Final Assessment Test – November 2022

VIT Vellore Institute of Technology

Course: **BMAT201L - Complex Variables and Linear Algebra**

Class NBR(s): **0287/ 0293 / 0303 / 0305 / 0307 / 0309 / 0311 / 0313 / 0317 / 0321 / 0323 / 0333 / 0335 / 0341 / 0344 / 0348 / 0351 / 0353 / 0355 / 0357 / 0358 / 0362 / 2216 / 2224 / 2270 / 2271 / 2299**

Slot: **C2+TC2+TCC2**

Time: **Three Hours**

Max. Marks: **100**

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Find if $\varphi = (x - y)(x^2 + 4xy + y^2)$ can represent the equipotential for an electric field. Find the corresponding complex potential $f(z) = \varphi + i\psi$ and also find ψ.

2. Determine the regular function $f(z) = P + iQ$, given that
$$P - Q = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}} \text{ and } f\left(\frac{\pi}{2}\right) = 0.$$

3. Show that the transformation $w = \frac{1}{z}$ maps the circle $|z - 3| = 5$ onto the circle $\left|w + \frac{3}{16}\right| = \frac{5}{16}$.

4. Find the bilinear transformation that maps the points $z_1 = 0, z_2 = 1$ and $z_3 = \infty$ into the points $w_1 = i$, $w_2 = 1$, and $w_3 = -i$ and also find its invariant points.

5. Find the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the region
(i) $|z + 1| < 1$,
(ii) $1 < |z + 1| < 2$,
(iii) $|z + 1| > 2$.

6. Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta$, by contour integration.

7. (a) Determine whether (1,1,1,1), (1,2,3,2), (2,5,6,4), (2,6,8,5) form a basis of R^4. If not, find the dimension of the subspace they span.
(b) Find a homogenous system of equation whose solution set W is spanned by (1 , -2, 0, 3, -1) , (2, -3, 2, 5, -3) and (1 , -2, 1, 2, -2).

8. Let $G : R^3 \to R^3$ be the linear mapping defined by
$$G(x;\ y;\ z) = (x + 2y - z;\ y + z;\ x + y - 2z).$$
Find a basis and the dimension of (i) the image of G, (ii) the kernel of G.

9. The vectors $u_1 = (1, 2, 0)$, $u_2 = (1, 3, 2)$, $u_3 = (0, 1, 3)$ form a basis S of R^3. Find
(i) The change-of-basis matrix P from the usual basis $E = \{e_1, e_2, e_3\}$ to S.
(ii)The change-of-basis matrix Q from S back to E.

10. Apply the Gram–Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace V of R^4 spanned by
$v_1 = (1, 1, 1, 1)$, $v_2 = (1, 2, 4, 5)$, $v_3 = (1, -3, -4, -2)$.

11. Find the eigen values and eigen vectors of the matrix

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix},$$ and hence state eigen values of A^{-1} and A^3.

12. Using Gauss-Jordan method, solve the system of equations

$x + 2y + 3z + 4w = 20,\ 3x - 2y + 8z + 4w = 26,$

$2x + y - 4z + 7w = 10,\quad 4x + 2y - 8z - 4w = 2.$

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