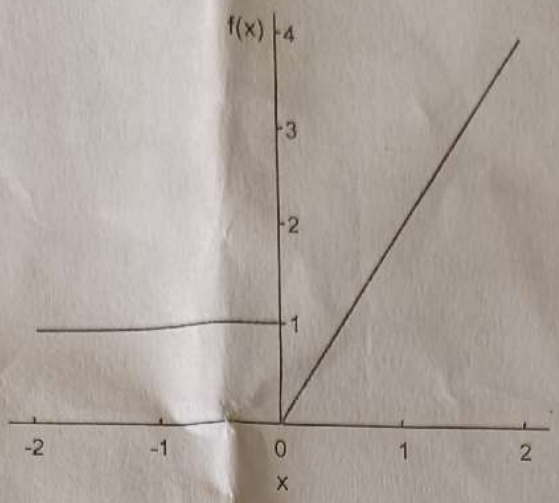
5. Find the Fourier series of the piecewise function $f(x)$ in the interval $(-2,2)$, for $f(x)$ as sketched in the below figure.



VIT
Vellore Institute of Technology

School of Advanced Sciences
Department of Mathematics
Winter Semester 2022-23
Continuous Assessment Test - II (May 2023)

Programme Name and Branch: B.Tech **Slot:** B2+TB2+TBB2
Course Code: BMAT102L **Course Title:** Differential Equations & Transforms
Class No(s): VL202223050 0399/1054/1067/4202/4233/4893/4895/4897/5013
Max. Time: 90 minutes **Max. Marks: 50**

The student is allowed to carry either one handwritten notebook and/or one textbook.

Answer all the questions. Each question carries 10 marks.

1. Find the Laplace transform of $\frac{\sinh(t)}{t}$ and hence show that $\int_{t=0}^{\infty}\int_{u=0}^{t} e^{-2t}\frac{\sinh(u)}{u}\,du\,dt = \frac{1}{4}\log 3$.

2. Find the inverse Laplace transform of $\frac{e^{-10s}s(s+3)}{(s^2+4)(s^2+6s+10)}$ by using the partial fractions.

3. The charge that is stored at any time t (in seconds) on the capacitor in an LC-series circuit, driven by an external emf $E(t)$ volts, is governed by the differential equation:

$$L\frac{d^2q}{dt^2}+\frac{1}{C}q = E(t),$$

where L and C are the inductance (in henrys) and the capacitance (in farads) respectively. If $L = 2$, $C = 0.02$, the initial charge and the initial current are zero, find the charge at any time in the circuit, when a voltage of $E(t) = \sin 5t$ is applied to the circuit and turned off abruptly at $t = 2\pi$.

4. Using the Laplace transform, solve

$$u_x + u_t + u = 0,$$

with

$$u(x,0) = x \text{ and } u(0,t) = 0, x > 0, y > 0.$$