DFT Numericals

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Matrix Method to calculate DFT (DFT property)

$$[X(k)]_{M\times 1} = [\quad]_{M\times M}\,[x(n)]_{M\times 1}$$

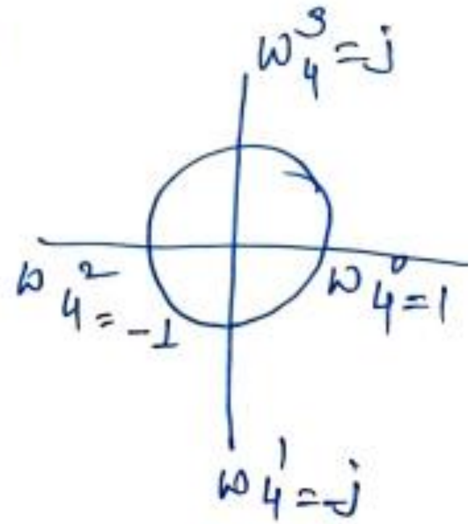
Twiddle matrix

$$W_N^{nk} = e^{\frac{-j2\pi nk}{M}} \quad \text{(Twiddle factor)}$$

Twiddle matrix (N = 4)

$$\Rightarrow \begin{array}{c|cccc} & n=0 & n=1 & n=2 & n=3 \\ \hline k=0 & e^{0} & e^{0} & e^{0} & e^{0} \\ k=1 & e^{0} & e^{\frac{-j2\pi}{4}} & e^{\frac{-j4\pi}{4}} & e^{\frac{-j6\pi}{4}} \\ k=2 & e^{0} & e^{\frac{-j4\pi}{4}} & e^{\frac{-j8\pi}{4}} & e^{\frac{-j12\pi}{4}} \\ k=3 & e^{0} & e^{\frac{-j6\pi}{4}} & e^{\frac{-j12\pi}{4}} & e^{\frac{-j18\pi}{4}} \end{array}_{4\times 4}$$

$$\Rightarrow \begin{array}{c|cccc} & n=0 & n=1 & n=2 & n=3 \\ \hline k=0 & \cancel{0}1 & 1 & 1 & 1 \\ k=1 & 1 & -j & -1 & j \\ k=2 & 1 & -1 & 1 & -1 \\ k=3 & 1 & j & -1 & -j \end{array}$$



Let $x(n) = \{1 \quad 2 \quad 3 \quad 4\}$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$X(k) = \begin{bmatrix} 10 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$X(k) = \{10, -2+2j, -2, -2-2j\}$$

<u>Problem</u> :- Find the convolution of the following sequences by DFT & IDFT method.

$$x_1(n) = \{2, 1, 2, 1\}$$

$$x_2(n) = \{1, 2, 3, 4\}$$

$$X_1(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$$X_2(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$DFT\left[x_1(n) \; N \; x_2(n)\right] = X_1(k) \cdot X_2(k) = X_3(k)$$

$$X_3(k) = \begin{bmatrix} 60 \\ 0 \\ -4 \\ 0 \end{bmatrix}$$

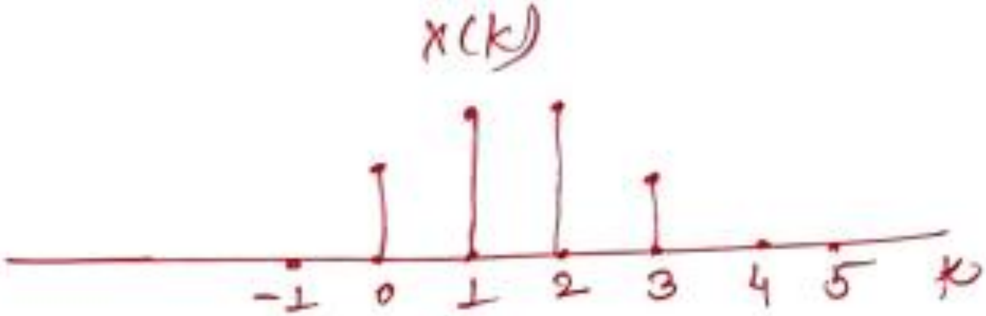
$$x_3(n) = \text{IDFT}[X_3(k)]$$

$$x_3(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 60 \\ 0 \\ -4 \\ 0 \end{bmatrix}$$

$$x_3(n) = \frac{1}{4}\begin{bmatrix} 56 \\ 64 \\ 56 \\ 64 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

Problem :- Consider the finite length sequence $x(n)$ shown below. The five point DFT of $x(n)$ is denoted by $X(k)$. plot the sequence whose DFT is

$$Y(k) = e^{\frac{-4\pi k}{5}} X(k)$$



Using circular time shift property of DFT;

$$DFT[x((n-m))_N] = e^{\frac{-j2\pi mk}{N}} X(k)$$

Here $m = 2$

$$DFT[x((n-2))_5] = e^{\frac{-j2\pi k(2)}{5}} X(k)$$

Hence $y(n) = x((n-2))_5$

$$y(0) = x((0-2))_5 = x(-2)$$

$$= x(-2+5) = x(3)$$

$$= 1 \text{ (periodicity)}$$

$$y(1) = x((1-2))_5 = x(-1) = x(-1+5) = x(4)$$

$$y(1) = 0$$

$$y(2) = x((2-2))_5 = x(0) = 1$$

$$y(3) = x((3-2))_5 = x(+1) = 2$$

$$y(4) = x((4-2))_5 = x(2) = 2$$

$$\cancel{y(5) =}$$

$$y(n) = \{1 \quad 0 \quad 1 \quad 2 \quad 2\}$$

Practice problem :- If the DFT of the sequence

$$x(n) = \{1, 2, 1, 1, 2, 1\} \text{ is } X(k).$$

Plot the sequence whose DFT is

$$Y(k) = e^{-jnk} X(k)$$

Problem :- Let $X(k)$ be a 14-point DFT of a length 14 real sequence of $x(n)$. The 1st 8 samples of $X(k)$ are given by $X(0) = 12$; $X(1) = -1+3j$; $X(2) = 3+j4$; $X(3) = 1-j5$; $X(4) = -2+j2$; $X(5) = 6+j3$; $X(6) = -2-j3$; $X(7) = 10$.

Determine the remeining samples of $X(k)$.

$$N = 14 \quad ; \quad X(k) = X^*(N-k)$$

$$X(8) = X^*(14-8) = X^*(6) = -2+j2$$

$$X(9) = X^*(14-9) = X^*(5) = 6-j3$$

$$X(10) = X^*(14-10) = X^*(4) = -2-j2$$

$$X(11) = X^*(14-11) = X^*(3) = 1+j5$$

$$X(12) = X^*(14-12) = X^*(2) = 3-j4$$

$$X(13) = X^*(14-13) = X^*(1) = -1-j3$$

THANK YOU