SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024

Continuous Assessment Test –I

Programme Name & Branch: B.Tech.

Slot: B1+TB1

Course Name & code: Probability and Statistics; BMAT202L

Class Number (s): VL2023240501677

Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s): Answer ALL Questions

<table>
<tr><th>Q. No.</th><th>Question</th><th>Max Marks</th><th>CO</th><th>BL</th></tr>
<tr><td>1.</td><td>In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.
<table>
<tr><td>Number of Mangoes</td><td>170-180</td><td>180-190</td><td>190-200</td><td>200-210</td><td>210-220</td><td>220-230</td><td>230-240</td><td>240-250</td></tr>
<tr><td>Number of Boxes</td><td>52</td><td>68</td><td>85</td><td>92</td><td>100</td><td>95</td><td>70</td><td>28</td></tr>
</table>
Find the mean, median and mode number of mangoes kept in a packing box.</td><td>10</td><td>CO1</td><td>BL1,
BL5</td></tr>
<tr><td>2.</td><td>Following are the observations showing the one-day sales of a shopping mall, where we determine the frequency of the first 50 customers of different age groups.
<table>
<tr><td>Age in Years</td><td>40-44</td><td>45-49</td><td>50-54</td><td>55-59</td><td>60-64</td><td>65-69</td></tr>
<tr><td>No. of Customers</td><td>5</td><td>8</td><td>11</td><td>10</td><td>9</td><td>7</td></tr>
</table>
For the given data, find the quartile deviation, standard deviation and hence compare your results for better dispersion measurement.</td><td>10</td><td>CO1</td><td>BL4,
BL5</td></tr>
<tr><td>3.</td><td>The joint probability density function of two random variables (X, Y) is given by
$$f_{XY}(x,y) = \begin{cases} cx(x-y), & 0 < x < 2;\ -x < y < x \\ 0 & \text{elsewhe} \end{cases}$$
(a) Determine the value of c. (b) Find the marginal distribution for X and marginal distribution for Y. (c) Evaluate $f_{Y/X}(y/x)$.</td><td>10</td><td>CO2</td><td>BL2,
BL1</td></tr>
<tr><td>4.</td><td>The joint probability mass function of (X,Y) is given by p(x,y)=k(2x+3y), x=0,1,2; and y=1,2,3. Find the marginal distribution for X and Y. Find the conditional probability distribution of X, given Y=1. Also find the probability distribution of (X+Y).</td><td>10</td><td>CO2</td><td>BL4</td></tr>
<tr><td>5.</td><td>A sample of 12 fathers and their eldest sons gave the following data about their height in inches:
Father : 65 63 67 64 68 62 70 66 68 67 69 71
Son : 68 66 68 65 69 66 68 65 71 67 68 70
Calculate correlation coefficient for the above data.</td><td>10</td><td>CO2</td><td>BL5</td></tr>
</table>

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES
Winter Semester 2023-2024
Continuous Assessment Test –I

Programme Name & Branch : **B. Tech**
Slot **:** **B2 + TB2 + TBB2**
Course Name & code **:** **Probability and Statistics & BMAT201L**
Exam Duration **:** **90 Min.** **Maximum Marks: 50**

Answer ALL Questions

<table>
<tr><th>Q.No.</th><th>Question</th><th>Max Marks</th><th>CO</th><th>BL</th></tr>
<tr><td>1.</td><td>Find mean, median and mode for the following data:
<table>
<tr><td>Class interval</td><td>150-154</td><td>155-159</td><td>160-164</td><td>165-169</td><td>170-174</td><td>175-179</td><td>180-184</td></tr>
<tr><td>Frequency</td><td>10</td><td>11</td><td>11</td><td>10</td><td>7</td><td>6</td><td>6</td></tr>
</table></td><td>10</td><td>CO1</td><td>BL5</td></tr>
<tr><td>2.</td><td>Life of bulbs produced by two factories A and B are given below:
<table>
<tr><td>Length of life (in hours)</td><td>550-650</td><td>650-750</td><td>750-850</td><td>850-950</td><td>950-1050</td></tr>
<tr><td>Factory A (No. of bulbs)</td><td>10</td><td>22</td><td>52</td><td>20</td><td>16</td></tr>
<tr><td>Factory B (No. of bulbs)</td><td>8</td><td>60</td><td>24</td><td>16</td><td>12</td></tr>
</table>
Find quartile deviation of A and B and then find its coefficients to know the bulbs of which factory are more consistent from the point of view of the length of life?</td><td>10</td><td>CO1</td><td>BL4</td></tr>
<tr><td>3.</td><td>The probability density function of a random variable X is given by
$$f_X(x) = \begin{cases} x\,, & 0 < x < 1 \\ k(2-x)\,, & 1 \le x \le 2 \\ 0\,, & \text{otherwise} \end{cases}$$
(i) Find the value of k
(ii) Find $P(0.2 < x < 1.2)$
(iii) What is $P[\ 0.5 < x < 1.5\ /\ x \ge 1\ ]$?
(iv) Find the distribution function of $f_X(x)$.</td><td>10</td><td>CO2</td><td>BL5</td></tr>
</table>

<table>
<tr>
<td>4.</td>
<td>
Find all the marginal and conditional distributions for the following table which represents the joint probability distribution of the discrete random variable (X, Y).
<table>
<tr><td rowspan="2">Y</td><td colspan="3">X</td></tr>
<tr><td>1</td><td>2</td><td>3</td></tr>
<tr><td>1</td><td>$\frac{1}{12}$</td><td>$\frac{1}{6}$</td><td>0</td></tr>
<tr><td>2</td><td>0</td><td>$\frac{1}{9}$</td><td>$\frac{1}{5}$</td></tr>
<tr><td>3</td><td>$\frac{1}{18}$</td><td>$\frac{1}{4}$</td><td>$\frac{2}{15}$</td></tr>
</table>
</td>
<td>10</td>
<td>CO2</td>
<td>BL2</td>
</tr>
<tr>
<td>5.</td>
<td>
Find the correlation coefficient between the two subjects: Mathematics and Statistics. The marks obtained by 10 students in those subjects are given below:
<table>
<tr><td>Marks in Mathematics</td><td>75</td><td>30</td><td>60</td><td>80</td><td>53</td><td>35</td><td>15</td><td>40</td><td>38</td><td>48</td></tr>
<tr><td>Marks in Statistics</td><td>85</td><td>45</td><td>54</td><td>91</td><td>58</td><td>63</td><td>35</td><td>43</td><td>45</td><td>44</td></tr>
</table>
</td>
<td>10</td>
<td>CO3</td>
<td>BL4</td>
</tr>
</table>

Answer Key

1.

Solution:

Class (1)	Frequency (f) (2)	Mid value (x) (3)	$d = \frac{x - A}{h} = \frac{x - 167}{5}$ $A = 167, h = 5$ (4)	$f \cdot d$ (5) = (2) × (4)	cf (7)
150 - 154	10	152	-3	-30	10
155 - 159	11	157	-2	-22	21
160 - 164	11	162	-1	-11	32
165 - 169	10	167=A	0	0	42
170 - 174	7	172	1	7	49
175 - 179	6	177	2	12	55
180 - 184	6	182	3	18	61
---	---	---	---	---	---
	$n = 61$	-----	-----	$\sum f \cdot d = -26$	-----

For Mean, it is not necessary to make continuous data- Median it is compulsory.

(If continuous data set framed then A=**167.5** instead of **167** --- Final answer for Mean also **165.4** instead of **164.9**)

Mean $\bar{x} = A + \frac{\sum fd}{n} \cdot h$

$= 167 + \frac{-26}{61} \cdot 5$

$= 167 + (-0.4262) \cdot 5$

$= 167 - 2.1311$

$= 164.8689$

Median $M = L + \frac{\frac{n}{2} - cf}{f} \cdot c$

$= 159.5 + \frac{30.5 - 21}{11} \cdot 5$

$= 159.5 + \frac{9.5}{11} \cdot 5$

$= 163.8182$

Here maximum frequency 11 is repeated
So mode can not be obtained directly
We have given Mean ($\bar{X}$) $= 164.8689$, Median(M) $= 163.8182$, Mode(Z) $= ?$

$Z = 3M - 2\bar{X}$

$Z = 3 \cdot 163.8182 - 2 \cdot 164.8689$

$Z = 491.4545 - 329.7377$

$Z = 161.7168$

Mean = 164.9 (4M)

Median = 163.8 (4M)

Mode = 161.7 (2M)

2.

Factory A

Class	Frequency f	cf
550 - 650	10	10
650 - 750	22	32
750 - 850	52	84
850 - 950	20	104
950 - 1050	16	120
---	---	---
	n = 120	--

$$Q_1 = L + \frac{\frac{n}{4} - cf}{f} \cdot c$$

$$= 650 + \frac{30 - 10}{22} \cdot 100$$

$$= 650 + \frac{20}{22} \cdot 100$$

$$= 650 + 90.9091$$

$$= 740.9091$$

$$Q_3 = L + \frac{\frac{3n}{4} - cf}{f} \cdot c$$

$$= 850 + \frac{90 - 84}{20} \cdot 100$$

$$= 850 + \frac{6}{20} \cdot 100$$

$$= 850 + 30$$

$$= 880$$

Factory B

Frequency f	cf
8	8
60	68
24	92
16	108
12	120
---	---
n = 120	--

$$Q_1 = L + \frac{\frac{n}{4} - cf}{f} \cdot c$$

$$= 650 + \frac{30 - 8}{60} \cdot 100$$

$$= 650 + \frac{22}{60} \cdot 100$$

$$= 650 + 36.6667$$

$$= 686.6667$$

$$Q_3 = L + \frac{\frac{3n}{4} - cf}{f} \cdot c$$

$$= 750 + \frac{90 - 68}{24} \cdot 100$$

$$= 750 + \frac{22}{24} \cdot 100$$

$$= 750 + 91.6667$$

$$= 841.6667$$

(3M+3M)

Factory A – CD under Q.D

$$\text{Quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{880 - 740.9091}{2} = \frac{139.0909}{2} = 69.5455 \text{ (Semi-InterQuartile range)}$$

$$\text{Coefficient of Quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{880 - 740.9091}{880 + 740.9091} = \frac{139.0909}{1620.9091} = 0.0858$$

(2M)

Factory B – CD under Q.D

$$\text{Quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{841.6667 - 686.6667}{2} = \frac{155}{2} = 77.5 \text{ (Semi-InterQuartile range)}$$

$$\text{Coefficient of Quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{841.6667 - 686.6667}{841.6667 + 686.6667} = \frac{155}{1528.3334} = 0.1014$$

(2M)

CV (A) < CV (B). Bulb produced by Factory A is more consistent than Factory B from the point of view of life Length.

3.

Given is pdf:

$$f_X(x) = \begin{cases} x\ , & 0 < x < 1 \\ k(2-x)\ , & 1 \leq x \leq 2 \\ 0\ , & \text{otherwise} \end{cases}$$

(i) $\int_0^1 x\,dx + \mathbf{k}\int_1^2 (2-x)\,dx$ **=1**

$$\frac{1}{2} + k\frac{1}{2} = 1$$
$$k = 1$$

(2M)

(iii) $P[\ 0.5 < x < 1.5 / x \geq 1\ ] = \dfrac{\int_1^{1.5}(2-x)dx}{\int_1^2 (2-x)dx}$

(3M)

(ii) $\int_{0.2}^{1} xdx + \int_1^{1.2} 2 - xdx$ **=0.66**

(2M)

(iv) $F(x) = \begin{cases} \frac{x^2}{2}\ , & 0 < x < 1 \\ -\frac{x^2}{2} + 2x - \frac{3}{2}\ , & 1 \leq x \leq 2 \\ 0\ , & \text{otherwise} \end{cases}$ **(3M)**

4.

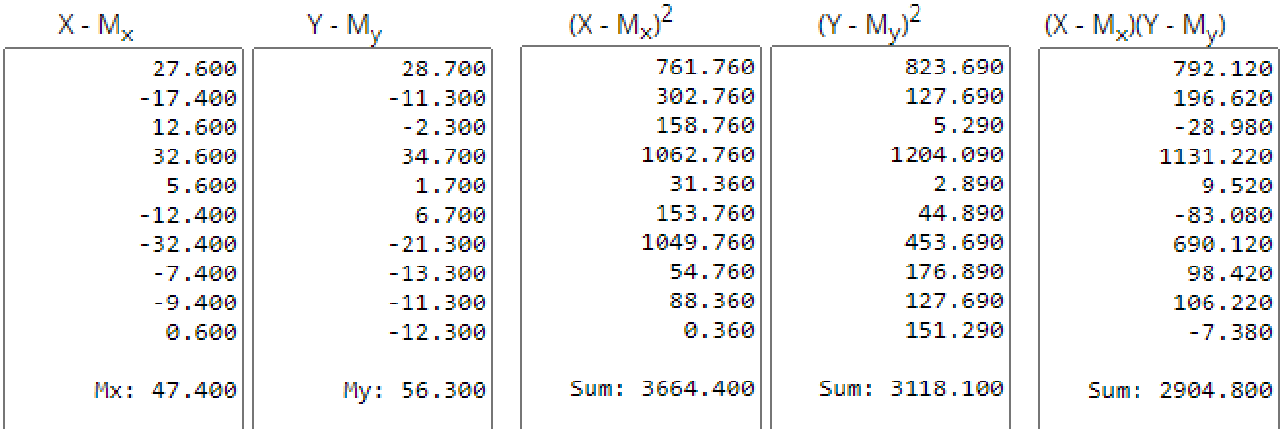
Y	X			MDF
	1	2	3	$P_Y(y)$
1	$\frac{1}{12}$	$\frac{1}{6}$	0	$\frac{1}{4}$
2	0	$\frac{1}{9}$	$\frac{1}{5}$	$\frac{14}{45}$
3	$\frac{1}{18}$	$\frac{1}{4}$	$\frac{2}{15}$	$\frac{79}{180}$
MDF $P_X(x)$	$\frac{5}{36}$	$\frac{19}{36}$	$\frac{1}{3}$	↓ → 1

(5M)

CDF: (Total column sum 1,1…) **(5M)**

	$\boldsymbol{for\ X = 1}$	$\boldsymbol{X = 2}$	$\boldsymbol{X = 3}$	**for** $\boldsymbol{Y = 1}$	$\boldsymbol{Y = 2}$	$\boldsymbol{Y = 3}$
1	$\frac{3}{5}$	$\frac{6}{19}$	0	$\frac{1}{3}$	0	$\frac{10}{79}$
2	0	$\frac{4}{19}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{5}{14}$	$\frac{45}{79}$
3	$\frac{2}{5}$	$\frac{9}{19}$	$\frac{2}{5}$	0	$\frac{9}{14}$	$\frac{24}{79}$

5.



$X - M_x$	$Y - M_y$	$(X - M_x)^2$	$(Y - M_y)^2$	$(X - M_x)(Y - M_y)$
27.600	28.700	761.760	823.690	792.120
-17.400	-11.300	302.760	127.690	196.620
12.600	-2.300	158.760	5.290	-28.980
32.600	34.700	1062.760	1204.090	1131.220
5.600	1.700	31.360	2.890	9.520
-12.400	6.700	153.760	44.890	-83.080
-32.400	-21.300	1049.760	453.690	690.120
-7.400	-13.300	54.760	176.890	98.420
-9.400	-11.300	88.360	127.690	106.220
0.600	-12.300	0.360	151.290	-7.380
Mx: 47.400	My: 56.300	Sum: 3664.400	Sum: 3118.100	Sum: 2904.800

(6M)

X Values

$\sum = 474$

Mean = 47.4

$\sum(X - M_x)^2 = SS_x = 3664.4$

Y Values

$\sum = 563$

Mean = 56.3

$\sum(Y - M_y)^2 = SS_y = 3118.1$

X and Y Combined

N = 10

$\sum(X - M_x)(Y - M_y) = 2904.8$

R Calculation

$r = \sum((X - M_y)(Y - M_x)) / \sqrt{((SS_x)(SS_y))}$

$r = 2904.8 / \sqrt{((3664.4)(3118.1))} = 0.8593$

(4M)

VIT

Vellore Institute of Technology

SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024

Continuous Assessment Test – I

Programme Name & Branch : **B.Tech**

Slot: **D1+TD1**

Course Name & code: **Probability and Statistics & BMAT202L**

Class Number (s): VL2023240501672/1745/1670/2282/2299/1664/1661

Exam Duration: 90 Min. **Maximum Marks: 50**

Answer ALL Questions

(Only calculator is to be permitted)

<table>
<tr><th>Q.No.</th><th>Question</th><th>Max Marks</th></tr>
<tr><td>1.</td><td>Calculate the missing frequency X and median for the following data:
<table>
<tr><th>No. of pills</th><th>No. of people cured</th></tr>
<tr><td>4 – 8</td><td>11</td></tr>
<tr><td>8 – 12</td><td>13</td></tr>
<tr><td>12 – 16</td><td>16</td></tr>
<tr><td>16 – 20</td><td>14</td></tr>
<tr><td>20 – 24</td><td>X</td></tr>
<tr><td>24 – 28</td><td>9</td></tr>
<tr><td>28 – 32</td><td>17</td></tr>
<tr><td>32 – 36</td><td>6</td></tr>
<tr><td>36 – 40</td><td>4</td></tr>
</table>
Given that the average number of pills to cure a person is 20.</td><td>10</td></tr>
</table>

2. Calculate the Quartile deviation and the Standard deviation of the number of children in 35 families for the following data:

No. of children	0	1	2	3	4	5
No of families	2	3	10	15	4	1

10

3. A random variable X has probability density function

$$f(x) = \begin{cases} kx^2 e^{-3x} & , x > 0 \\ 0 & , x \le 0 \end{cases}$$

Find (i) the constant k

(ii) $P(1 < X < 2)$

(iii) $P(X \geq 3)$

(iv) $P(X < 1)$

10

4. The joint probability mass function of two random variables X and Y is specified as follows,

$$P[X = x, Y = y] = k\,(2x + 3y), \qquad x = 1,2\,; y = 0,1,2.$$

Obtain (i) the constant k.

(ii) the marginal probability distribution of X.

(iii) the conditional probability of Y given X = 1.

(iv) $P(X > 1, Y < 2)$.

(v) Check whether X and Y are independent or not.

10

5. A sample of 10 fathers and their sons gave the following data about their heights in inches:

Father X	65	63	67	64	68	62	70	66	68	67
Son Y	68	66	68	65	69	66	68	65	71	67

Calculate the correlation coefficient of X and Y and comment on the result.

10

CAT-I BMAT202L (P&S)

① Given $\overline{X} = 20 = \frac{\sum_i f_i x_i}{\sum_i f_i}$

$\sum f_i x_i = 1772 + 22X$

$\sum f_i = 90 + X$

$\therefore \overline{X} = \frac{1772 + 22X}{90 + X} = 20$

$\Rightarrow \boxed{X = 14}$ (ie) Missing frequency

Median class : 16-20

$\text{Median} = 16 + \left[\frac{52 - 40}{14}\right] \times 4$

$= 19.4286.$

② $N = \sum_i f_i = 35$

$N/4 = 8.75 ; \frac{3N}{4} = 26.25$

$Q_1 = 2 ; Q_3 = 2$

$Q.D. = \frac{Q_3 - Q_1}{2} = 0.5.$

$\sum_i f_i x_i^2 = 267. \quad \sum f_i x_i = 89$

$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$

$= \sqrt{7.6286 - (2.5429)^2}$

$= \sqrt{1.1623} = 1.0781.$

③ As $f(x)$ is a p.d.f, $\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_0^{\infty} k x^2 e^{-3x}\,dx = 1 \Rightarrow k = 27/2.$

(ii) $P(1 < x < 2) = \frac{35}{2}(e^{-3} - e^{-6})$

$= \frac{35(e^3 - 1)}{2e^6}.$

(iii) $P(x \geq 3) = \frac{101}{2e^9}.$

(iv) $P(x > 1) = 1 - 17/2e = \frac{2e - 17}{2e}.$

④ $P(X = x, Y = y) = k(2x + 3y)$, $x = 1, 2$ $y = 0, 1, 2.$

X\Y	0	1	2	P_{i*}
1	2k	5k	8k	15k
2	4k	7k	10k	21k
P_{*j}	6k	12k	18k	1

$\sum_{i=1}^{2} \sum_{j=1}^{3} P_{ij} = 1 \Rightarrow k = 1/36.$

(ii) Marginal prob. distn. of X:

X	1	2
P_{i*}	15/36	21/36

(iii) Condl. prob. fn. of Y gn. X=1.

$P(Y=0 \mid X=1) = 2/15 ; P(Y=2 \mid X=1) = \frac{8}{15}$

$P(Y=1 \mid X=1) = 5/15$

(iv) $P(X > 1, Y < 2) = 4k + 7k = 11/36$

(v) X & Y are not independent.

⑤ $\sum X = 660 ; \sum X^2 = 43616$

$\sum Y = 673 ; \sum Y^2 = 45325$

$\sum XY = 44445 ; n = 10$

There is a positive correlation.

$0.6368 = \frac{270}{423.9811} = \frac{270}{\sqrt{560 \times 321}} = r_{xy} = \frac{n \sum XY - (\sum X)(\sum Y)}{\sqrt{\{n(\sum X^2) - (\sum X)^2\}\{n \sum Y^2 - (\sum Y)^2\}}}$

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Slot: D2+TD2

Course Name & code: Probability and Statistics- BMAT202L

Class Number (s): VL2023240501665, VL2023240502271, VL2023240502291, VL2023240501744, VL2023240501662, VL2023240502275, VL2023240502278.

Faculty Name (s): MURUGAN V , GOURANGA MALLIK, PADMA R, DEBAROTI DAS, POORNIMA T, RAMU G, DHARANI S.

Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s): Answer all questions 5×10=50

<table>
<tr><th>Q.No.</th><th>Question</th><th>Max Marks</th><th>CO</th><th>BL</th></tr>
<tr><td>1.</td><td>Calculate the mean, median and mode for the following distribution.
<table>
<tr><td>Marks</td><td>30-39</td><td>40-49</td><td>50-59</td><td>60-69</td><td>70-79</td><td>80-89</td><td>90-99</td></tr>
<tr><td>No. of students</td><td>8</td><td>87</td><td>190</td><td>304</td><td>211</td><td>85</td><td>20</td></tr>
</table></td><td>10</td><td>CO1</td><td>BL3</td></tr>
<tr><td>2.</td><td>Find the coefficient of mean deviation from mean, coefficient of variation for the following data.
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>f</td><td>4</td><td>36</td><td>100</td><td>232</td><td>280</td><td>204</td><td>112</td><td>28</td><td>4</td></tr>
</table></td><td>10</td><td>CO1</td><td>BL3</td></tr>
<tr><td>3.</td><td>Let X and Y be two random variables having the joint probability mass function $f(x,y)=\frac{1}{27}(2x+y)$ where x and y can assume only the integer values 0, 1, 2.
(i) Find all marginal distributions and means of X and Y.
(ii) Determine the value of $P[X \le 1/Y=1]$ and $P[X \ge 1, Y<2]$</td><td>10</td><td>CO2</td><td>BL3</td></tr>
</table>

<table>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4.</td><td>Let X and Y have the joint probability density function

$$f(x,y)=\begin{cases} x^2+\frac{xy}{3}, 0\le x\le 1,\ 0\le y\le 2 \\ 0, \qquad otherwise \end{cases}$$

Then find (i) $P\left(X>\frac{1}{2}\right)$ (ii) $P(Y<X)$ (iii) $P\left(Y<\frac{1}{2}/X<\frac{1}{2}\right)$</td><td>10</td><td>CO2</td><td>BL3</td></tr>
<tr><td>5.</td><td>Calculate the Karl-Pearson’s coefficient of correlation for the following percentage of marks in Economics (E) and Statistics (S)
<table>
<tr><td>S.No</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>E</td><td>78</td><td>36</td><td>98</td><td>25</td><td>75</td><td>82</td><td>90</td><td>62</td><td>65</td><td>39</td></tr>
<tr><td>S</td><td>84</td><td>51</td><td>91</td><td>60</td><td>68</td><td>62</td><td>86</td><td>58</td><td>53</td><td>47</td></tr>
</table></td><td>10</td><td>CO3</td><td>BL2</td></tr>
</table>

CAT- I

D_2 - Slot Key

BMAT 202L — Probability & Statistics

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①

Marks	f	Mid (x)	x f	Cum- fr
29.5 – 39.5	8	34.5	276	8
39.5 – 49.5	87	44.5	3871.5	95
49.5 – 59.5	190	54.5	10355	285
59.5 – 69.5	304	64.5	19608	589
69.5 – 79.5	211	74.5	15719.5	800
79.5 – 89.5	85	84.5	7182.5	885
89.5 – 99.5	20	94.5	1890	905
	$\Sigma f = 905$		58902.5	

→ Mean $\frac{\Sigma f x}{\Sigma f} = 65.0856$

→ Median Class $\frac{N}{2} = 452.5$

Class 59.5 — 69.5

$L = 59.5, \ f = 304, \ C.f = 285, \ i = 10$

$$\text{Median} = L + \left(\frac{N}{2} - C.f\right)\frac{i}{f} = 65.0099$$

→ Modal Class 59.5 — 69.5

$L = 59.5, \ f_1 = 304, \ f_0 = 190, \ f_2 = 211$

$i = 10$

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 65.0072$$

②

$$\text{Mean} = \frac{\sum fx}{N} = \frac{3972}{1000} = 3.972$$

x	f	$D = \lvert x - \bar{x} \rvert$	fD	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0	4	3.972	15.88	15.776	63.104
1	36	2.972	106.992	8.832	317.952
2	100	1.972	197.2	3.888	388.8
3	232	0.972	225.504	0.944	219.008
4	280	0.028	7.84	0.00078	0.2184
5	204	1.028	209.712	1.056	215.424
6	112	2.028	227.136	4.112	460.544
7	28	3.028	84.784	9.168	256.704
8	4	4.028	16.112	16.224	64.896

$$\text{Mean deviation} = \frac{\sum f\lvert D \rvert}{N} = \frac{1091.168}{1000}$$

$$= 1.091168$$

$$\text{Coefficent of Mean deviation} = \frac{\text{M.D}}{\text{Mean}} = \frac{1.091168}{3.972}$$

$$= 0.2747$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f(x-\bar{x})^2}{N}} = \sqrt{\frac{1986.6504}{1000}}$$

$$= 1.4094$$

$$\text{Coefficent of S.D} = \frac{\text{S.D}}{\text{mean}} = 0.35498$$

$$\text{Coefficent of variation} = 100 \times \frac{\text{S.D}}{\text{mean}} = 35.498$$

③

Marginal distribution of X,

$$P_X(x) = \sum_y P(x,y) = P(x,0) + P(x,1) + P(x,2)$$

$$P_X(x) = \begin{cases} \frac{1}{9} & \text{if } x = 0 \\ \frac{1}{3} & \text{if } x = 1 \\ \frac{5}{9} & \text{if } x = 2 \end{cases}$$

$$P_Y(y) = \begin{cases} \frac{2}{9} & \text{if } y = 0 \\ \frac{1}{3} & \text{if } y = 1 \\ \frac{4}{9} & \text{if } y = 2 \end{cases}$$

Mean $E(X) = \sum x \, P(x,y) = \frac{13}{9}$

$E(Y) = \sum y \, P(x,y) = \frac{11}{9}$

$$P(X \le 1 \mid Y = 1) = \frac{P(X,Y)}{P_Y(Y)} = \frac{P(X \le 1, 1)}{P_Y(1)}$$

$$= \frac{P(0,1) + P(1,1)}{P_Y(1)}$$

$$= 4/9$$

$$P(X \geq 1, Y < 2) = P(1,0) + P(2,0) + P(1,1) + P(2,1)$$

$$= \frac{2}{27} + \frac{4}{27} + \frac{3}{27} + \frac{5}{27} = \frac{14}{27}$$

④ (i) $P(X > \frac{1}{2}) = \int_{x=1/2}^{1} \int_{y=0}^{2} \left(x^2 + \frac{xy}{3}\right) dy\, dx$

$= 5/6$

(ii) $P(Y < X) = \int_{x=0}^{1} \int_{y=0}^{x} \left(x^2 + \frac{xy}{3}\right) dy\, dx$

$= \frac{7}{24}$

(iii) $P(Y < 1/2 \mid X < 1/2) = \frac{P(Y < 1/2) \cap P(X < 1/2)}{P(X < 1/2)}$

$P(Y < 1/2) \cap P(X < 1/2) = \int_{x=0}^{1/2} \int_{y=0}^{1/2} \left(x^2 + \frac{xy}{3}\right) dy\, dx$

$= \frac{5}{192}$

$P(X < 1/2) = \int_{0}^{1/2} \int_{0}^{2} \left(x^2 + \frac{xy}{3}\right) dy\, dx$

$= 1/6$

$\therefore \quad P(Y < \frac{1}{2} \mid X < \frac{1}{2}) = \frac{5}{32}$

⑤

X	Y	$X-\bar{X}$	$Y-\bar{y}$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$	$(X-\bar{X})(Y-\bar{Y}$
78	84	13	18	169	324	234
36	51	−29	−15	841	225	435
98	91	33	25	1089	625	825
25	60	−40	−6	1600	36	240
75	68	10	2	100	4	20
82	62	17	−4	289	16	−68
90	86	25	20	625	400	500
62	58	−3	−8	9	64	24
65	53	0	−13	0	169	0
39	47	−26	−19	676	361	494
			Total =	5938	2224	2704

$$\bar{x} = \frac{\sum x}{n} = \frac{650}{10} = 65$$

$$\bar{y} = \frac{\sum y}{n} = \frac{660}{10} = 66$$

$$r = \frac{\mathrm{Cov}(x,y)}{\sigma_x \sigma_y} = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2}\sqrt{\sum (y-\bar{y})^2}}$$

$$= \frac{2704}{73.47 \times 47.16}$$

$$= 0.7804$$

+Vely Correlated.

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SCHOOL OF ADVANCED SCIENCES

DEPARTMENT OF MATHEMATICS

SLOT F1

CONTINUOUS ASSESMENT TEST-I (January 2023)

WINTER SEMESTER 2022-23

Programme Name & Branch: B.Tech

Course Code: BMAT202L

Course Name: Probability and Statistics

Exam Duration: 90 minutes **Maximum Marks: 50**

General instruction(s): Answer all questions 5×10=50

<table>
<tr><th>Sl.No.</th><th>Question</th><th>Marks</th></tr>
<tr><td>1.</td><td>Calculate the lower quartile, median and upper quartile for the following distribution.
<table>
<tr><td>Age</td><td>54-57</td><td>58-61</td><td>62-65</td><td>66-69</td><td>70-73</td><td>74-77</td><td>78-81</td><td>82-85</td></tr>
<tr><td>No. of employees</td><td>5</td><td>7</td><td>10</td><td>12</td><td>6</td><td>5</td><td>4</td><td>1</td></tr>
</table></td><td>10</td></tr>
<tr><td>2.</td><td>Find the coefficient of mean deviation from mean, coefficient of variation for the following data.
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>f</td><td>4</td><td>36</td><td>100</td><td>232</td><td>280</td><td>204</td><td>112</td><td>28</td><td>4</td></tr>
</table></td><td>10</td></tr>
<tr><td>3.</td><td>Let X and Y are two random variables having the joint probability mass function $f(x,y)=\frac{1}{27}(2x+y)$ where x and y can assume only the integer values 0, 1 and 2. (i) Find all marginal distributions and means of X and Y. (ii) Determine the value of $P[X \le 1/Y=1]$.</td><td>10</td></tr>
<tr><td>4.</td><td>Let X and Y have the joint probability density function
$$f(x,y)=\begin{cases} x^2+\frac{xy}{3}, 0 \le x \le 1, 0 \le y \le 2 \\ 0, \quad otherwise \end{cases}$$
Then Find (i) $P\left(X>\frac{1}{2}\right)$ (ii) $P(Y<X)$ (iii) $P\left(Y<\frac{1}{2}/X<\frac{1}{2}\right)$
(iv) Verify whether X and Y are independent?</td><td>10</td></tr>
<tr><td>5.</td><td>Obtain the correlation coefficient for the following ages of husbands(X) and wives (Y).
<table>
<tr><td>X</td><td>23</td><td>27</td><td>28</td><td>28</td><td>29</td><td>30</td><td>31</td><td>33</td><td>35</td><td>36</td></tr>
<tr><td>Y</td><td>18</td><td>20</td><td>22</td><td>27</td><td>21</td><td>29</td><td>27</td><td>29</td><td>28</td><td>29</td></tr>
</table></td><td>10</td></tr>
</table>

CAT- I BMAT 202L

QN ①

		f	cf
	53.5 - 57.5	5	5
	57.5 - 61.5	7	12
$Q_1 \rightarrow$	61.5 - 65.5	10	22
$Q_2 \rightarrow$	65.5 - 69.5	12	34
$Q_3 \rightarrow$	69.5 - 73.5	6	40
	73.5 - 77.5	5	45
	77.5 81.5	4	49
	81.5 85.5	1	50

$\Sigma f = 50$

$Q_1 \rightarrow \frac{N}{4} = 12.5$

$Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - cf\right)$

$= 61.5 + \frac{4}{10}(12.5 - 12) = 61.7$

$Q_2 - \frac{N}{2} = 25$

$\text{median}/Q_2 = L + \frac{h}{f}\left(\frac{N}{2} - cf\right)$

$= 65.5 + \frac{4}{12}(25 - 22)$

$= 66.5$

$Q_3 - \frac{3N}{4} = 37.5$

$Q_3 = 69.5 + \frac{4}{6}(37.5 - 34)$

$= 71.83$

Q ②

x	0	1	2	3	4	5	6	7	8
f	4	36	100	232	280	204	112	28	4

$\Sigma f = 1000$

mean = 3.972

mean deviation from mean $= \frac{1.091}{3.972} = 0.27467$

S.D. = 1.410

$C.V. = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.41 \times 100}{3.972} = 35.495$

8/12 IV ③ $f(x,y) = \frac{1}{27}(2x+y)$

X\Y	0	1	2	
0	0	1/27	2/27	3/27
1	2/27	3/27	4/27	9/27
2	4/27	5/27	6/27	15/27
	6/27	9/27	12/27	1

marginal of X

X	0	1	2
mg	$\frac{3}{27}$	$\frac{9}{27}$	$\frac{15}{27}$

marg of Y

Y	0	1	2
margin	$\frac{6}{27}$	$\frac{9}{27}$	$\frac{12}{27}$

$$E(X) = 0 \times \frac{3}{27} + 1 \times \frac{9}{27} + 2 \times \frac{15}{27} = \frac{39}{27}$$

$$E(Y) = 0 \times \frac{6}{27} + 1 \times \frac{9}{27} + 2 \times \frac{12}{27} = \frac{33}{27}$$

$$P(X \le 1/Y=1) = \frac{P(X \le 1, Y=1)}{P(Y=1)} = \frac{\frac{1}{27} + \frac{3}{27}}{9/27} = \frac{4}{\cancel{27}} \times \frac{\cancel{27}}{9} = \frac{4}{9}$$

Qn ④ $f(x,y) = x^2 + \frac{xy}{3}, \quad 0 \le x \le 1, \quad 0 \le y \le 2$

(i) $P(X > \frac{1}{2}) = \int_{1/2}^{1} \int_{0}^{2} \left(x^2 + \frac{xy}{3}\right) dy\, dx = \frac{5}{6}$

(ii) $P(Y < X) = \int_{0}^{1} \int_{y=0}^{x} \left(x^2 + \frac{xy}{3}\right) dy\, dx = \frac{7}{24}$

(iii) $$P(Y < \tfrac{1}{2} / X < \tfrac{1}{2}) = \frac{\int_0^{1/2} \int_0^{1/2} \left(x^2 + \frac{xy}{3}\right) dx\, dy}{\int_0^{1/2} \int_0^{2} \left(x^2 + \frac{xy}{3}\right) dy\, dx} = \frac{5/192}{1/6} = \frac{5}{32}$$

(iv) $f_x(x) = \int_0^2 f(x,y)\, dy$

$= 2x^2 + \frac{2}{3}x$

$f_y(y) = \int_0^1 f(x,y)\, dx = \left(\frac{1}{3} + \frac{y}{6}\right)$

$f(x) f(y) \neq f(x,y)$

Not Independent

⑤ $$r = \frac{n\Sigma xy - \Sigma x\,\Sigma y}{\sqrt{\left[n\Sigma x^2 - (\Sigma x)^2\right]\left[n\Sigma y^2 - (\Sigma y)^2\right]}} = 0.81$$

$\Sigma xy = 7623$ $\Sigma x = 300$ $\Sigma y = 250$

$\Sigma x^2 = 9138$ $\Sigma y^2 = 6414$, $n = 10$

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SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024 Continuous Assessment Test –I

Programme Name & Branch : B.Tech(common) **Slot : G1+TG1**

Course Name : Probability and Statistics **Course Code : BMAT202L**

Exam Duration: 90 Min. **Maximum Marks: 50**

Answer ALL Questions(5x10=50 Marks)

1. Find the value of Mean, Median and Mode from the data given below

Weight(kg) :	20-40	40-60	60-80	80-100	100-120	120-140	140-160	160-180	180-200
No of Students :	8	12	20	30	40	35	18	7	5

2. The scores of two bats man A and B in a series of matches are as follows:

A : 37 43 28 62 59 20 83 48 52 47

B : 35 52 77 38 26 58 63 31 40 46

Which of the two batsman do you consider the more consistent and more efficient?

3a) A discrete random variable has the following probability distribution

x	0	1	2	3	4	5	6	7	8
p(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) Find the value of a (ii)Find $P(x \geq 7)$ (iii) Find $P(3 < x < 7/x > 5)$

2 2 2

b) A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of number of good components in the sample. (4)

4. Two electronic components of a missile system work in harmony for the success of the total system. Let *X* and *Y* denote the life in hours of the two components. The joint density of *X* and *Y* is

$$f(x,y) = \begin{cases} ye^{-y(1+x)} & x, y \geq 0 \\ 0, & elsewhere \end{cases}$$

(i) Give the marginal density functions for both random variables. → 6

(ii) What is the probability that the lives of both components will exceed 2 hours? → 4

5. Calculate the correlation coefficient between X and Y

X: 22 53 46 67 43 35 88 11 95 13

Y: 18 39 31 42 55 64 82 10 96 14

$\bar{x} = ?$, median = ?, mode = ?.

Weight	f.	x	c.f	fixi
20-40	8	30	8	240.
40-60	12	50	20	600.
60-80	20	70	40	1400
80-100	30	90	70	2700
100-120	40	110	110	4400
120-140	35	130	145	4550.
140-160	18	150	163	2700.
160-180	7	170	170.	1190
180-200	5	190.	175	950.
	N=175			$\Sigma f_i x_i = 18730$

- Mean = $\bar{x} = \frac{\Sigma f_i x_i}{N}$

$= \frac{18730}{175}$

$\Rightarrow \boxed{\text{Mean} = 107.03}$.

- $\frac{N}{2} = \frac{175}{2} = 87.5$.

c.f greater than 87.5 is 110.

∴ Median class = 100-120.

$\Rightarrow l = 100,\ c.f = 70,\ f = 40,\ h = 20.$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - c.f\right)}{f} \times h$$

(frequency of median class) ← f

$= 100 + \frac{(87.5 - 70)}{40} \times 20.$

$\Rightarrow \boxed{\text{Median} = 108.75}$

- Maximum frequency = 40.

$\Rightarrow$ Modal class = 100-120.

frequency of modal class ← f_1; f_2 → frequency of class succeeding modal class

$\Rightarrow l = 100,\ f_1 = 40,\ f_0 = 30,\ f_2 = 35,\ h = 20.$

f_0 → frequency of class preceding modal class.

$$\text{Mode} = l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h.$$

$= 100 + \frac{(40-30)}{(80-30-35)} \times 20 = 113.33 \Rightarrow \boxed{\text{Mode} = 113.33}$

A. (x_1)	B. (x_2)	$\lvert D\rvert_A = \lvert x_1 - \bar{x}_A\rvert$	$\lvert D\rvert_A^2$	$\lvert D\rvert_B = \lvert x_2 - \bar{x}_B\rvert$	$\lvert D\rvert_B^2$
37	35	+10.9.	118.81.	11.6.	134.56.
43.	52	4.9	24.01.	5.4.	29.16.
28	77	19.9	396.01.	~~21.7~~ 30.4.	924.16.
62	38	14.1	198.81.	8.6	73.96.
59.	26.	11.1.	123.21.	20.6	424.36.
20.	58	27.9	778.41.	11.4	129.96
83.	63.	35.1.	1232.01.	16.4	268.96.
48	31	0.1.	0.01.	15.6.	243.36.
52	40	4.1.	16.81.	6.6.	43.56.
47.	46.	0.9.	0.81.	0.6.	0.36.
479.	466.		$\Sigma\lvert D\rvert_A^2 = 2888.9$		$\Sigma\lvert D\rvert_B^2 = 2272.4$

$\bar{x}_A = \frac{479}{10} = 47.9.$ $\left(\bar{x} = \frac{\text{Sum of observations}}{\text{No. of observations}}\right)$

$\bar{x}_B = \frac{466}{10} = 46.6.$

$\sigma_A = \sqrt{\frac{\Sigma\lvert D\rvert_A^2}{N.}} = \sqrt{\frac{2888.9}{10}} = 16.99.$

$\sigma_B = \sqrt{\frac{\Sigma\lvert D\rvert_B^2}{N}} = \sqrt{\frac{2272.4}{10}} = 15.07.$

$(C.V)_A = \frac{\sigma_A}{\bar{x}_A} \times 100$

$= 35.46\%$

$(C.V)_B = \frac{\sigma_B}{\bar{x}_B} \times 100$

$= 32.33\%.$

Here,

$(C.V)_A > (C.V)_B \Rightarrow$ B is more consistent than A.

$\bar{x}_A > \bar{x}_B \Rightarrow$ A is a better player than B. i.e., A is more efficient than B.

3) a). $\sum_{i=0}^{8} p(x_i) = 1.$

$\Rightarrow.\ a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1.$

$\Rightarrow 81a = 1$

$\Rightarrow \boxed{a = \frac{1}{81}}$

x	0	1	2	3	4	5	6	7	8
$P(x)$	$\frac{1}{81}$	$\frac{3}{81}$	$\frac{5}{81}$	$\frac{7}{81}$	$\frac{9}{81}$	$\frac{11}{81}$	$\frac{13}{81}$	$\frac{15}{81}$	$\frac{17}{81}$.

ii) $P(x \geq 7) = P(X=7) + P(X=8)$

$= \frac{15}{81} + \frac{17}{81} = \frac{32}{81}.$

iii) $P(3<x<7 \mid x>5) = \frac{P(3<x<7,\ x>5)}{P(x>5)}.$

$= \frac{P(5<x<7)}{P(x>5)} = \frac{P(X=6)}{P(X=6)+P(X=7)+P(X=8)}$

$= \frac{\frac{13}{81}}{\frac{13}{81}+\frac{15}{81}+\frac{17}{81}} = \frac{13}{45}$

b). 4 good components and 3 defective components.

X	0	1	2	3.
P(X).	$\frac{1}{35}$.	$\frac{12}{35}$	$\frac{18}{35}$	$\frac{4}{35}$

X → random variable denoting the no. of good components when 3 components are taken

④

$$f(x,y) = \begin{cases} y e^{-y(1+x)} & x, y \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

i) marginal density function of $x = f_x(x)$

$$f_x(x) = \int_0^{\infty} f(x,y)\, dy$$

$$= \int_0^{\infty} y e^{-y(1+x)}\, dy$$

$$= \left[\frac{y e^{-y(1+x)}}{-(1+x)} - \left[\frac{e^{-y(1+x)}}{(1+x)^2} \right] \right]_0^{\infty}$$

$$= 0 - \left[0 - \frac{1}{(1+x)^2} \right]$$

$$= \frac{1}{(1+x)^2}.$$

marginal density function of $x = \frac{1}{(1+x)^2}.$

marginal distributive function of $y = f_y(y)$

$$f_y(y) = \int_0^\infty f(x,y)\,dx$$

$$= \int_0^\infty y e^{-y(1+x)}\,dx.$$

$$= \int_0^\infty y e^{-y} \cdot e^{-yx}\,dx.$$

$$= y e^{-y} \int_0^\infty e^{-yx}\,dx$$

$$= y e^{-y} \left[\frac{e^{-yx.}}{-y} \right]_0^\infty$$

$$= y e^{-y} [0 - 1/-y]$$

$$= e^{-y}.$$

marginal distributive function of $y = e^{-y}$.

ii) ~~propabi~~ probability that both live exceed 2 hours $= \int_2^\infty \int_2^\infty y e^{-y(1+x)}\,dx\,dy$

$$= \int_2^\infty \int_2^\infty y e^{-y(1+x)}\,dx\,dy.$$

$$= \int_2^\infty y e^{-y} \int_2^\infty e^{-xy}\,dx\,dy.$$

$$= \int_{2}^{\infty} ye^{-y}\left[\frac{e^{-xy}}{-y}\right]_{2}^{\infty} dy.$$

$$= \int_{2}^{\infty} ye^{-y}\left[0 - \frac{e^{-2y}}{-y}\right] dy.$$

$$= \int_{2}^{\infty} \cancel{y}e^{-y}\left[\frac{e^{-2y}}{\cancel{y}}\right] dy$$

$$= \int_{2}^{\infty} e^{-3y}\, dy$$

$$= \left.\frac{e^{-3y}}{-3}\right]_{2}^{\infty} = \left[0 - \frac{e^{-6}}{-3}\right]$$

$$= 1/3e^{6}.$$

$\therefore$ probability for both exceeding 2 hours $\Big\}$ $= \frac{1}{3e^{6}}$.

⑤

x	y	xy	x^2	y^2
22	18	396	484	324
53	39	2067	2809	1521
46	31	1426	2116	961
67	42	2814	4489	1764
43	55	2365	1849	3025
35	64	2240	1225	4096
88	82	7216	7744	6724
11	10	110	121	100
95	96	9120	9025	9216
13	14	182	169	196

$\underline{\Sigma x = 473}$ $\underline{\Sigma y = 451}$ $\underline{\Sigma xy = 27936}$ $\underline{\Sigma x^2 = 30031}$ $\underline{\Sigma y^2 = 27{,}927}$

Correlation co-efficient = $\dfrac{Cov(x,y)}{\sigma_x \ \sigma_y}$.

$$= \frac{\frac{\Sigma xy}{N} - \bar{x}\bar{y}}{\sqrt{\frac{\Sigma x^2}{N} - (\bar{x})^2} \sqrt{\frac{\Sigma y^2}{N} - (\bar{y})^2}}$$

Now $\Sigma xy = 27,936$

$\Sigma x^2 = 30,031$

$\Sigma y^2 = 27,971$

$\Sigma x = 473 \implies \bar{x} = 47.3$.

$\Sigma y = 451 \implies \bar{y} = 45.1$

$N = 10$.

Correlation co-efficient = $\dfrac{\frac{27,936}{10} - (47.3)(45.1)}{\sqrt{\frac{30,031}{10} - (47.3)^2}\sqrt{\frac{27,971}{10} - (45.1)^2}}$

$$= \frac{2793.6 - 2133.23.}{\sqrt{3003.1 - 2237.29}\sqrt{2797.1 - 2034.01}}$$

$$= \frac{660.37}{\sqrt{765.81}\sqrt{763.09}} = \frac{660.37}{27.67 \times 27.62.}$$

Correlation co-efficient $= \dfrac{660 \cdot 37}{764 \cdot 25}$

$= 0 \cdot 864 \cdot$

$\therefore$ Correlation co-efficient $= 0 \cdot 864$

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SCHOOL OF ADVANCED SCIENCES
Winter Semester 2023-2024
Continuous Assessment Test –I

Programme Name & Branch: **B.Tech**
Slot: **G2+TG2**
Course Name & code: **Probability and Statistics & BMAT202L**
Exam Duration: 90 Min. **Maximum Marks: 50**

Answer ALL the Questions

Q. No.	**Question**	**Max Marks**
1.	Find the mean, median and mode for the following data 48.61, 44.66,, 34.94	10

Class	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	3	7	13	17	12	10	8	8	6	6

2.	Find the quartiles Q_1, Q_2, Q_3 and coefficient of quartile deviation for the following data. $Q_1 = 23.52$ $Q_2 = 33.91$ $Q_3 = 44.28$ (0.3	10

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	20	34	46	28	14	10

3.	The following table represents the joint probability distribution function of the discrete random variables X and Y	10

Y \ X	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

(i) Find the conditional distribution of X *given* $Y = 2$. (0, 5/14, 9/14)
(ii) Find the conditional distribution of Y *given* $X = 3$. (0, 3/5, 2/5)
(iii) Find $P(X \leq 2, Y = 3)$ 11/36
(iv) Find $P(X + Y < 4)$ 1/4

4.	If X and Y are two random variables having joint probability density function:	10

$$f(x,y) = \begin{cases} \frac{1}{k}(6 - x - y), 0 \leq x < 2, 2 \leq y < 4 \\ 0, \qquad\qquad otherwise \end{cases}$$

8 3/18 5/24 3/5

Find (i)Find the value of k (ii) $P(X < 1 \cap Y < 3)$ (iii) $P(X + Y < 3)$ and (iv) $P(X < 1 / Y < 3)$

.	Calculate the coefficient of correlation between x and y from the following data: 0.9159	10

x	60	34	40	50	45	41	22	43
y	75	32	34	40	45	33	12	30

G12- CAT-I BMAT202L Prob & S

Q ①

Class	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
f	3	7	13	17	12	10	8	8	6	6

$n = \Sigma f = 90$, Assumed mean $= 45.5$ $\Sigma fd = 28$

Mean $= 48.61$, median class $40.5 - 50.5$

Median $= 44.6667$

Mode $= 34.944$ Modal class : $30.5 - 40.5$

Qn ②

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
f	8	20	34	46	28	14	10

$Q_1 = 23.52$

$Q_2 = 33.913$

$Q_3 = 44.28$

Coeff^n $\frac{Q_3 - Q_1}{Q_3 + Q_1} = 0.31$

Qn ③

Y \ X	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

(i) Conditional dis^n of $X / Y=2$

$P(X=1/Y=2) = 0$

$P(X=2/Y=2) = 1/9 / 14/45 = \frac{5}{14}$

$P(X=3/Y=2) = \frac{1/5}{14/45} = \frac{9}{14}$

(ii) $P(Y/X=3)$

$P(Y=1/X=3) = 0$

$P(Y=2/X=3) = 1/5 / 1/3 = 3/5$

$P(Y=3/X=3) = 2/15 / 1/3 = 2/5$

(iii) $P(X \le 2, Y=3) = \frac{1}{18} + \frac{1}{4} = \frac{11}{36}$

(iv) $P(X+Y<4) = P(X=1, Y=1) + P(X=2, Y=1) + P(X=1, Y=2)$

$= \frac{1}{12} + \frac{1}{6} + 0 = \frac{1}{4}$

Qu ④ $f(x,y) = \begin{cases} \frac{1}{k}(6-x-y), & 0 \le x < 2,\ 2 \le y < 4 \\ 0 & \text{Othe} \end{cases}$

(i) $k = 8$

(ii) $P(X<1 \cap Y<3) = \int_0^1 \int_2^3 \frac{1}{8}(6-x-y)\,dx\,dy = \frac{3}{8}$

(iii) $P(X+Y<3) = \int_2^3 \int_0^{3-y} \frac{1}{8}(6-x-y)\,dx\,dy = \frac{5}{24}$

(iv) $P(X<1/Y<3) = \frac{P(X<1 \cap Y<3)}{P(Y<3)} = \frac{3/8}{5/8} = \frac{3}{5}$

Qu ⑤

x	60	34	40	50	45	41	22	43
y	75	32	34	40	45	33	12	30

$\Sigma x = 335$

$\Sigma y = 301$

$\bar{x} = 41.87$

$\bar{y} = 37.62738$

$\Sigma(x-\bar{x}) = -1, \ \Sigma(y-\bar{y}) = -3$

$\Sigma(x-\bar{x})^2 = 867, \ \Sigma(y-\bar{y})^2 = 2239$

$\Sigma(x-\bar{x})(y-\bar{y}) = 1276$

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}} = 0.9159$$

$\Sigma x^2 = 14895$

$\Sigma y^2 = 13563$

$\Sigma xy = 13880$

$n = 8$

$\sigma_x = 11.12$

$\sigma_y = 17.88$