F - Test is used to carry out the test for the equality of two population's/sample's variances.

F - Test :- (Variance ratio test)

Let $x_1, x_2, \ldots x_n$ and $y_1, y_2, \ldots y_n$ be the values of two independent random samples of size n_1 and n_2 $(n_1, n_2 \leq 30)$ ie ~~variances~~ and mean $\bar{x}$ and $\bar{y}$, drawn from the normal population with mean μ and S.D σ. The test statistic of F-test is given by

$$F = \frac{S_1^2}{S_2^2}, \quad \text{where } S_1^2 > S_2^2$$

$$F = \frac{\text{larger estimate of variance}}{\text{smaller estimate of variance}}$$

where :-

$$S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1} \quad \text{--- (1)} \qquad S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1} \quad \text{--- (2)}$$

ie in $F = \frac{S_1^2}{S_2^2}$, numerator has degree of freedom $n_1 - 1$ and denominator has degree of freedom $n_2 - 1$.

* If s_1 and s_2 are S.D of samples then

$$s_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1}, \quad s_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2}$$

ie $\Sigma(x-\bar{x})^2 = n_1 s_1^2$ & $\Sigma(y-\bar{y})^2 = n_2 s_2^2$

ie from (1) & (2)

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}, \quad S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$$

* $\boxed{F > 1}$ as $F = \frac{S_1^2}{S_2^2}$ & $S_1^2 > S_2^2$

Test of significance for Ratio of Variance

significance test is performed by means of F-table which provide 5% and 1% of point of significance for F.

How to see F-Table :-

The F-value is found by looking at the degrees of freedom in the numerator and denominator.

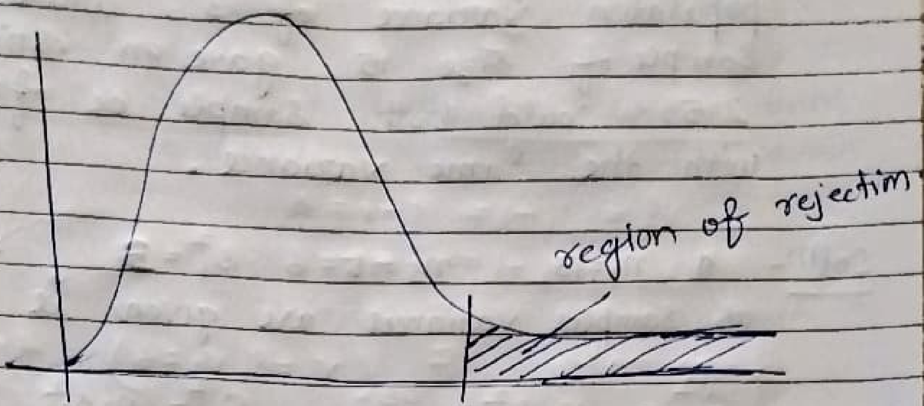
* column headings indicate the numerator degree of freedom. (DF1)
* Row heading indicate the denominator degree of freedom. (DF2)
* cell in the table, for intersection of these two value represent the critical F-value for right tailed test.

numerator D.O.F

Denominator D.O.F

DF2 = ↓ / DF1 =	1	2	3	4	- - -
1	-	-	-	-	
2	-	-	-	-	
3	-	-	-	-	
4	-	-	-	-	
5	-	-	-	-	

F - test curve



① The F - distribution curve is not symmetrical but skewed to the right.

② For different degree of freedoms, there is different curve.

③ As the degree of freedom for numerator and denominator gets larger, the curve approximates the normal.

Test of significance for ratio of variances :-

(1) $H_0 : \sigma_1^2 = \sigma_2^2$

(2) $H_1 : \sigma_1^2 \neq \sigma_2^2$

(3) select α

(4) calculate $F = \frac{S_1^2}{S_2^2}$, $S_1^2 > S_2^2$

(5) calculate F_α, for given α, at degree of freedom $\nu_1 = n_1 - 1$

$\nu_2 = n_2 - 1$

(6) if $F < F_\alpha$, H_0 is accepted otherwise rejected

Q:- A Sample of size 13 gave an estimated population variance of 3, while another Sample of size 15 gave an estimate of 2.5. Could both sample be from populations with the same variance.

Soln:- $n_1 = 13$, $n_2 = 15$ $s_1^2 = 3$ $s_2^2 = 2.5$
as samples variances are given ie

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} \qquad S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$$

$$S_1^2 = \frac{13 \times 3}{12} \qquad S_2^2 = \frac{15 \times 2.5}{14}$$

$$S_1^2 = 3.25 \qquad S_2^2 = 2.67$$

Null Hypo: $H_0 : \sigma_1^2 = \sigma_2^2$ ie variance of two populations are equal.

Alternate $H_1 : \sigma_1^2 \neq \sigma_2^2$

level of significance $\alpha = .05$

Critical value:-

$$V_1 = n_1 - 1 = 12$$

$$V_2 = n_2 - 1 = 14$$

$F_{.05}(12, 14) = 2.53$ (from Table)

as Test statistic

$$F = \frac{S_1^2}{S_2^2} = \frac{3.25}{2.67} = 1.217$$

as $F < F_{0.05}$

ie H_0 is accepted. ie two samples could have been taken from populations with equal variances.

~~Q2. Two random samples are drawn from two populations and the~~

Q In a test given to two group of students drawn from two normal populations, the marks obtained were as follows:-

Group I:- 18 20 36 50 49 36 34 49 41

Group II: 29 28 26 35 30 44 46

examine at 5% level, whether the two populations have same variances.

solution:- $n_1 = 9$, $n_2 = 7$

$$\left.\begin{array}{l} \bar{x} = 37, \quad \bar{y} = 34 \\ s_1 = 11.225 \quad s_2 = 7.426 \end{array}\right\} \text{using calculator.}$$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = 141.75 \qquad S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = 64.3$$

H_0:- $\sigma_1^2 = \sigma_2^2$, ie two populations having same variances.

H_1: $\sigma_1^2 \neq \sigma_2^2$.

$\alpha = .05$

Test statistic:-

$$F = \frac{S_1^2}{S_2^2} = \frac{141.75}{64.33} = 2.203$$

Critical value:-

$$\nu_1 = n_1 - 1 = 9 - 1 = 8$$
$$\nu_2 = n_2 - 1 = 7 - 1 = 6$$

from table $F_{.05}(\nu_1 = 8, \nu_2 = 6) = 4.15$

as $F < F_{.05}$

ie H_0 is accepted. ie two populations have same variances.

Q:- A group of 10 rats fed on diet A and another group of 8 rats fed on diet B recorded following increase in weight:

(gm) Diet A:	5	6	8	1	12	4	3	9	6	10
(gm) Diet B:	2	3	6	8	1	10	2	8		

Find if variances are significantly different.

Sol^n:- $n_1 = 10$, $n_2 = 8$

$s_1 = 3.2$ $s_2 = 3.23$ } calculate

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = 11.38$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = 11.92$$

Null Hypo : $\sigma_1^2 = \sigma_2^2$

Alter Hypo : $\sigma_1^2 \neq \sigma_2^2$

$\alpha = .05$

Test Statistic :- $F = \dfrac{S_2^2}{S_1^2}$

$$F = \frac{11.92}{11.33} = 1.05$$

Critical value :-

$\nu_1 = 9, \quad \nu_2 = 7$

$\cancel{F_{.05}(9,7)}$ $F_{.05}(\nu_2 = 7, \nu_1 = 9) = 3.68$

as $F_{cal} < F_{.05}$

ie H_0 is accepted ie variances are not significantly different.

Q:- Two samples of sizes 9 and 8. have been drawn and gave the sum of squares of deviation from their respective means equal to 160 and 91 respectively. can they be regarded as drawn from the same population.

Solⁿ:- $n_1 = 9, \quad n_2 = 8$

$$\Sigma(x - \bar{x})^2 = 160, \quad \Sigma(y - \bar{y})^2 = 91$$

as $s_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1}$ & $s_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2}$

ie

$n_1 s_1^2 = 160$ ie $n_2 s_2^2 = 91$

Now $S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$, $S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$

$S_1^2 = \frac{160}{8}$ $S_2^2 = \frac{91}{7}$

$= 20$ $= 13$

Null Hypo:- $\sigma_1^2 = \sigma_2^2$

Alternate Hypo: $\sigma_1^2 \neq \sigma_2^2$

$\alpha = .05$

Test statistic $F = \frac{S_1^2}{S_2^2}$

$F = \frac{20}{13} = 1.54$

Critical value:-

$F_{.05}(\nu_1 = 8, \nu_2 = 7) = 3.73$

Since $F_{cal} < F_{.05}$

ie H_0 is accepted.

ie two samples could have come from two normal population with equal variances.