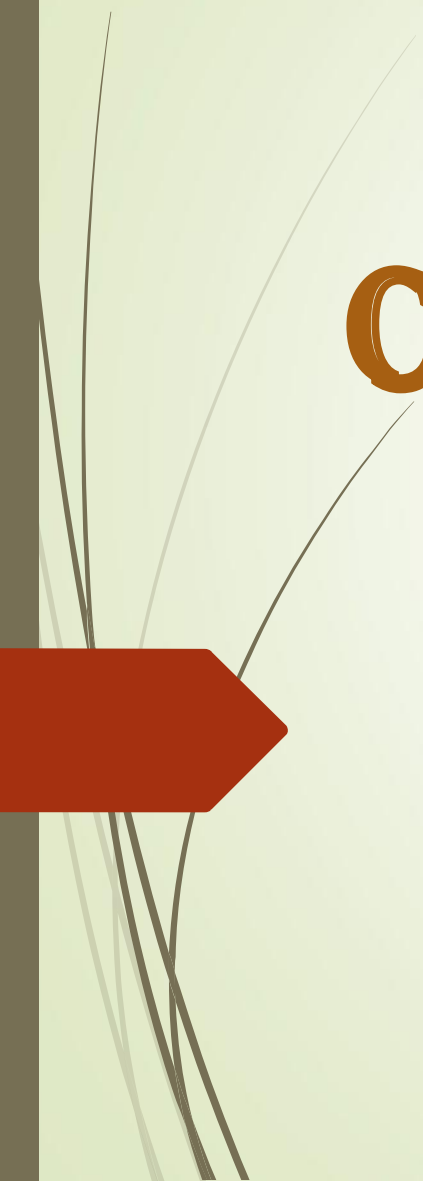




# **BEEE203L**

# **CIRCUIT THEORY**

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# **MODULE – IV**

## **Analysis of Transient Response of Circuits**

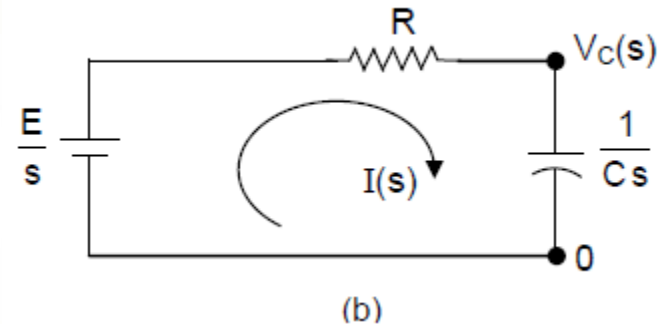
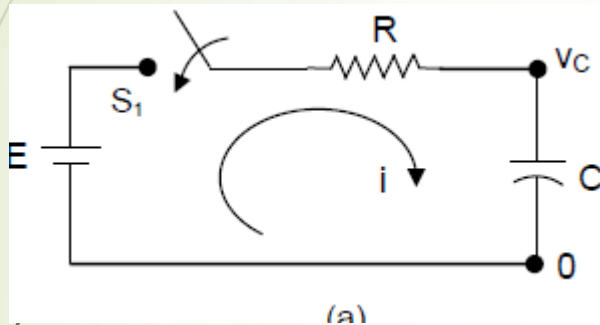
# Contents

- Review of Laplace transformation;
- Laplace transform of network
- Time domain solution for RL,RC, RLC networks for DC excitation
- Time domain solution for RL,RC, RLC networks for AC excitation
- Transient Behaviour of circuit elements under switching conditions and their representations
- Evaluation of initial and final conditions in RL,RC,RLC circuits with DC excitation
- Evaluation of initial and final conditions in RL,RC,RLC circuits with AC excitation

# Transient Analysis of RC circuits

4

- Consider the RC circuit shown in Fig (a). Assume that the switch is closed at time  $t = 0$  and assume that the voltage across the capacitor at the time of switching is zero.



- The transform circuit for time  $t > 0$  is shown in Fig. (b). From this

$$I(s) = \frac{E/s}{R + \frac{1}{Cs}} = \frac{EC}{RCs + 1} = \frac{E/R}{s + \frac{1}{RC}}$$

- Taking inverse LT

$$i(t) = \frac{E}{R} e^{-\frac{1}{RC}t}$$

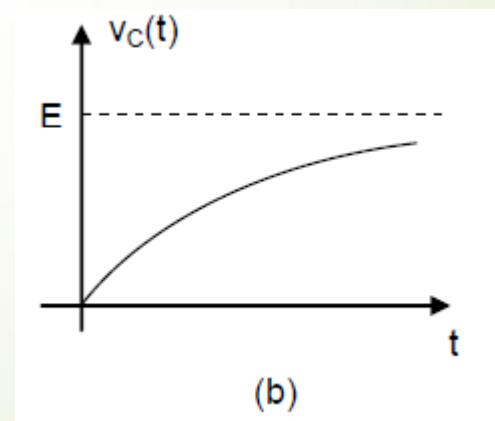
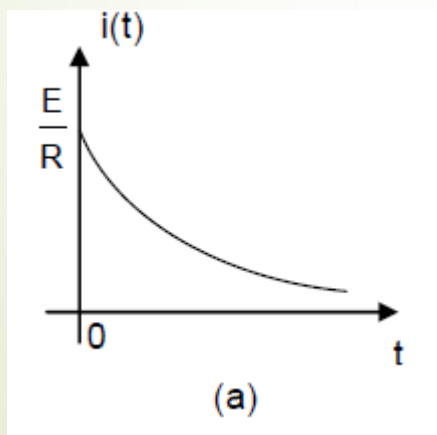
- Voltage across the capacitor is

$$V_C(s) = \frac{1}{Cs} I(s) = \frac{E/RC}{s(s + \frac{1}{RC})} = E \left( \frac{1}{s} - \frac{1}{s + \frac{1}{RC}} \right)$$

- Taking inverse LT, we get the capacitor voltage as

$$v_C(t) = E (1 - e^{-\frac{1}{RC}t})$$

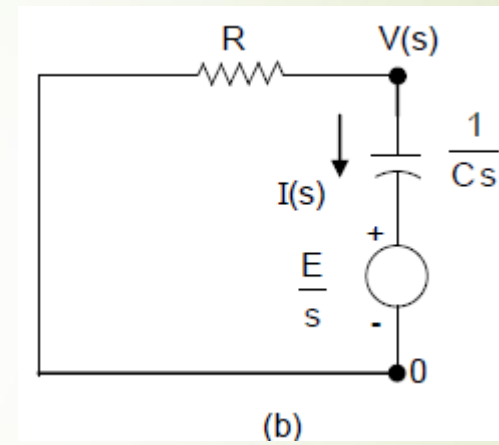
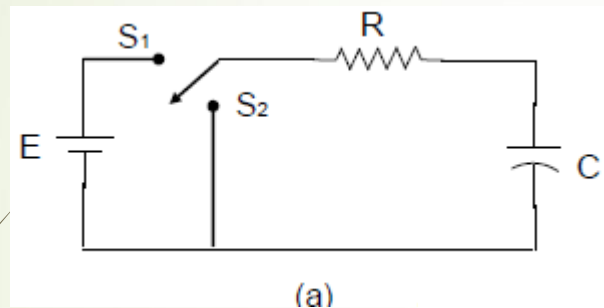
- The circuit current and the voltage across the capacitor vary as shown in Fig. below



# Transient response of RC circuit –

## 6 With initial conditions applied

Consider the circuit shown in Fig.(a). Let us say that with the switch in position  $S_1$ , steady state condition is reached. Now, the capacitor is charged to voltage  $E$  and will act as open circuit.



$$V_C(0+) = V_C(0-) = E$$

At time  $t = 0$ , the switch is turned to position  $S_2$ . The transform circuit for time  $t > 0$  is shown in Fig. (b).

Considering the transformed circuit

$$I(s) = - \frac{E/s}{R + \frac{1}{Cs}} = - \frac{EC}{RCs + 1} = - \frac{E/R}{s + \frac{1}{RC}}$$

Taking inverse LT

$$i(t) = - \frac{E}{R} e^{-\frac{1}{RC}t}$$

The capacitor current exponentially changes from  $-E/R$  to zero with time constant  $1/RC$ .

Since  $R I(s) + V_C(s) = 0$  the voltage across the capacitor is

7

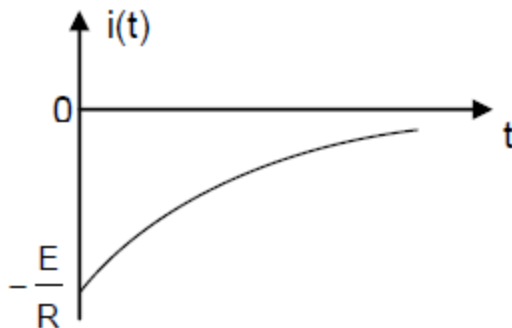
$$V_C(s) = -R I(s) = \frac{E}{s + \frac{1}{RC}}$$

Taking inverse LT

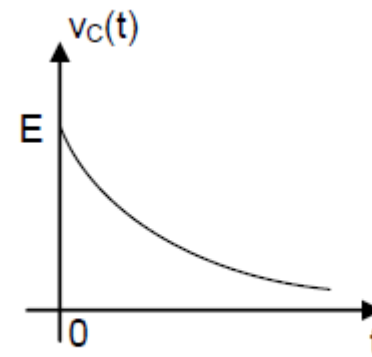
$$v_C(t) = E e^{-\frac{1}{RC}t}$$

The Capacitor voltage decays exponentially with time constant  $1 / RC$ .

The current and voltage transients are given by the above two equations are shown in Figure



(a)

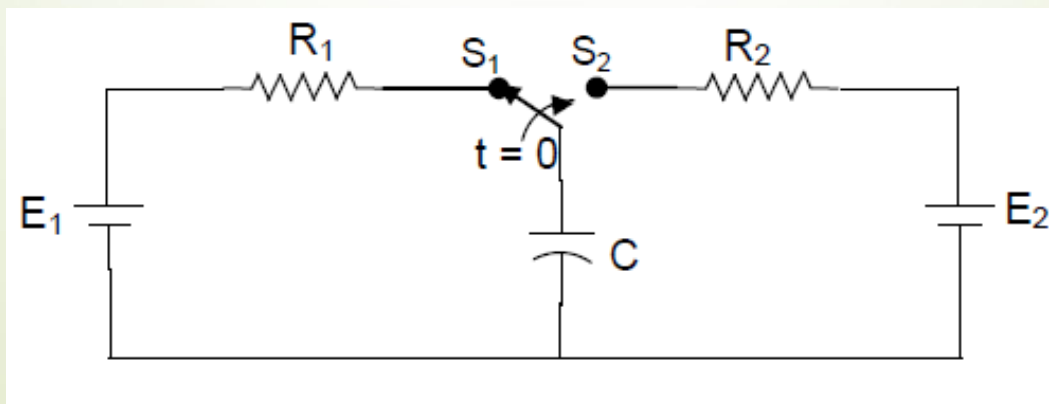


(b)

# General case of finding the Capacitor Voltage

8

- In the previous discussion, it was assumed that the initial capacitor voltage  $V_C(0) = 0$ .
- There may be initial charge in the capacitor resulting non-zero initial capacitor voltage. Further, the circuit arrangements can also cause non-zero initial capacitor voltage.
- For this purpose consider the circuit shown below. The switch was in position  $S_1$  for a long time. It is moved from position  $S_1$  to  $S_2$  at time  $t = 0$ .



We shall assume the following:

1. At time  $t = 0^-$  the circuit was at steady state condition with the switch in position  $S_1$
2. After switching to position  $S_2$ , the circuit is allowed to reach the steady state condition. Thus, we are interested about the transient analysis for one switching period only.

Initial capacitor voltage  $V_C(0)$  is  $E_1$  and the final capacitor current  $V_C(\infty)$ , will be  $E_2$ .

The more general expression for the Capacitor voltage can be obtained as

$$V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)]e^{-(1/CR_2)t}$$

# Circuit with multiple resistances

10

In order to calculate the time constant  $1/RC$  for circuits with multiple resistances, following steps to be followed

1. Calculate the Thevenin's Equivalent resistance across the Inductor (i.e. inductor as the load )
2. Time constant is given by  $1/CR_{th}$

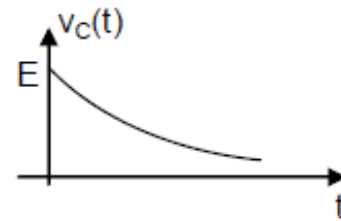
# Summary of formulae useful for transient analysis on RC circuits

11

1. Time constant  $\tau = RC$      $\alpha = 1 / RC$

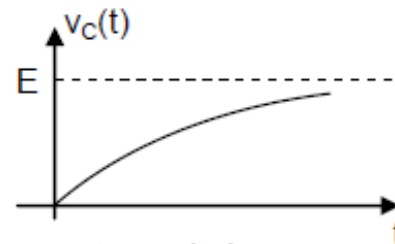
2. When the capacitor is discharging from the initial voltage of E

$$v_C(t) = E e^{-\frac{1}{RC} t}$$



3. When the capacitor is charged from zero initial voltage to final voltage of E

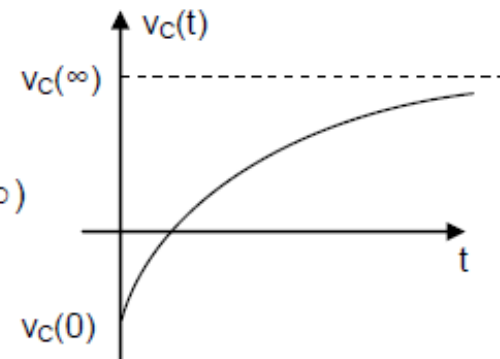
$$v_C(t) = E (1 - e^{-\frac{1}{RC} t})$$



4. When the capacitor voltage changes from  $v_C(0)$  to  $v_C(\infty)$

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)] e^{-\frac{1}{RC} t}$$

Plot of  $v_C(t)$  depends on values of  $v_C(0)$  and  $v_C(\infty)$



5. Capacitor current  $i_C(t) = C \frac{dv_C(t)}{dt}$

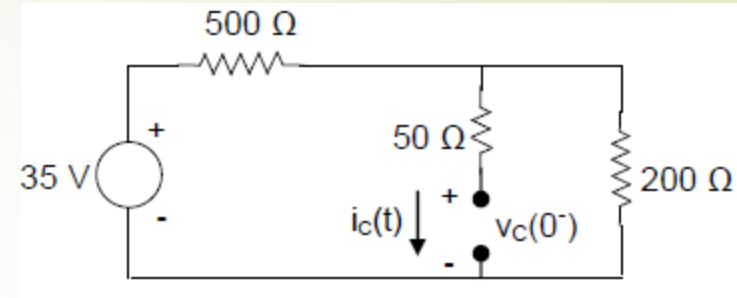
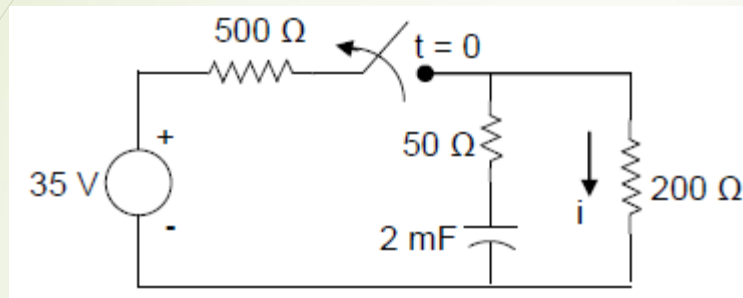
$$i(t) = \frac{E}{R} e^{-\frac{1}{RC} t}$$

# Problems

12

P.4.15

Consider the circuit shown below. The switch was in closed position for a long time. It is opened at time  $t = 0$ . Find the current  $i(t)$  for  $t > 0$ .



- Solution Circuit at time  $t = 0^-$  is shown.

$$v_C(0^-) = 35 \times \frac{200}{200 + 500} = 10 \text{ V}$$

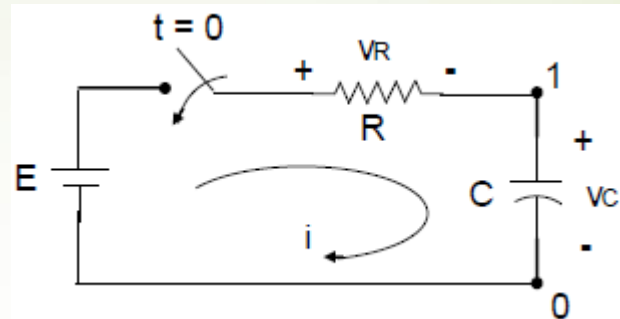
- For time  $t > 0$ , capacitor voltage of 10 V is discharged through a resistor of 250  $\Omega$ .
- Time constant  $RC = 250 \times 2 \times 10^{-3} = 0.5 \text{ s}$ ;

$$\begin{aligned} \text{So } V_C(t) &= V_C(\infty) + [V_C(0) - V_C(\infty)]e^{-(1/CR)t} \\ &= 0 + (10 - 0)e^{-(1/CR)t} = 10e^{-(1/0.5)t} \end{aligned}$$

$$i_C(t) = (E/R)e^{-t/RC} = (10/250)e^{-t/0.5} = 0.04e^{-2t} \text{ A}$$

**P.4.16** A series RC circuit has a constant voltage of  $E$ , applied at time  $t = 0$  as shown in Fig. below. The capacitor has no initial charge. Find the equations for  $i$ ,  $V_R$  and  $V_C$ . Sketch the wave shapes.  $E = 100 \text{ V}$   
 $R = 5000 \Omega$   $C = 20 \mu\text{F}$

**13**



**Solution**

Since there is no initial charge,  $V_C(0^+) = V_C(0^-) = 0$

For  $t > 0$ , capacitor is charged to final voltage of  $100 \text{ V}$ .

Time constant  $RC = 5000 \times 20 \times 10^{-6} = 0.1 \text{ sec}$ .

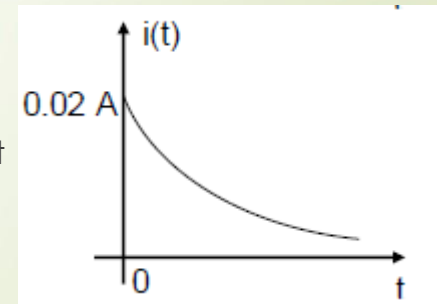
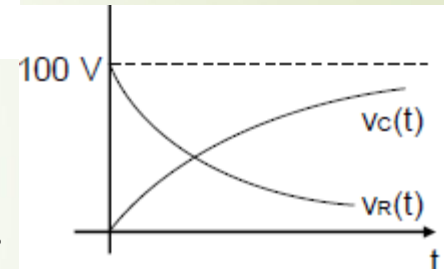
$$V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)]e^{-(1/CR)t}$$

$$V_C(0) = 0 ; V_C(\infty) = 100 ; \text{ So } V_C(t) = 100 + [0 - 100] e^{-(1/0.1)t}$$

$$= 100 (1 - e^{-10t}) \text{ V}$$

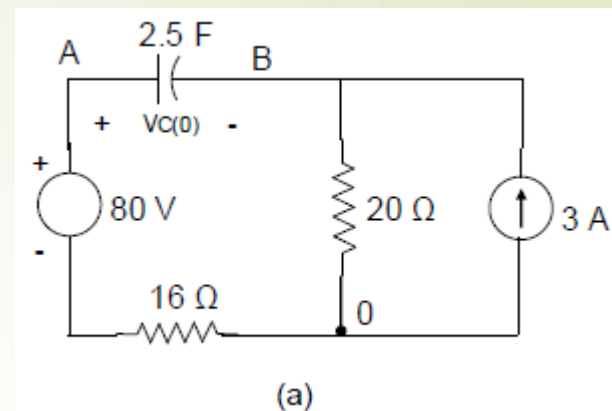
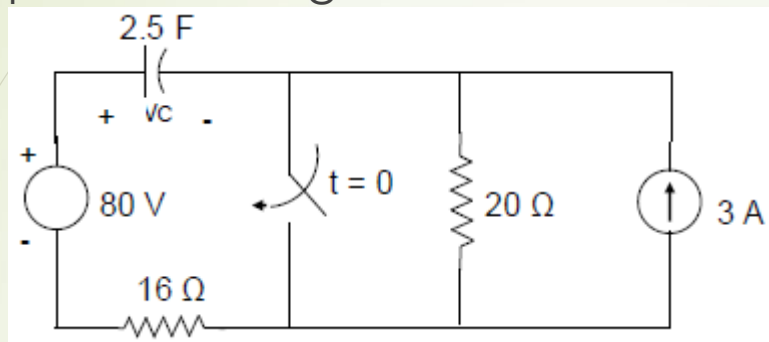
$$i_C(t) = (E/R) e^{-t/RC} = (100/5000) e^{-t/0.1} = 0.02 e^{-10t} \text{ A}$$

$$\text{Voltage across the resistor is } V_R(t) = R i(t) = 5000 \times 0.02 e^{-10t} = 100 e^{-10t} \text{ A}$$



**P.4.17****14**

Consider the circuit shown. The switch was in open position for a long time. It is operated as shown. Compute and plot the capacitor voltage for  $t > 0$ . Also find the time at which the capacitor voltage is 50 V.



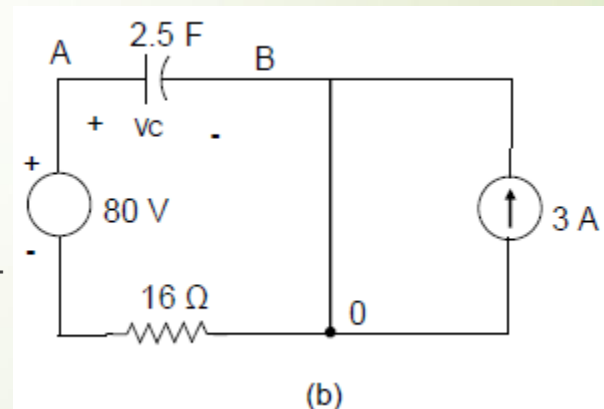
**Solution** Circuit at time  $t = 0^-$  is shown in Fig. (a).

Capacitor acts as open circuit.  $I_{16\ \Omega} = 0$ .

Voltage  $V_A = 80\text{ V}$  and voltage  $V_B = 60\text{ V}$

$$\text{Thus } V_C(0) = 20\text{ V}$$

With the switch is in closed position, the circuit will be as shown in Fig. (b).



With the steady state reached, Capacitor acts as open circuit.

$I_{16\ \Omega} = 0$ . Voltage  $V_A = 80\text{ V}$  and voltage  $V_B = 0\text{ V}$ .

$$\text{Thus } V_C(\infty) = 80\text{ V}$$

15

$$RC = 16 \times 2.5 = 40 \text{ s}$$

$$\text{Using } V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)]e^{-(1/CR)t}$$

$$\begin{aligned} V_C(t) &= 80 + [20 - 80] e^{-(1/40)t} \\ &= 80 - 60 e^{-0.025t} \text{ V} \end{aligned}$$

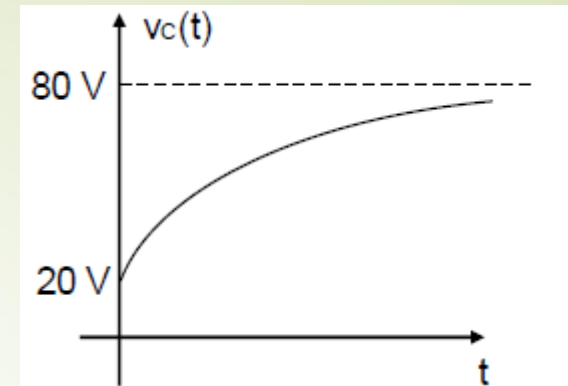
Plot of the capacitor voltage is shown.

Let  $t_1$  be the time at which the capacitor voltage = 50 V.

$$\text{Then } 80 - 60 e^{-0.025t_1} = 50$$

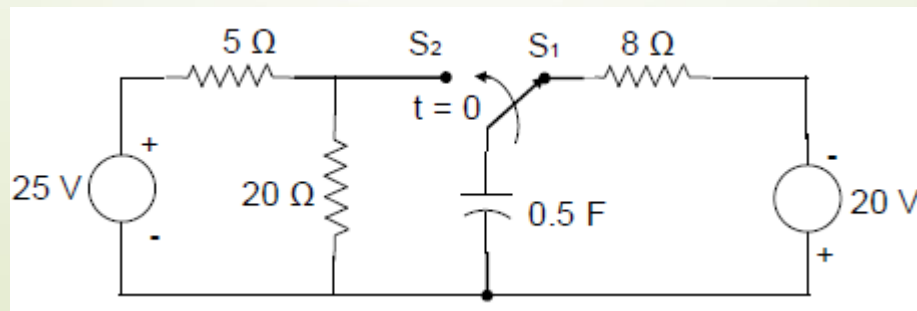
$$\text{i.e. } 60e^{-0.025t_1} = 30 \text{ or } e^{-0.025t_1} = 0.5$$

$$\text{Thus } t_1 = 27.728 \text{ s}$$



P.4.18

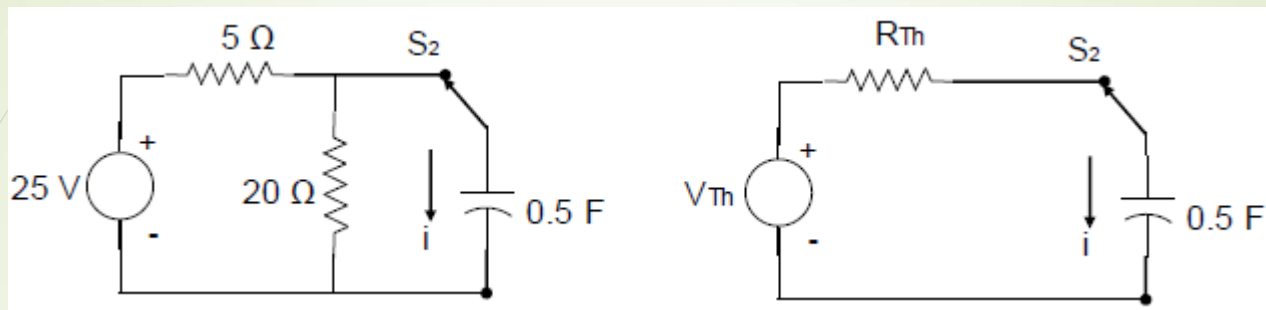
Consider the circuit shown below. The switch was in position S1 for a long time. It is operated as shown. Compute and plot the capacitor voltage for  $t > 0$ . Also find the time at which the capacitor voltage becomes zero.



## Solution

16

Voltage  $V_C(0) = -20$  V Circuit for time  $t > 0$  and its Thevenin's equivalent are shown below.



$$V_{Th} = \frac{20}{20 + 5} \times 25 = 20 \text{ V} \quad \text{So } V_C(\infty) = 20 \text{ V}$$

$$R_{Th} = 5 \parallel 20 = 4 \Omega;$$

$$RC = 4 \times 0.5 = 2 \text{ s}$$

$$\text{Using } V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)]e^{-(1/RC)t}$$

$$V_C(t) = 20 + [-20 - 20] e^{-(1/2)t}$$

$$= 20 - 40e^{-0.5t} \text{ V}$$

$$i_C(t) = (E/R) e^{-t/RC} = (40/4)e^{-0.5t} = 10 e^{-0.5t} \text{ A}$$

Let  $t_1$  be the time at which the capacitor voltage reaches zero value.

Then

$$20 - 40e^{-0.5t_1} = 0; \quad \text{i.e. } e^{-0.5t_1} = 0.5$$

$$\text{Or } -0.5 t_1 = -0.6931; \text{ Thus } t_1 = 1.3863 \text{ s}$$

Capacitor voltage reaches zero value at time  $t_1 = 1.3863 \text{ s}$

| RC Circuit                                                                                                 | RL Circuit                                                                                                |
|------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| $\tau = RC ; \alpha = 1 / RC$                                                                              | $\tau = L / R ; \alpha = R / L$                                                                           |
| Switching at $t = 0 ; V_C(0^+) = V_C(0^-)$                                                                 | Switching at $t = 0 ; i_L(0^+) = i_L(0^-)$                                                                |
| With DC, at SS capacitor acts as open circuit                                                              | With DC, at SS inductor acts as short circuit                                                             |
| $V_C(0) \neq 0; V_C(\infty) = 0$ ; Then<br>$V_C(t) = V_C(0) e^{-t/\tau}$                                   | $i_L(0) \neq 0; i_L(\infty) = 0$ ; Then<br>$i_L(t) = i_L(0) e^{-t/\tau}$                                  |
| $V_C(0) = 0; V_C(\infty) \neq 0$ ; Then<br>$V_C(t) = V_C(\infty) (1 - e^{-t/\tau})$                        | $i_L(0) = 0; i_L(\infty) \neq 0$ ; Then<br>$i_L(t) = i_L(\infty) (1 - e^{-t/\tau})$                       |
| $V_C(0) \neq 0; V_C(\infty) \neq 0$ ; Then<br>$V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)] e^{-(1/RC)t}$ | $i_L(0) \neq 0; i_L(\infty) \neq 0$ ; Then<br>$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)] e^{-(R/L)t}$ |
| $i_C(t) = C \frac{dV(t)}{dt}$                                                                              | $V_L(t) = L \frac{di(t)}{dt}$                                                                             |