VIT
Vellore Institute of Technology

DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
FALL SEMESTER 2022-2023

CONTINUOUS ASSESSMENT TEST – II

Programme Name & Branch	: B.Tech
Course Code	: BMAT101L
Course Name	: Calculus
Slot	: A1+TA1
Date of the Examination	: 11.12.2022
Duration	: 90 minutes

Max. Marks : 50

General instruction(s): Students are permitted to bring one text book and one hand written note book

Q. No	Question	Marks
1.	Find the second order Taylor expansion of $f(x,y) = \sqrt{1+4x^2+y^2}$ about $(x_0, y_0) = (1,2)$ and use it to compute approximately $f(1.1, 2.05)$.	10
2.	A toy manufacturer estimates a production function to be $f(x,y) = 100x^{3/4}y^{1/4}$, where x represents the units of labour (at Rs. 150 per unit) and y represents the units of capital (at Rs. 250 per unit). The total cost of labour and capital is equal to Rs. 50,000. Find the maximum production level for this manufacturer.	10
3.	Evaluate $\int_0^1 \int_{-\sqrt{1-x^2}}^{0} 2x \cos\left(y - \frac{y^3}{3}\right) dy\, dx$ by reversing the order of integration.	10
4.	Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2 = z$.	10
5.	Evaluate the following using either Beta or Gamma functions.	
	a) $\int_0^\infty \sqrt[4]{x} e^{-\sqrt{x}} dx$	5
	b) $\int_0^2 \frac{x^2}{\sqrt{2-x}} dx$	5