion of a rigid body rotating about a fixed axis is said to be known when we can express its angular coordinate θ as a known function of t . In practice, however, we can seldom describe the rotation of a rigid body by a relation between θ and t . More often, the conditions of motion are specified by the angular acceleration of the body. For example, α may be given as a function of t , as a function of θ , or as a function of ω .

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\alpha = \omega \frac{d\omega}{d\theta}$$

Equation defining Rotation of rigid body about fixed axis

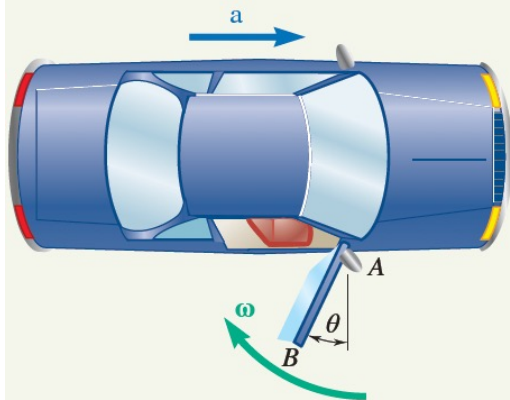
Two particular cases of rotation occur frequently:

1. *Uniform Rotation.* This case is characterized by the fact that the angular acceleration is zero, so the angular velocity is constant and the angular position is given by

$$\theta = \theta_0 + \omega t \quad (15.15)$$

2. *Uniformly Accelerated Rotation.* In this case, the angular acceleration is constant. We can derive the following formulas relating angular velocity, angular position, and time in a manner similar to that described in Sec. 11.2B. The similarity between the formulas derived here and those obtained for the rectilinear uniformly accelerated motion of a particle is apparent.

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ \theta &= \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0)\end{aligned} \quad (15.16)$$



A driver starts his car with the door on the passenger's side wide open ($\theta = 0$). As the car moves forward with constant acceleration, the angular acceleration of the door is $\alpha = 2.5 \cos \theta$, where α is in rad/s^2 . Determine the angular velocity of the door as it slams shut ($\theta = 90^\circ$).

STRATEGY: You are given the angular acceleration as a function of θ , so use the kinematic relationships between angular acceleration, angular velocity, angular position, and time.

MODELING and ANALYSIS: Model the door as a rigid body. Using the basic kinematic relationship gives

$$\alpha = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta} = 2.5 \cos \theta$$

Separating variables gives

$$\omega d\omega = 2.5 \cos \theta d\theta$$

Integrating, using $\omega = 0$ when $\theta = 0$, you have

$$\int_0^\omega \omega d\omega = \int_0^\theta 2.5 \cos \theta d\theta$$

$$\frac{1}{2} \omega^2 = 2.5 \sin \theta \bigg|_0^{\pi/2} = 2.5$$

$$\omega = 2.24 \text{ rad/s} \quad \curvearrowright \quad \blacktriangleleft$$