Department of Mathematics
School of Advanced Sciences
BMAT 101P – Calculus (MATLAB)
Experiment 5–B
Verification of Green's theorem

Aim:

To write MATLAB code to demonstrate Green's theorem.

Green's theorem

Let C be a piecewise smooth simple closed curve bounding a region R. If $M, N, \frac{\partial M}{\partial y}, \frac{\partial N}{\partial x}$ are continuous on R, then

$$\oint_C M(x,y)dx + N(x,y)dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy \,, \quad (1)$$

the integration being carried in the positive direction (counter clockwise) of C.

Verification Procedure:

1. Represent the parametric form $[x(t), y(t)]$ of the curve C and obtain the limits for t.
2. Now evaluate the LHS of (1), substituting the parametric curves.
3. Now, find the expressions $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$ and evaluate the double integral of RHS of (1).
4. Show that LHS=RHS.

MATLAB CODE:

```
clear
clc
syms x y t
F=input('Enter the vector function M(x,y)i+N(x,y)j in the form [M N]: ');
M(x,y)=F(1); N(x,y)=F(2);
r=input('Enter the parametric form of the curve C as [r1(t) r2(t)]: ');
r1=r(1);r2=r(2);
P=M(r1,r2);Q=N(r1,r2);
dr=diff(r,t);
F1=sum([P,Q].*dr);
T=input('Enter the limits of integration for t [t1,t2]: ');
t1=T(1);t2=T(2);
LHS=int(F1,t,t1,t2);
yL=input('Enter limits for y in terms of x: [y1,y2]: ');
xL=input('Enter limits for x as constants: [x1,x2]: ');
y1=yL(1);y2=yL(2);x1=xL(1);x2=xL(2);
F2=diff(N,x)-diff(M,y);
RHS=int(int(F2,y,y1,y2),x,x1,x2);
if(LHS==RHS)
    disp('LHS of Greens theorem=')
    disp(LHS)
    disp('RHS of Greens theorem=')
    disp(RHS)
    disp('Hence Greens theorem is verified.');
end
```

Example 1. Verify Green's theorem for the vector field $F = -yi + xj$ over a unit circle C.

Input/Output:

```
Enter the vector function M(x,y)i+N(x,y)j in the form [M N]: [-y,x]
Enter the parametric form of the curve C as [r1(t) r2(t)]:[cos(t) sin(t)]
Enter the limits of integration for t [t1,t2]: [0,2*pi]
Enter limits for y in terms of x: [y1,y2]: [-sqrt(1-x^2) sqrt(1-x^2)]
Enter limits for x as constants: [x1,x2]: [-1,1]
LHS of Greens theorem=
2*pi

RHS of Greens theorem=
2*pi

Hence Greens theorem is verified.
```

Exercise.

1. Verify Green's theorem for the vector field $F = (x^2 - y^3)i + (x^3 + y^2)j$, over the ellipse $C: x^2 + 4y^2 = 64$.
2. Verify Green's theorem for the vector field $F = (x + y)i + (y - x)j$, over a unit circle $C: x^2 + y^2 = 1$.
3. Evaluate the line integral $\oint_C e^x(siny\ dx + cosy\ dy)$, where C is the ellipse $4(x + 1)^2 + 9(y - 3)^2 = 36$.