

Quiz-1 Answer key

Set A

Q1. What are the types of waves based on the motion of the particles of the medium? Give examples of each type and mention three important characteristics of each type with suitable diagrams. 3M

Q2. What are the magnitudes of reflection and transmission coefficients for the following cases when a wave enters from one medium to another? (i) $v_1 = v_2/3$ (ii) $v_2 = 0$ (iii) $v_2 = 2v_1$ where v_1 and v_2 are the wave velocities in medium 1 and 2 respectively. 3M

Q3. The frequency of the third harmonic playing on a guitar string is 350 Hz. What is the value of fundamental frequency? If the wavelength for 2nd harmonic is 26 cm, how long is the string? How fast are the waves moving on the string? 4M

Q1 $\rightarrow$ Based on the motion of the particles of the medium, there are two types of waves:

1) Longitudinal waves

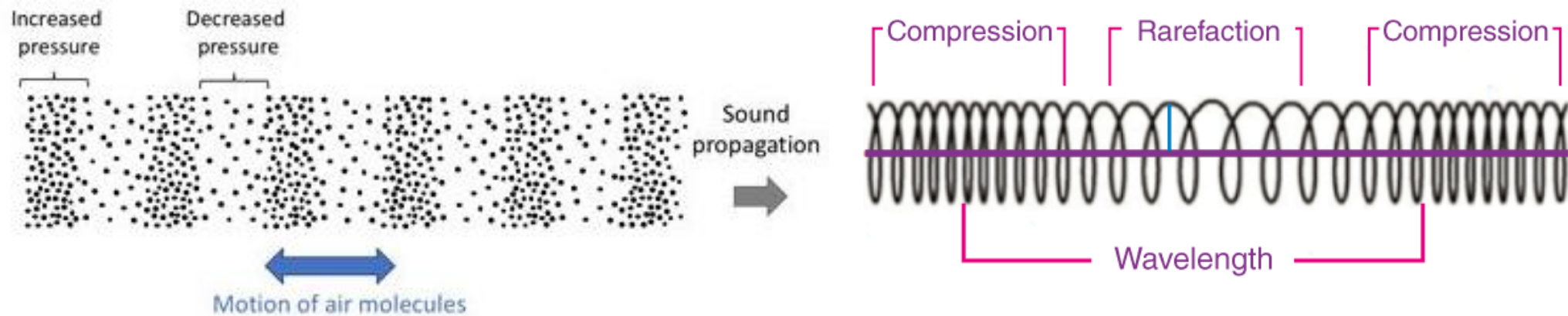
2) Transverse waves

Longitudinal waves $\rightarrow$ In longitudinal waves, the particles of the medium vibrate back & forth along the direction of the wave propagation about their mean position.

which leads to formation of compression and rarefactions. Compressions are the regions where particles of the medium come closer while in rarefaction particles go away from each other.

Examples! → sound waves, waves in a spring,

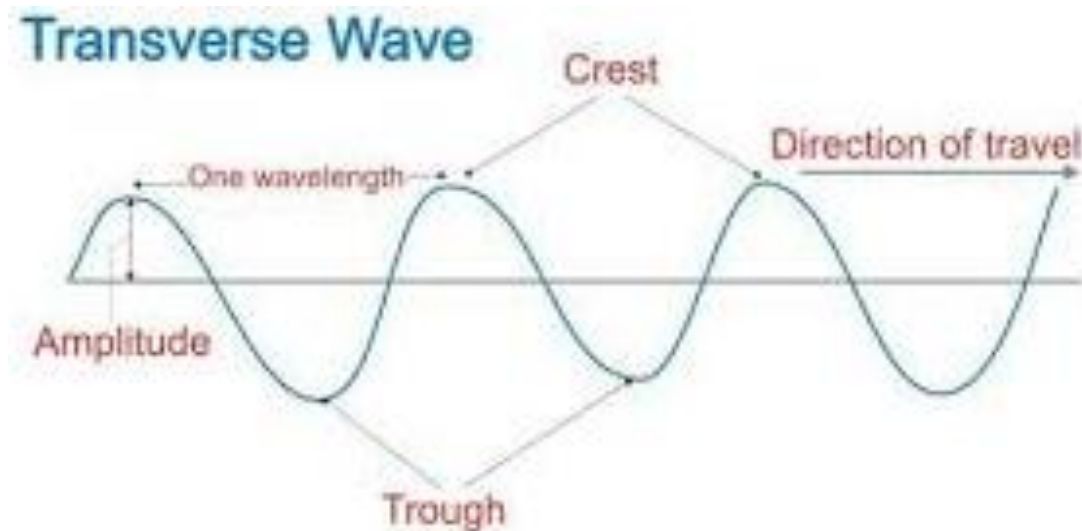
Diagram :-



Transverse waves $\rightarrow$ In transverse waves, the particles of the medium vibrate perpendicular to the direction of wave propagation which leads to formation of +ve amplitude regions called crest and -ve amplitude regions called trough.

Examples $\rightarrow$ Electromagnetic waves, Ripples in a pond

Diagram $\rightarrow$



Q2:- Reflection coefficient, $R = \frac{v_2 - v_1}{v_1 + v_2}$ or $\frac{v_1 - v_2}{v_1 + v_2}$

Transmission coefficient, $T = \frac{2v_2}{v_1 + v_2}$

Case I: $v_1 = v_2/3$ which implies $v_2 > v_1$

Therefore, $R = \frac{v_2 - v_1}{v_1 + v_2} = \frac{v_2 - \frac{v_2}{3}}{\frac{v_2}{3} + v_2} = \frac{\frac{2}{3}v_2}{\frac{4}{3}v_2}$

$$T = \frac{2v_2}{\frac{v_2}{3} + v_2} = \frac{2v_2}{\frac{4}{3}v_2} = \frac{3}{2}$$

$$T = \frac{3}{2}$$

$$R = \frac{1}{2}$$

Case II $\Rightarrow v_2 = 0$

which means $v_1 > v_2$

$$\therefore R = \frac{v_1 - v_2}{v_1 + v_2} = \frac{v_1}{v_1} = 1, \quad T = 0$$

$$\boxed{R=1} \quad \boxed{T=0}$$

No transmitted wave!

Case III $\Rightarrow v_2 = 2v_1$

which means $v_2 > v_1$

$$\therefore R = \frac{v_2 - v_1}{v_1 + v_2} = \frac{2v_1 - v_1}{2v_1 + v_1} = \frac{\cancel{v_1}}{3\cancel{v_1}} = \frac{1}{3}$$

$$\boxed{R = \frac{1}{3}}$$

$$T = \frac{2v_2}{v_1 + v_2} = \frac{2 \times 2v_1}{v_1 + 2v_1} = \frac{4\cancel{v_1}}{3\cancel{v_1}} = \frac{4}{3}$$

$$\boxed{T = \frac{4}{3}}$$

Q3 :->

$$v_3 = 350 \text{ Hz}$$

$$v_n = \frac{nv}{2l}$$

$$v_1 = ?$$

$$v_3 = 3v_1 \Rightarrow v_1 = \frac{v_3}{3} = \frac{350}{3} = 116.7 \text{ Hz}$$

wavelength for 2nd harmonic, $\lambda_2 = 26 \text{ cm}$

$$\lambda_n = \frac{2l}{n}$$

$l \rightarrow$ length of the string

$$\lambda_2 = \frac{2l}{2} \Rightarrow$$

$$l = 26 \text{ cm}$$

Wave velocity, $v = ?$

$$v_3 = \frac{3v}{2l}, \quad 3^{\text{rd}} \text{ harmonic frequency}$$

$$350 = \frac{3v}{2 \times 26} \quad \left| \quad l = 26 \text{ cm} \right.$$

$$v = \frac{350 \times 2 \times 26}{3} = \underline{\underline{6067 \text{ cm s}^{-1}}}$$

Set B

Q1. Write five important differences between a progressive wave and a standing wave.

3M

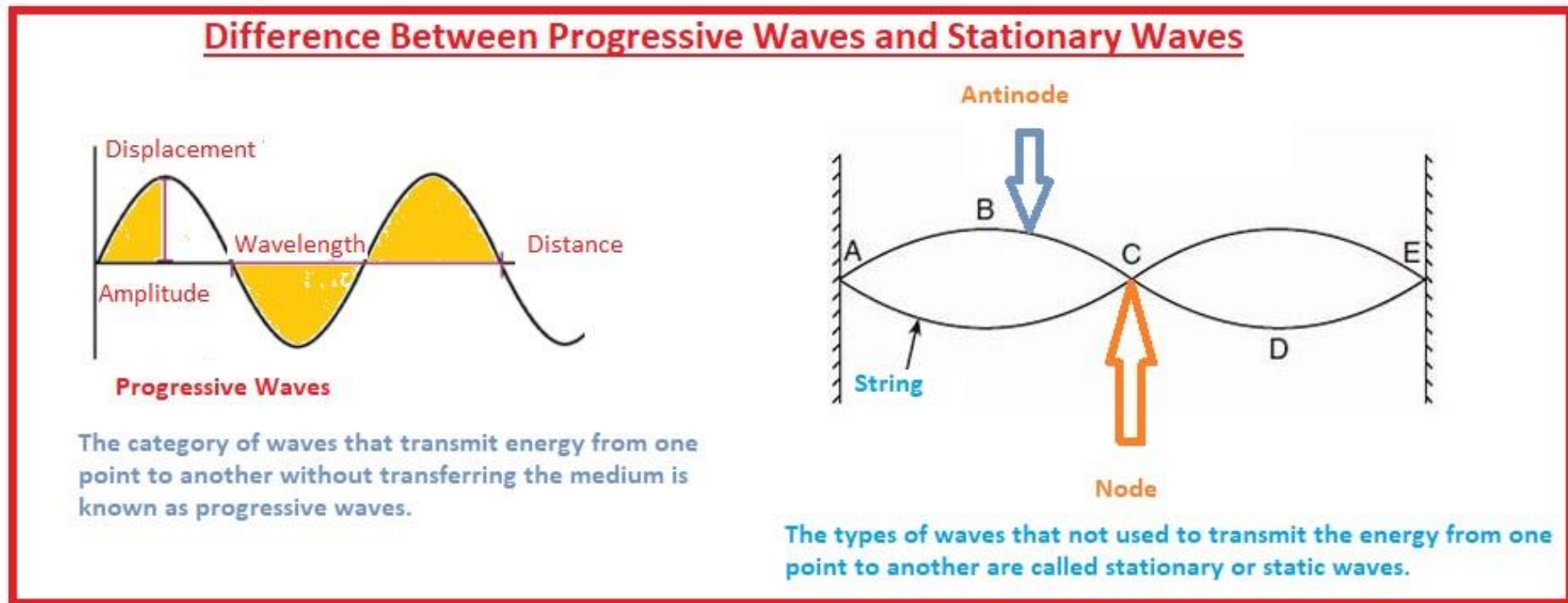
Q2. What are the magnitudes of reflection and transmission coefficients for the following cases when a wave enters from one medium of impedance, Z_1 to another medium of impedance, Z_2 ? (i) $Z_2 = \infty$ (ii) $Z_2 = 0$ (iii) $Z_2 = 2Z_1$

3M

Q3. A standing wave is given by the function $y = 5 \sin(\pi x/3) \cos(40\pi t)$. Find the amplitude and velocity of its component waves and also the velocity of particles of the medium at $t = 1.125$ s and $x = 1.5$ cm. (use CGS units)

4M

Q1 :-



S. No.	Progressive Wave	Stationary Wave
1.	This is an advancing wave, which moves in the medium continuously with finite velocity.	There is no advancement of the wave in any direction.
2.	These waves are produced due to continuous periodic vibrations of the particles of a portion of a medium.	These waves are produced due to the superposition of two identical progressive waves moving along the same line but in opposite direction in a medium.
3.	Each particle of the medium executes identical periodic motion about the mean position of rest.	The particles between two successive nodes are in the same phase of vibration, while the particles on opposite sides of a nodes are in opposite phase of vibration.
4.	The phase of vibration varies continuously from point to point.	The particles between two successive nodes are in the same phase of vibration, while the particles on opposite sides of a node are in opposite phase of vibration.
5.	All the particles do not pass through their mean positions or reach their outer most positions simultaneously.	All the particles pass through their mean positions or reach their outer most positions simultaneously twice in a periodic time.
6.	Wave curve advances without change of shape.	Wave curve changes its shape from time to time-it shrinks to straight line twice in each period.
7.	The amplitude of each particles is same.	The amplitude varies continuously from point to point-it is maximum at the antinodes and minimum at nodes.

Q2 $\Rightarrow R = \left| \frac{Z_1 - Z_2}{Z_1 + Z_2} \right| \quad T = \frac{2Z_1}{Z_1 + Z_2}$

Case I $\Rightarrow Z_2 = \infty \Rightarrow Z_1 < Z_2$

$\therefore R = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\cancel{Z_2} \left(1 - \frac{Z_1}{Z_2}\right)}{\cancel{Z_2} \left(1 + \frac{Z_1}{Z_2}\right)} = \frac{(1-0)}{(1+0)} = 1$

$T = \frac{2Z_1}{Z_1 + \infty} = 0 \quad \text{No Transmitted wave!}$

Case II $\Rightarrow Z_2 = 0 \Rightarrow R = 1, T = 2$

Case III :-

$$Z_2 = 2Z_1$$

$$\Rightarrow Z_2 > Z_1$$

$$\therefore R = \frac{Z_2 - Z_1}{Z_1 + Z_2} =$$

$$\frac{2Z_1 - Z_1}{Z_1 + 2Z_1} = \frac{\cancel{Z_1}}{\cancel{3Z_1}} = \frac{1}{3}$$

$$T = \frac{2Z_1}{Z_1 + 2Z_1} =$$

$$\frac{\cancel{2Z_1}}{\cancel{3Z_1}} = \frac{2}{3}$$

$$R = \frac{1}{3}, T = \frac{2}{3}$$

Q3. A standing wave is given by the function $y = 5 \sin(\pi x/3) \cos(40\pi t)$. Find the amplitude and velocity of its component waves and also the velocity of particles of the medium at $t = 1.125$ s and $x = 1.5$ cm. (use CGS units)

Ans: $\rightarrow$

$$y = 5 \sin\left(\frac{\pi x}{3}\right) \cos(40\pi t) \quad \text{--- (1)}$$

Wave function of a standing wave is $y = 2A \sin(kx) \cos(\omega t)$ --- (2)

Comparing eqns. (1) & (2) we get

$$2A = 5 \Rightarrow A = \underline{\underline{5/2 \text{ cm}}} \rightarrow \text{Amplitude of component waves}$$

$$k = \frac{\pi}{3} \text{ cm}^{-1} \rightarrow \text{wave no. of component waves}$$

$$\omega = 40\pi \text{ rad/s}^{-1} \rightarrow \text{frequency of component waves}$$

Therefore, velocity of component wave = $\frac{\omega}{k} = \frac{40\pi}{\pi/3} = 120 \text{ cm s}^{-1}$

$$v = 120 \text{ cm s}^{-1}$$

velocity of particles of the medium = $\frac{dy}{dt}$

$$y = 5 \sin\left(\frac{\pi x}{3}\right) \cos(40\pi t)$$

$$\frac{dy}{dt} = 5 \sin\left(\frac{\pi x}{3}\right) \sin(40\pi t) (-40\pi)$$

$$\frac{dy}{dt} = -200\pi \sin\left(\frac{\pi x}{3}\right) \sin(40\pi t)$$

at $t = 1.125 \text{ s}$ and $x = 1.5 \text{ cm}$

$$\frac{dy}{dt} = -200\pi \sin\left(\frac{\pi}{3} \times 1.5^{0.5}\right) \sin(40\pi \times 1.125)$$

$$= -200\pi \sin\left(\frac{\pi}{2}\right) \sin(45\pi)$$

$$\boxed{\frac{dy}{dt} = 0}$$

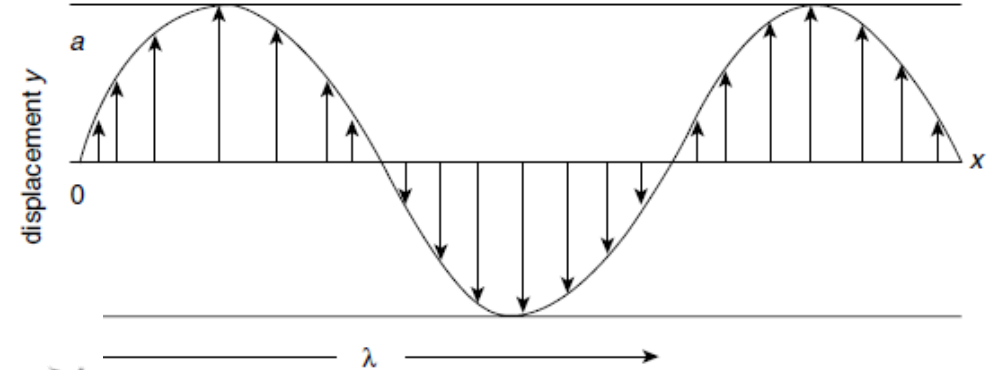
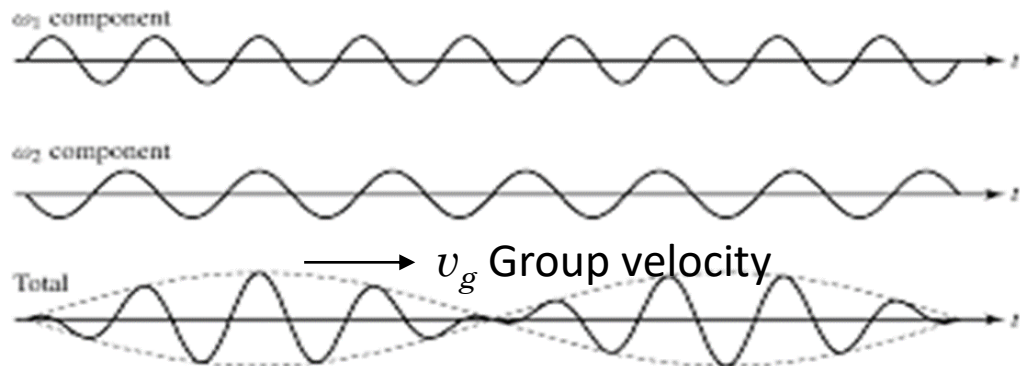
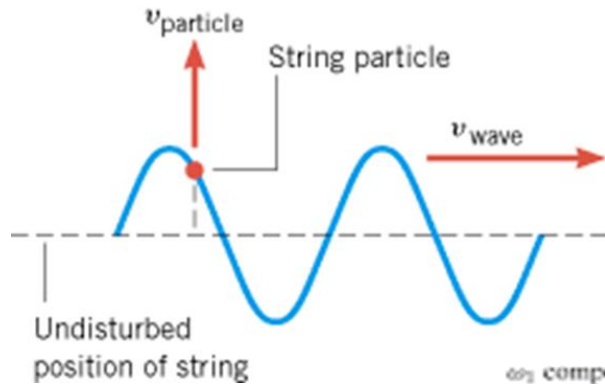
Ans

Set C

- Q1.** Explain different types of velocities in wave motion with suitable examples and diagrams. **3M**
- Q2.** When a wave encounters a boundary while going from one medium to another, in which scenario, transmission coefficient will be twice of reflection coefficient? Justify your answer. **3M**
- Q3.** The wave function of a stationary wave generated in a 100 cm long string is $y = 0.8 \cos (\pi x/20) \sin (200\pi t)$. What is the separation between two consecutive nodes? Also find the frequency of 3rd overtone. (Use CGS units) **4M**

Q1. Explain different types of velocities in wave motion with suitable examples and diagrams.

- The particles or constituents of the medium which make up the medium do not progress through the medium with the waves.
- Their motion is simple harmonic, limited to oscillations, transverse or longitudinal, about their equilibrium positions.
- It is their phase relationships we observe as waves, not their progressive motion through the medium.
- There are three velocities in wave motion which are quite distinct although they are connected mathematically. They are:
 - ❖ The particle velocity, which is the simple harmonic velocity of the oscillator about its equilibrium position.
 - ❖ The wave or phase velocity, the velocity with which planes of equal phase, crests or troughs, progress through the medium.
 - ❖ The group velocity. A number of waves of different frequencies, wavelengths and velocities may be superposed to form a group.



Q2. When a wave encounters a boundary while going from one medium to another, in which scenario, transmission coefficient will be twice of reflection coefficient? Justify your answer.

Ans: When a wave encounters a boundary while going from one medium to another, it may get partly reflected and partly transmitted. However depending upon the impedance of the second medium, the wave may be completely reflected back in the first medium or may be completely transmitted in the second medium.

There are two scenarios in which transmission coefficient becomes twice of reflection coefficient.

In first scenario, when impedance of the second medium is zero. Then

$$T = \frac{A_T}{A_I} = \frac{2v_2}{v_1 + v_2} = \frac{2Z_1}{Z_1 + Z_2}, \text{ For } Z_2 = 0, T = 2.$$

In second scenario, when impedance of the second medium is twice that of first medium i.e. $Z_2 = 2Z_1$

Which will give $R = 1/3$ and $T = 2/3$

Q3. The wave function of a stationary wave generated in a 100 cm long string is $y = 0.8 \cos(\pi x/20) \sin(200\pi t)$. What is the separation between two consecutive nodes? Also find the frequency of 3rd overtone. (Use CGS units)

Ans! →

$$L = 100 \text{ cm}$$

$$y = 0.8 \cos(\pi x/20) \sin(200\pi t) \quad \text{--- ①}$$

Separation b/w ^{two} consecutive nodes is $\lambda/2$

$$\text{From eqn ①} \quad k = \frac{\pi}{20}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\cancel{\pi} \times 20}{\cancel{\pi}} = 40 \text{ cm}$$

∴ separation b/w two consecutive nodes is $\frac{40}{2} = 20 \text{ cm}$

(n=4) frequency of 3rd overtone, $\nu_4 = \frac{4v}{2l} = \frac{2v}{l}$

wave velocity, $v = \frac{\omega}{k} = \frac{200\pi}{\pi/20} = 4000 \text{ cm s}^{-1}$

$l = 100 \text{ cm}$

$\Rightarrow \boxed{\nu_4 = \frac{2 \times 4000}{100} = 80 \text{ Hz}}$

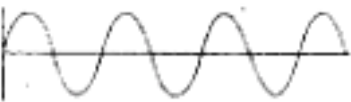
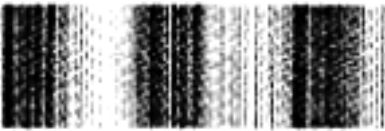
Set D

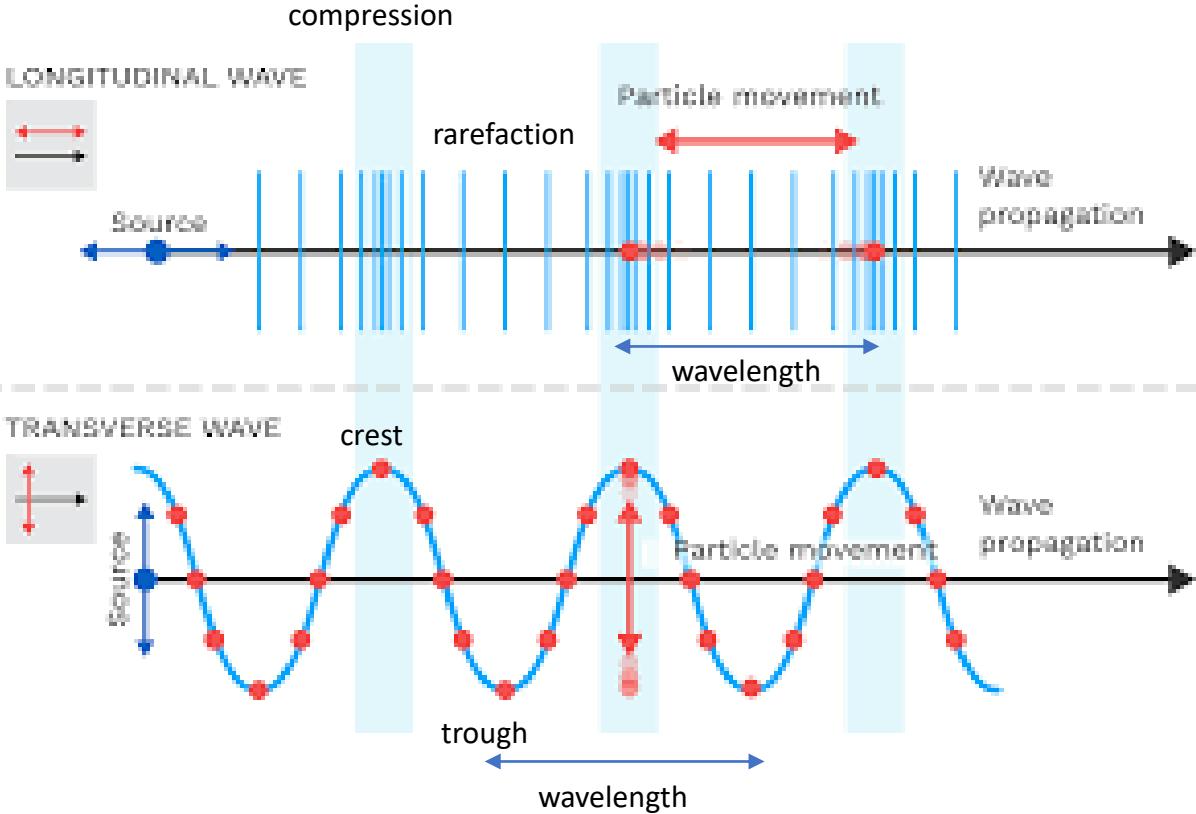
Q1. Differentiate between a longitudinal and transverse wave. Give two examples of each and also draw suitable and well labelled diagrams to represent both types of waves. **3M**

Q2. What are the magnitudes of reflection and transmission coefficients for the following cases when a wave enters from one medium to another? (i) $k_1 = k_2/2$ (ii) $k_2 = 0$ (iii) $k_2 = k_1/3$ where k_1 and k_2 are the wave numbers of the wave for medium 1 and medium 2 respectively. **3M**

Q3. The vibrations of a 60 cm long string fixed at both the ends are represented by the function $y = 4 \sin (\pi x/15) \cos (96\pi t)$. What are the locations of the nodes on the string? What is the wave velocity and amplitude of the component waves? (use CGS units) **4M**

Q1. Differentiate between a longitudinal and transverse wave. Give two examples of each and also draw suitable and well labelled diagrams to represent both types of waves.

Transverse wave	Longitudinal wave
1. 	1. 
2. Particles vibrate in a direction perpendicular to the direction of propagation of the wave	2. Particles vibrate in a direction parallel to the direction of propagation of the wave
3. Crests and troughs are formed	3. Compressions and rarefactions are formed
4. Formed on the surface of solids and liquids	4. Formed in solids, liquids and gases
5. May be elastic waves or non elastic waves eg. light wave, radio wave	5. Only elastic waves (mechanical) eg. sound wave, seismic wave
6. Do not create pressure difference in the medium	6. Creates pressure difference in the medium



Q2. What are the magnitudes of reflection and transmission coefficients for the following cases when a wave enters from one medium to another? (i) $k_1 = k_2/2$ (ii) $k_2 = 0$ (iii) $k_2 = k_1/3$ where k_1 and k_2 are the wave numbers of the wave for medium 1 and medium 2 respectively

Ans:-

$$R = \frac{k_1 - k_2}{k_1 + k_2} \quad \text{OR} \quad \frac{k_2 - k_1}{k_1 + k_2}, \quad T = \frac{2k_1}{k_1 + k_2}$$

Case I $\rightarrow k_1 = k_2/2 \Rightarrow k_2 > k_1$

$$\therefore R = \frac{k_2 - k_1}{k_1 + k_2} = \frac{k_2 - k_2/2}{k_2/2 + k_2} = \frac{\cancel{k_2}/2}{3\cancel{k_2}/2} = \frac{1}{3}$$

$$T = \frac{2k_2/2}{k_2/2 + k_2} = \frac{\cancel{k_2}}{3\cancel{k_2}/2} = \frac{2}{3}$$

$$\boxed{R = \frac{1}{3}, \quad T = \frac{2}{3}}$$

Case II :- $k_2 = 0$

$$R = \frac{k_1 - k_2}{k_1 + k_2} = \frac{k_1 - 0}{k_1 + 0} = 1$$

$$T = \frac{2k_1}{k_1 + k_2} = 2$$

$$R = 1, T = 2$$

Case III :- $k_2 = k_1/3$ $k_1 > k_2$

$$R = \frac{k_1 - k_2}{k_1 + k_2} = \frac{k_1 - k_1/3}{k_1 + k_1/3} = \frac{\cancel{2/3} k_1}{\cancel{4/3} k_1} = \frac{1}{2}$$

$$T = \frac{2k_1}{k_1 + k_2} = \frac{2k_1}{k_1 + k_1/3} = \frac{\cancel{2} k_1}{2 \cancel{k_1}/3} = \frac{3}{2}$$

$$R = 1/2, T = 3/2$$

Q3. The vibrations of a 60 cm long string fixed at both the ends are represented by the function $y = 4 \sin(\pi x/15) \cos(96\pi t)$. What are the locations of the nodes on the string? What is the wave velocity and amplitude of the component waves? (use CGS units)

Sol:-

$$l = 60 \text{ cm}$$

$$y = 4 \sin(\pi x/15) \cos(96\pi t) \quad \text{--- (1)}$$

Nodes will form at those locations where $y=0$

$$\text{for that } kx = n\pi \quad n = 0, 1, 2, 3$$

$$x = \frac{n\pi}{k}$$

$$x = \frac{n\pi}{\pi/15} = 15n$$

Nodes will form at $x = 0, 15, 30, 45, 60 \text{ cm}$.

we know that for a standing wave

$$y = 2A \sin bx \cos \omega t$$

— (2)

comparing eqns (1) and (2)

$$2A = 4 \Rightarrow \underline{A = 2 \text{ cm}}$$

$$\omega = 96\pi \text{ rad s}^{-1}, \quad b = \frac{\pi}{15} \text{ cm}^{-1}$$

wave velocity, $v = \frac{\omega}{b} = \frac{96\pi}{\pi/15} = 1440 \text{ cm s}^{-1}$