SCHOOL OF ELECTRONICS ENGINEERING
FALL SEMESTER _ 2023-24
CONTINUOUS ASSESSMENT TEST (CAT)-1
BECE202L – SIGNALS AND SYSTEMS

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Programme: B.Tech (ECE) **Class Nbrs:** VL2023240102251, 53, 55, 57, 59, 61
Slot: B1+TB1 **Max Marks: 50** **Duration: 90 Min**
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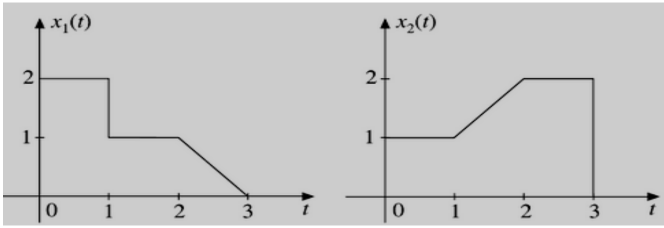
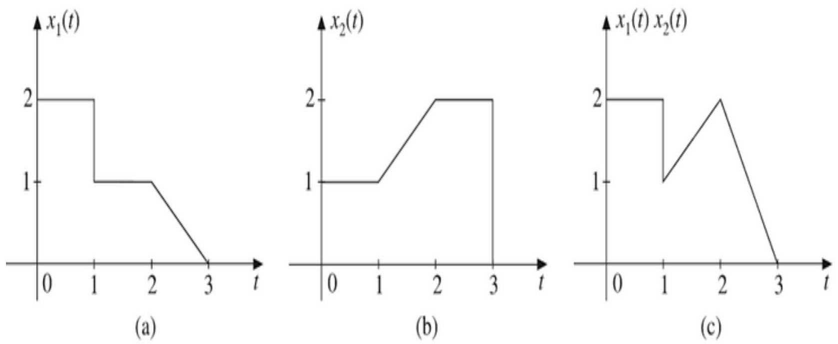
ANSWER ALL QUESTIONS

<table>
<tr><th>Q.No</th><th>Question</th><th>Mark</th></tr>
<tr><td>1.</td><td>

(a). For the signals shown below, find $x_1(t) \times x_2(t)$



ANSWER



(b). Find the ℓ_1 and ℓ_2 norms of the vector $\boldsymbol{a} = [3, -2]^T$.

Question changed by Moderator Dr.Ramachandra Reddy Sir.

Answer not available with course chair

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[5]

[5]

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Classify the following signals:

a. $x_1(t) = \cos t + \sin\sqrt{3}t$ as periodic or non-periodic. If periodic, find the fundamental period.

b. $x_2(t) = Ae^{-\alpha t}\, u(t), \alpha > 0$ as energy or power signals. Also, find the power/energy.

c. $x_3[n] = 2e^{j3n}$ as energy or power signals. Also, find the power/energy.

d. $x_4[n] = \cos\left(\frac{1}{5}\pi n\right) \sin\left(\frac{1}{3}\pi n\right)$ as periodic or non-periodic. If periodic find the fundamental period.

ANSWER

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[10]

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Solution : i) $x(t) = \cos t + \sin \overset{\sqrt{3}}{\cancel{\sqrt{2}}}\, t$

Compare with, $\quad x(t) = \cos 2\pi f_1 t + \sin 2\pi f_2 t$

$\therefore \quad 2\pi f_1 t = t \quad \Rightarrow \quad f_1 = \frac{1}{2\pi} = \frac{1}{T_1}$, Hence $T_1 = 2\pi$

and $\quad 2\pi f_2 t = \overset{\sqrt{3}\,t}{\cancel{\sqrt{2}\,t}} \quad \Rightarrow \quad f_2 = \frac{\sqrt{\cancel{2}\,3}}{2\pi} = \frac{1}{T_2}$, Hence $T_2 = \cancel{\frac{2\pi}{\sqrt{2}} = \sqrt{2}\,\pi} = \frac{2\pi}{\sqrt{3}}$

The ratio of two periods is, $\frac{T_1}{T_2} = \frac{2\pi}{\sqrt{2}\,\pi} = \sqrt{\cancel{2}\,3}$ Since the ratio $\frac{T_1}{T_2}$ is not ratio of two integers (i.e. not rational number), the signal is non-periodic.

(ii).

This signal is non periodic and of infinite duration. It can be energy signal. Therefore calculate its energy directly as per step 3.

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$= \int_{0}^{\infty} [A\, e^{-\alpha t}]^2\, dt \qquad \text{Since } u(t) = 1 \text{ for } 0 \le t \le \infty$$

$$= A^2 \int_0^{\infty} e^{-2\alpha t}\, dt = A^2 \cdot \left[\frac{e^{-2\alpha t}}{-2\alpha}\right]_0^{\infty}$$

$$= \frac{A^2}{2\alpha}$$

The energy is finite and non-zero, hence the given signal is energy signal with $E = \frac{A^2}{2\alpha}$

(iii).

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Here $\quad \left|2e^{j3n}\right| = 2\,|\cos 3n + j \sin 3n| = 2$

$$\therefore \quad P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 2$$

$$= 2 \lim_{N \to \infty} \frac{1}{2N+1} (2N+1)$$

$$= 2$$

Since power is finite and non zero, the signal is power signal.

(iv).

ii) $x(n) = \cos\left(\frac{\pi n}{5}\right)\sin\left(\frac{\pi n}{3}\right)$

$$= \frac{1}{2}\left\{\sin\left(\frac{\pi n}{3}+\frac{\pi n}{5}\right)+\sin\left(\frac{\pi n}{3}-\frac{\pi n}{5}\right)\right\}$$

$$= \frac{1}{2}\left\{\sin\frac{8\pi n}{15}+\sin\frac{2\pi n}{15}\right\} = A\,\{\sin 2\pi f_1 n + \sin 2\pi f_2 n\}$$

Here $f_1 = \frac{4}{15} = \frac{k_1}{N_1} \Rightarrow N_1 = 15$

and $f_2 = \frac{1}{15} = \frac{k_2}{N_2} \Rightarrow N_2 = 15$

Since both frequencies are rational, and $\frac{N_1}{N_2} = 1$, the signal is periodic. The fundamental period is,

$$N = N_1 = N_2 = 15$$

3. **(a).** A system is represented by the following difference equation: **[8+2]**

$y(n) = 3y^2(n-1) - nx(n) + 4x(n-1) - x(n+1), n \geq 0$

Is the system linear, time-invariant, stable and causal?

ANSWER

a. The system is non-linear as the expression contains square terms
b. If the time is varied in the given expression from n to $n-k$
   $y(n-k) = 3y^2(n-k-1) - (n-k)x(n-k) + 4x(n-k-1) - x(n-k+1)$ (1)
   Instead, the expression should be

$y(n-k) = 3y^2(n-k-1) - nx(n-k) + 4x(n-k-1) - x(n-k+1)$ (2)

Hence $(1) \neq (2)$. Thus, the system is time variant

c. The fourth term in expression specifies that the system depends on the future inputs hence, denoting that the system is non-causal

(b). Show that the system described by the following equation is linear.

$$\frac{dy}{dt} + 3y(t) = x(t)$$

ANSWER

To check linearity, consider the following equations

$\frac{dy_1}{dt} + 3y_1(t) = f_1(t);$ (1)

$\frac{dy_2}{dt} + 3y_2(t) = f_2(t)$ (2)

Multiplying equation (1) with $K_1$and (2) with $K_2$and summing them to establish the homogeneity and superposition properties yields equation (3)

$\frac{d}{dt}[K_1y_1(t) + K_2y_2(t)] + 3[K_1y_1(t) + K_2y_2(t)] = K_1f_1(t) + K_2f_2(t)$ (3)

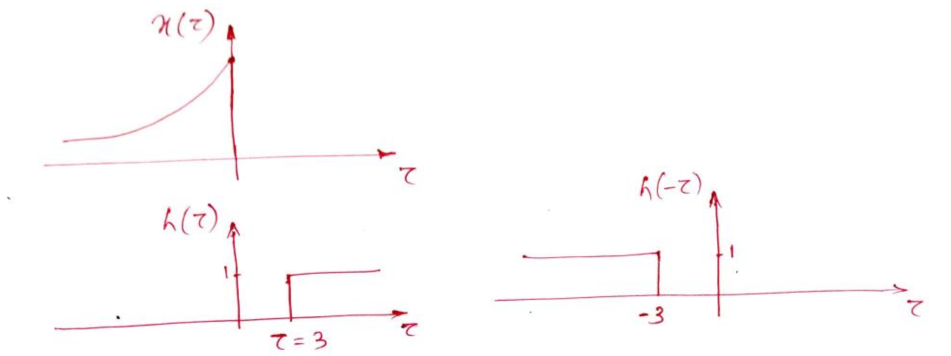
From equation (3) it can be inferred that

$y(t) = K_1y_1(t) + K_2y_2(t)$ (4)

$f(t) = K_1f_1(t) + K_2f_2(t)$ (5)

	Therefore, when the input is (5) the response is (4) which proves that the system is linear.	
4.	Determine the response of an LTI system characterized by an impulse response $h(t) = u(t-3)$, when a signal $x(t) = e^{2t}u(-t)$ is given as input.	**[10]**

ANSWER



Case-I

When $t-3 \leq 0$,

the product of $x(\tau)$ and $h(t-\tau)$ is non-zero for $-\infty < \tau < t-3$.

$$y(t) = \int_{-\infty}^{t-3} e^{2\tau}\, d\tau$$

$$= \frac{1}{2} e^{2\tau} \Big|_{-\infty}^{t-3} = \frac{1}{2} e^{2(t-3)}$$

Case-II

When $t-3 \geqslant 0$,

the product of $x(\tau)$ and $h(t-\tau)$ is non-zero for $-\infty < \tau < 0$

$$y(t) = \int_{-\infty}^{0} e^{2\tau}\, d\tau = \frac{1}{2} e^{2\tau} \Big|_{-\infty}^{0} = \frac{1}{2}$$

5.	For the periodic signal $x(t)$ shown in Fig. 1 below (i) Determine the compact trigonometric Fourier series.	**[10]**

(ii) Sketch the graph of trigonometric Fourier spectra.

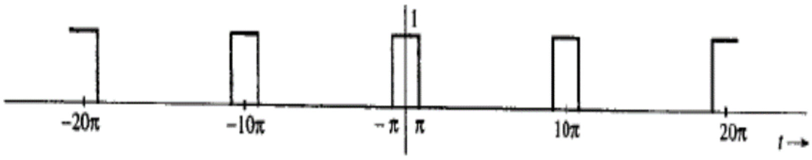


Fig. 1.

ANSWER

i) Compact trigonometric Fourier series.

$T_0 = 10\pi$, $\omega_0 = \frac{2\pi}{T_0} = \frac{1}{5}$. Because of even symmetry, all the sine terms are zero.

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n}{5}t\right) + b_n \sin\left(\frac{n}{5}t\right)$$

$$a_0 = \frac{1}{5} \qquad \text{(by inspection)}$$

$$a_n = \frac{2}{10\pi}\int_{-\pi}^{\pi} \cos\left(\frac{n}{5}t\right)\,dt = \frac{1}{5\pi}\left(\frac{5}{n}\right)\sin\left(\frac{n}{5}t\right)\Big|_{-\pi}^{\pi} = \frac{2}{\pi n}\sin\left(\frac{n\pi}{5}\right)$$

$$b_n = \frac{2}{10\pi}\int_{-\pi}^{\pi} \sin\left(\frac{n}{5}t\right)\,dt = 0 \qquad \text{(integrand is an odd function of } t\text{)}$$

Here $b_n = 0$, and we allow C_n to take negative values. Note that $C_n = a_n$ for $n = 0, 1, 2,$

ii) Trigonometric Fourier spectra: