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Basis

The set of vectors $S = \{v_1, v_2, \cdots v_n\}$ in a vector space V is called a basis for V if

(i) S is linearly independent.
(ii) S spans V.

The set S spans (generate) the whole space V. It means that every vector in V can be written as a linear combination of vectors in S and this combinate is unique.

* In the basis set, their is never a null vector because null vector is always linearly dependent, while basis set is always linearly independent.

standard basis for $\mathbb{R}^n$

vectors $e_1 = (1, 0, \cdots 0)$, $e_2 = (0, 1, 0 \cdots 0)$, $e_3 = (0, 0, 1, \cdots 0)$ -- $e_n = (0, 0, \cdots, 1)$ in $\mathbb{R}^n$ are linearly independent and they span $\mathbb{R}^n$.

i.e for $x \in \mathbb{R}^n$ $\quad x = (x_1, x_2, \cdots x_n)$

$$x = x_1 e_1 + x_2 e_2 + \cdots + x_n e_n$$

set $S = \{e_1, e_2, \cdots e_n\}$ is basis for $\mathbb{R}^n$ and also called standard basis for $\mathbb{R}^n$.

<u>Note</u>: Basis set for a vector space is not unique.

<u>for Ex</u>:- The set of vectors $v_1 = (1,1,0)$, $v_2 = (0,1,1)$, $v_3 = (0,0,1)$ is linearly independent set and also spans $\mathbb{R}^3$.

$\Rightarrow$ $\{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$. Also set $\{e_1, e_2, e_3\}$ is a basis for $\mathbb{R}^3$. This shows that there may exist many number of basis for a vector space.

Set $\{v_1, v_2, v_3\}$, $v_1 = (1,1,0)$, $v_2 = (0,1,0)$, $v_3 = (1,0,$ can't span $\mathbb{R}^3$ as they are not L.I.

$(1,0,0)$, $(0,1,1)$, $(1,0,1)$, $(0,1,0)$ are not L.I but spans $\mathbb{R}^3$.

$(1,0,0)$, $(0,1,0)$ are L.I but cannot spans $\mathbb{R}^3$.

<u>Zero space</u>: Zero space contains only zero vector. It has zero basis i.e. no basis because it is always L.D.

Uniqueness

Theorem :- If $S = \{v_1, v_2, \dots v_n\}$ is a basis for vector space V, then every vector v in V can be uniquely expressed as a linear combinations of $v_1, \dots v_n$ i.e there are unique scalar's c_i's $i = 1, 2, \dots n$

s.t $v = c_1 v_1 + c_2 v_2 + \dots + c_n v_n$

Pf :- S is a basis for V

ie (i) S is L.I

(ii) S spans V

since S span V

Let $v = c_1 v_1 + c_2 v_2 + \dots + c_n v_n$ —— (1)

if possible, let

$$v = d_1 v_1 + d_2 v_2 + \dots + d_n v_n \quad \text{—— (2)}$$

(1) − (2)

$$0 = (c_1 - d_1) v_1 + (c_2 - d_2) v_2 + \dots + (c_n - d_n) v_n$$

Since S is L.I

ie $c_1 - d_1 = 0$ ie $c_1 = d_1$

$c_2 - d_2 = 0$ $\quad$ $c_2 = d_2$

$\vdots$ $\quad$ $\vdots$

$c_n - d_n = 0$ $\quad$ $c_n = d_n$

This shows that if S is a basis for vector space V, then ∃ a unique linear combination for each vector v.

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Q: Determine whether the given vectors
$v_1 = (1,-1,1)$ $v_2 = (0,1,2)$, $v_3 = (3,0,-1)$
forms a basis for $\mathbb{R}^3$.

Ans: to check $S = \{v_1, v_2, v_3\}$
↓
(i) S is L.I
(ii) S spans V.

consider $c_1 v_1 + c_2 v_2 + c_3 v_3 = (0,0,0)$

$c_1(1,-1,1) + c_2(0,1,2) + c_3(3,0,-1) = (0,0,0)$
$(c_1 + 3c_3, -c_1 + c_2, c_1 + 2c_2 - c_3) = (0,0,0)$

$c_1 + 3c_3 = 0$
$-c_1 + c_2 = 0$
$c_1 + 2c_2 - c_3 = 0$

homo. sys. $AX = 0$ — (1) where $X = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$ $n=3$

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

$R_2 \to R_2 + R_1$, $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 2 & -4 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & -10 \end{bmatrix}$$

as Rank(A) = 3 = n (no. of unknowns)

ie system (1) has trivial solⁿ
ie $c_1 = 0, c_2 = 0, c_3 = 0$
ie S is L.I

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Now
to check span(S) = V

say $(x, y, z) \in V$
consider
$(x, y, z) = c_1(1,-1,1) + c_2(0,1,2) + c_3(3,0,-1)$

$c_1 + 3c_3 = x$
$-c_1 + c_2 = y$ $\rightarrow$ $AX = B$
$c_1 + 2c_2 - c_3 = z$

$[A|B] = \left[\begin{array}{ccc|c} 1 & 0 & 3 & x \\ -1 & 1 & 0 & y \\ 1 & 2 & -1 & z \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & x \\ 0 & 1 & 3 & y+x \\ 0 & 2 & -4 & z-x \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & x \\ 0 & 1 & 3 & y+x \\ 0 & 0 & -10 & -3x-2y+z \end{array}\right]$

Rank(A) = 3
Rank(A/B) = 3
= no. of unknown

ie sys. $AX = B$ is consistent. ie $\boxed{\text{Span}(S) = V}$

$c_3 = \dfrac{-3x-2y+z}{-10}$, $\quad c_2 + 3c_3 = y + x$

$c_2 = (y+x) - 3\left(\dfrac{3x+2y-z}{10}\right) = \dfrac{x+4y+3z}{10}.$

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$c_1 + 3c_3 = x$

$c_1 = x - 3c_3$

$c_1 = x - 3\left(\frac{-3x-2y+z}{-10}\right)$

$= \frac{10x - 9x - 6y + 3z}{10}$

$c_1 = \frac{x - 6y + 3z}{10}$

ie S is L.I and span(S) = V

ie S is a basis for $\mathbb{R}^3$.

Q:- which of the following sets of vectors are basis for P_2.

(a) $S_1 = \{1 + 2x + x^2, 2 + 9x, 3 + 3x + 4x^2\}$

(Ans. yes)

(b) $S_2 = \{2 - 3x + x^2, 4 + x + x^2, 7x + x^2\}$

(NO)

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Q:- Determine whether (1, 1, 1, 1), (1, 2, 3, 2), (2, 5, 6, 4), (2, 6, 8, 5) form a basis for $\mathbb{R}^4$.

Soln:- $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = 0$

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 5 & 6 \\ 1 & 3 & 6 & 8 \\ 1 & 2 & 4 & 5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 3 & 4 \\ 0 & 2 & 4 & 6 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ i.e echelon form has zero row.

~~Ra~~ $\rho(A) = 3$ = no. of linearly Inde. ~~vector~~ row are 3.

Hence ~~ve~~ given four vectors are L.D and do not ~~f~~ form basis of $\mathbb{R}^4$.

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~~Theorem on Bases~~

~~Ca~~ ·

Dimension of vector space :-

The no. of vectors in a basis of a vector space V is called the dimension of V and denoted by dim(V).

i.e.

* A non zero vector space V is called finite dimensional if ~~its~~ no. of element in basis set are finite.
  say if $S = \{v_1, v_2, \ldots v_n\}$ is a basis for Vector space V then
  $\boxed{\dim(V) = n}$ and V is finite dimensional

* If a vector space V, does not have finite no. of basis. Then V is said to be of infinite dimensional.

* dimension of zero space $\{0\}$ is 0 as it has no basis.

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Standard basis :-

* Standard Basis for $\mathbb{R}^2 = \{(1,0),(0,1)\}$

" " " $\mathbb{R}^3 = \{(1,0,0),(0,1,0),(0,0,1)\}$

$\dim(\mathbb{R}^2) = 2 \quad \dim(\mathbb{R}^3) = 3$

similarly $\dim(\mathbb{R}^n) = n$

* Standard basis for vector space $M_{m\times n}$:-

Ex $M_{2\times 3}$

$$S = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

$\dim(M_{2\times 3}) = 6$

$\dim(M_{m.n}) = m.n$

* standard basis for $P_n(t)$ of all polynomial of degree $\leq n$:-

$S = \{1, t, t^2, -- t^n\}$

$\boxed{\dim(P_n(t)) = n+1}$

Note:- vector space $P(t)$ of all polynomials :-

$\dim(P(t))$ is infinite as any set

$S = \{1, t, t^2, ------\}$ consisting of

all powers of t, spans $P(t)$ and L.I also.

ie $P(t)$ is infinite dimensional vectorspace

Theorem on Bases:-

Theo:- Let V be a vector space of dimension n. then

(1) Any $(n+1)$ or more vector are L.D.

(2) Any L.I ~~set~~ set $S = \{v_1, ---, v_n\}$ with n element is basis of V.

(3) Any spanning set $S = \{v_1 --- v_n\}$ of V with n element is a basis of V.

Theo:- Let V be a finite-dimensional vector space, let $S = \{v_1, -- v_n\}$ be any basis

(a) If S has more than n vectors, then it is linearly dependent.

(b) If a set S has fewer than n vectors, then it does not span V.

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Dimension and subspace:-

Let W be a subspace of n-dimensional vector space V. Then $\dim(W) \le n$
in particular if $\dim(W) = n$
then $\boxed{W = V}$

Ex:- Let W be a subspace of $V = \mathbb{R}^3$.
Note that $\dim \mathbb{R}^3 = 3$
ie $\dim W$ can only be 0, 1, 2, 3.

(a) $\dim(W) = 0$ then $W = \{0\}$ a point.

(b) $\dim(W) = 1$, then W is a line through origin.

(c) $\dim(W) = 2$, then W is a plane through origin.

(d) $\dim(W) = 3$, then W is entire space $\mathbb{R}^3$.

i.e (a) $W = \{(0,0,0)\}$, zero space
$\dim W = 0$

(b) $W = \{(x,0,0) \mid x \in \mathbb{R}\}$
$\dim(W) = 1$.

(c) $W = \{(x,y,0) \mid x,y \in \mathbb{R}\}$, $\dim(W) = 2$

(d) $W = \{(x,y,z) \mid x,y,z \in \mathbb{R}\}$ $\dim(W) = 3$.

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Ex:- dimension of vector space of all diagonal $n \times n$ matrices is $\underline{n}$.

Soln: say
we have
$$W = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

W is a subspace of $M_{2\times2}$.

Basis for $W = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$$\boxed{\dim W = 2}$$

* Dimension of vector space of all symmetric matrices of order $n \times n$:- $\frac{n(n+1)}{2}$

eg:- 2×2 $W = \left\{ \begin{bmatrix} a & b \\ b & c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$

standard basis for $W = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$$\boxed{\dim W = 3}$$

* $W = \left\{ \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, a, b, c, d, e, f \in \mathbb{R} \right\}$

Basis set $= \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$

$\underline{\dim(W) = 6}$

(c) The vector space of all upper triangular matrices of order $n \times n$ has dim $\frac{n(n+1)}{2}$.

Ex:-

$$W = \left\{ \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}, \ a, b, c, d, e, f \in \mathbb{R} \right\}$$

$\boxed{\dim W = 6}$

Q:- Find a basis and dim. of subspace

(a) $W = \{(a, b, c) : a+b+c=0\}$

~~dim W ≠ IR~~

Note $W \neq \mathbb{R}^3$
as $(1, 2, 3) \notin W$
thus $\dim(W) < 3$

$(a, b, -a-b) \in W$

Note that
$v_1 = (1, 0, -1)$, $v_2 = (0, 1, -1)$ are two L.I ~~set~~ vectors in W. Thus $\dim W = 2$.
ie $S = \{v_1, v_2\}$ is a basis for W.

(b) $W = \{(a, b, c) : a=b=c\}$
$W \neq V$ ie $\dim(W) < 3$

~~dim~~

say $(a, a, a) \in W$, be any vector in W.
ie only 1 vector $(1, 1, 1)$ spans W and also L.I.
ie $\boxed{\dim(W) = 1}$.

Q:- Let V be vector space of 2x2 matrices over field $\mathbb{R}$. Let W be subspace of symmetric matrices. show that $\dim(W)=3$ and find basis for W.

Soln

$$W = \left\{ \begin{bmatrix} a & b \\ b & c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$$

Basis can be found as

$$e_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Say $S = \{e_1, e_2, e_3\}$ is basis for W.

check :-

(1) L.I

(2) $\text{span}(S) = W$

L.I

$$c_1 e_1 + c_2 e_2 + c_3 e_3 = 0 \quad \text{——— (1)}$$

$$c_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\cancel{c_1 = 0}$, $\begin{bmatrix} c_1 & c_2 \\ c_2 & c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow c_1 = 0, \; c_2 = 0, \; c_3 = 0$

ie (1) has only trivial solⁿ

ie S is L.I.

check for span :

say $\begin{bmatrix} a & b \\ b & c \end{bmatrix} = c_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ ——— (2)

$$\Rightarrow \begin{bmatrix} c_1 & c_2 \\ c_2 & c_3 \end{bmatrix} = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$\Rightarrow c_1 = a, \quad c_2 = b, \quad c_3 = c$

ie (2) is consistent

ie $\operatorname{span}(S) = W$

ie S spans W.

ie S is a basis for W.

Finding a basis for subspace W, spanned by $v_1, v_2, -- v_n$ ie $W = span(v_1, v_2, -- v_n)$

Row space Algo:-

(1) Form the matrix M whose rows are given vectors.

(2) ~~Ro~~ Reduce M to echelon form, using elementary row operations.

(3) output the nonzero rows of echelon form.

sometime we want a basis that only comes from the original given vectors; ~~so~~ that can be done by following algo.

Casting out Algo:-

(1) form the matrix M, whose column are the given vectors

(2) Reduce M to echelon form.

(3) For each column in the echelon form without a pivot delete the vectors v_k from list s of given vectors.

(4) output the remaining vectors in s.

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Q:- Let W be a subspace of $\mathbb{R}^4$ spanned by ~~E~~ vectors

$v_1 = (1,-2,5,3)$, $v_2 = (0,1,1,4)$, $v_3 = (1,0,1,0)$

find the basis for W.

Solⁿ:- Note W can be spanned by three vectors as $\underline{\dim W \le 4}$

consider $A = \begin{bmatrix} 1 & -2 & 5 & 3 \\ 0 & 1 & 1 & 4 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Reduce A to echelon form

$\sim \begin{bmatrix} 1 & -2 & 5 & 3 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 5/6 \end{bmatrix} = U$

three non-zero rows of U are L.I and they span W.

ie $S = \{(1,-2,5,3), (0,1,1,4), (0,0,1,5/6)\}$ is basis for W

or as ~~Ran~~ Rank(A) = 3

ie v_1, v_2, v_3 are L.I

ie v_1, v_2, v_3 forms a basis for W.

$S = \{v_1, v_2, v_3\}$ " " " "

ie $\boxed{\dim(W) = 3}$

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Q:-2 Let W be a subspace of $\mathbb{R}^5$ spanned by $v_1 = (1, 2, 1, 3, 2)$, $v_2 = (1, 3, 3, 5, 3)$,

$v_3 = (3, 8, 7, 13, 8)$, $v_4 = (1, 4, 6, 9, 7)$

$v_5 = (5, 13, 13, 25, 19)$

soln:- using casting out Algo:-

$$M = \begin{bmatrix} 1 & 1 & 3 & 1 & 5 \\ 2 & 3 & 8 & 4 & 13 \\ 1 & 3 & 7 & 6 & 13 \\ 3 & 5 & 13 & 9 & 25 \\ 2 & 3 & 8 & 7 & 19 \end{bmatrix}$$

reduced M to echelon form

$$\sim \begin{bmatrix} 1 & 1 & 3 & 1 & 5 \\ 0 & 1 & 2 & 2 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Pivots appear in column c_1, c_2, c_4. ie we cast out the vectors v_3 and v_5.

ie $S = \{v_1, v_2, v_4\}$ form a basis for W.

$$\boxed{\dim(W) = 3}$$

Imp.

Theo:- Let V be a finite dimensional vector space.

(1) any linearly independent set in V can be extended to a basis by adding more vectors if necessary.

(2) Any set of vectors that span V, can be reduced to basis by discarding some vectors if necessary.

~~Ex:- Q1 we found that~~

Ex:- Let W be a subspace of $\mathbb{R}^4$ spanned by vectors

$v_1 = (1, -2, 5, 3)$, $v_2 = (0, 1, 1, 4)$, $v_3 = (1, 0, 1, 0)$

find the basis for W and extend it to basis for $\mathbb{R}^4$.

Soln: Q1 we noticed (last page)

dim $W = 3$

and non-zero rows of U are basis for W

i.e. $S = \frac{r}{2}$.
$$U = \begin{bmatrix} 1 & -2 & 5 & -3 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 5/6 \end{bmatrix}$$

ie we can add a non zero vector of the form

$v_4 = (0, 0, 0, t)$ to the rows of U, to get basis of $\mathbb{R}^4$.

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Q: W be subspace of $\mathbb{R}^4$ spanned by vectors $u_1 = (1, -2, 5, -3)$, $u_2 = (2, 3, 1, -4)$
$u_3 = (3, 8, -3, -5)$

(a) find basis and dim of W.

(b) extend the basis of W to a basis of $\mathbb{R}^4$.

Sol^n W is spanned by u_1, u_2, u_3

$$\begin{bmatrix} 1 & -2 & 5 & -3 \\ 2 & 3 & 1 & -4 \\ 3 & 8 & -3 & -5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 5 & -3 \\ 0 & 7 & -9 & 2 \\ 0 & 14 & -18 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 5 & -3 \\ 0 & 7 & -9 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U$$

Non zero rows of U are L.I and form a basis for W

Basis for W = $\{(1, -2, 5, -3), (0, 7, -9, 2)\}$

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or

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 8 \\ 5 & 1 & -3 \\ -3 & -4 & -5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 7 & 14 \\ 0 & -9 & -18 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 7 & 14 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Pivot apperas in column 1 and 2.

ie $S = \{(1, -2, 5, -3), (2, 3, 1, -4)\}$ is L.I and form basis of W.

(b) as we Know non-zero rows of U form a basis for W

ie
$$U = \begin{bmatrix} 1 & -2 & 5 & -3 \\ 0 & 7 & -9 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

we can extend a basis for W to basis for $\mathbb{R}^4$ by adding two Vectors $(0, 0, 1, 0)$, $(0, 0, 0, 1)$.

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$S' = \{(1, -2, 5, -3), (0, 7, -9, 2), (0, 0, 1, 0), (0, 0, 0, 1)\}$
is a basis for $\mathbb{R}^4$.