Linear Transformation

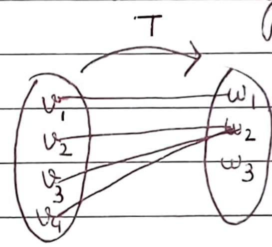
$f: X \to Y$ reffered as transformation or mapping.

Here instead of sets, we have vector spaces

$$T : U \longrightarrow W \quad \text{or} \quad T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

Linear transformation is a function that associates each element of $\mathbb{R}^n$ with exactly one element of $\mathbb{R}^m$.
represented by.

$T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ we say that T maps $\mathbb{R}^n$ into $\mathbb{R}^m$.



Linear transformation:-

Let V and W be two vector spaces. A linear transformation $T : V \longrightarrow W$ is a function from V to W s.t

(a) $T(v_1 + v_2) = T(v_1) + T(v_2)$

(b) $T(\alpha v) = \alpha T(v)$

for all vectors $v_i \in V$, for all scalars k.

The above two properties can be combined to single

$$T(\alpha v_1 + \beta v_2) = \alpha T(v_1) + \beta T(v_2)$$

Q-1 Show that the following transformation are Linear transformations.

(i) $T: \mathbb{R}^2 \to \mathbb{R}^2$.

$T(x, y) = (x + 2y, 3x - y)$

Let $u = (x_1, y_1)$, $v = (x_2, y_2)$ be two vectors in $\mathbb{R}^2$, & α be any scalar

$T(u) = (x_1 + 2y_1, 3x_1 - y_1)$
$T(v) = (x_2 + 2y_2, 3x_2 - y_2)$

$u + v = (x_1 + x_2, y_1 + y_2)$

$T(u + v) = T(x_1 + x_2, y_1 + y_2)$

$= (x_1 + x_2 + 2(y_1 + y_2), 3(x_1 + x_2) - (y_1 + y_2))$

$= (x_1 + 2y_1 + x_2 + 2y_2, 3x_1 - y_1 + 3x_2 - y_2)$

$= (x_1 + 2y_1, 3x_1 - y_1) + (x_2 + 2y_2, 3x_2 - y_2)$

$= T(u) + T(v)$

$\boxed{T(u + v) = T(u) + T(v)}$

$T(\alpha u) = T(\alpha(x_1, y_1))$

$= T(\alpha x_1, \alpha y_1)$

$= (\alpha x_1 + 2\alpha y_1, 3\alpha x_1 - \alpha y_1)$

$= \alpha(x_1 + 2y_1, 3x_1 - y_1)$

$= \alpha \, T(u)$

$\boxed{T(\alpha u) = \alpha \, T(u)}$

(2) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$T(x, y, z) = (x+1, y, z)$, check whether T is linear transformation or not.

Sol^n: Let $\left.\begin{array}{l} u = (x_1, y_1, z_1) \\ v = (x_2, y_2, z_2) \end{array}\right\} \in \mathbb{R}^3$

$T(u) = T(x_1, y_1, z_1) = (x_1+1, y_1, z_1)$

$T(v) = T(x_2, y_2, z_2) = (x_2+1, y_2, z_2)$

$u + v = (x_1, y_1, z_1) + (x_2, y_2, z_2)$

$= (x_1 + x_2, y_1 + y_2, z_1 + z_2)$

To show

(I) $T(u+v) = T(u) + T(v)$

(II) $T(\alpha u) = \alpha T(u)$

$T(u+v) = T((x_1, y, z_1) + (x_2, y, z_2))$

$= T(x_1 + x_2, y_1 + y_2, z_1 + z_2)$

$T(u+v) = (x_1 + x_2 + 1, y_1 + y_2, z_1 + z_2)$ —— (1)

$T(u) + T(v) = (x_1+1, y_1, z_1) + (x_2+1, y_2, z_2)$ (2)

$= (x_1 + x_2 + 2, y_1 + y_2, z_1 + z_2)$

from (1) & (2)

$T(u+v) \neq T(u) + T(v)$

ie given transformation is not linear transformation.

(3) $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

given by

$$T(x, y, z) = (2x - y + z,\ y - 4z)$$

sol^n Let $u = (x_1, y_1, z_1)$, $v = (x_2, y_2, z_2)$

$$u + v = (x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2)$$

$$T(u) = T(x_1, y_1, z_1) = (2x_1 - y_1 + z_1,\ y_1 - 4z_1)$$
$$T(v) = T(x_2, y_2, z_2) = (2x_2 - y_2 + z_2,\ y_2 - 4z_2)$$

To show $\boxed{T(u+v) = T(u) + T(v)}$

$$T(u+v) = T(x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2)$$
$$= \left(2(x_1 + x_2) - (y_1 + y_2) + (z_1 + z_2),\ (y_1 + y_2) - 4(z_1 + z_2)\right)$$
$$= \left(2x_1 + 2x_2 - y_1 - y_2 + z_1 + z_2,\ y_1 - 4z_1 + y_2 - 4z_2\right)$$
$$= \left(2x_1 - y_1 + z_1 + 2x_2 - y_2 + z_2,\ y_1 - 4z_1 + y_2 - 4z_2\right)$$
$$= (2x_1 - y_1 + z_1,\ y_1 - 4z_1) + (2x_2 - y_2 + z_2,\ y_2 - 4z_2)$$
$$= T(u) + T(v)$$

ie $\boxed{T(u+v) = T(u) + T(v)}$ —— (1)

$$T(\alpha u) = T(\alpha(x_1, y_1, z_1))$$
$$= T(\alpha x_1, \alpha y_1, \alpha z_1)$$
$$= (2\alpha x_1 - \alpha y_1 + \alpha z_1,\ \alpha y_1 - 4\alpha z_1)$$

$= \alpha\,(2x_1 - y_1 + z_1,\ \ y_1 - 4z_1)$

$= \alpha\, T(u)$

ie $\boxed{T(\alpha u) = \alpha\, T(u)}$ — (2)

from (1) & (2) it's clear that given transformation is linear.

check whether the following transformations are linear or not

4. $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$T(x, y) = (xy,\ x^2+1)$ (NO)

5. $T : \mathbb{R}^2 \longrightarrow \mathbb{R}$

$T(x) = x^2 - x$ (NO)

6) $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$T(x, y) = (0, -x)$ yes

7. $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

$T(x, y, z) = (|x|, 0)$ NO

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Q:- Show that the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (\sin x, y)$ is not linear.

Sol^n. Let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$$u + v = (x_1 + x_2, y_1 + y_2)$$

$$T(u + v) = T(x_1 + x_2, y_1 + y_2)$$

$$= (\sin(x_1 + x_2), y)$$
$$= (\sin x_1 \cos x_2 + \cos x_1 \sin x_2, y)$$

$$T(u) + T(v) = T(x_1, y_1) + T(x_2, y_2)$$

$$= (\sin x_1, y_1) + (\sin x_2, y_2)$$

$$= (\sin x_1 + \sin x_2, y_1 + y_2)$$

it is clear that

$$T(u + v) \neq T(u) + T(v)$$

is given transformation is not linear.

Q:- Check whether the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x + 1, 2y, x + y)$ is linear or not.

Ans (not linear)