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DEPARTMENT OF MATHEMATICS

SCHOOL OF ADVANCED SCIENCES

Fall Semester 2022-23

Continuous Assessment Test –II (October-2022)

Slot: A2+TA2+TAA2

Course Code: BMAT201L **Course Title:** Complex Variables and Linear Algebra

Max. Time: 90 minutes **Max. Marks:** 50

Answer *all* the Questions

1. Find the Laurent's series expansion of $f(z) = \frac{z}{(z^2+1)(z^2+4)}$ in the region

   $(a)\ 1 < |z| < 2$ $(b)\ |z| > 2$ **(10 M)**

2. (a) For the function $f(z) = \frac{1-e^{2z}}{z^4}$, find the poles and residues at each of the poles.

   (b) Using Cauchy's integral formula,

   evaluate $\int_C \frac{z \sec z}{1-z^2}\, dz$ where C is the ellipse $4x^2 + 9y^2 = 9$. **(5+5 M)**

3. Using contour integration evaluate $\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta$. **(10 M)**

4. For a matrix $A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 3 \end{pmatrix}$. Find the eigenvalues of A and hence verify

   eigenvalues of A^{-1}. **(10 M)**

5. Find the values of a and b for which the following system of equations has (i) no solution and (ii) unique solution:

   $x + 2y + 2z = 10;$

   $2x - 2y + 3z = 1;$

   $4x - 3y + az = b.$

   Solve the system by Gauss-Elimination method for $a = 5$ and $b = 4$. **(10 M)**