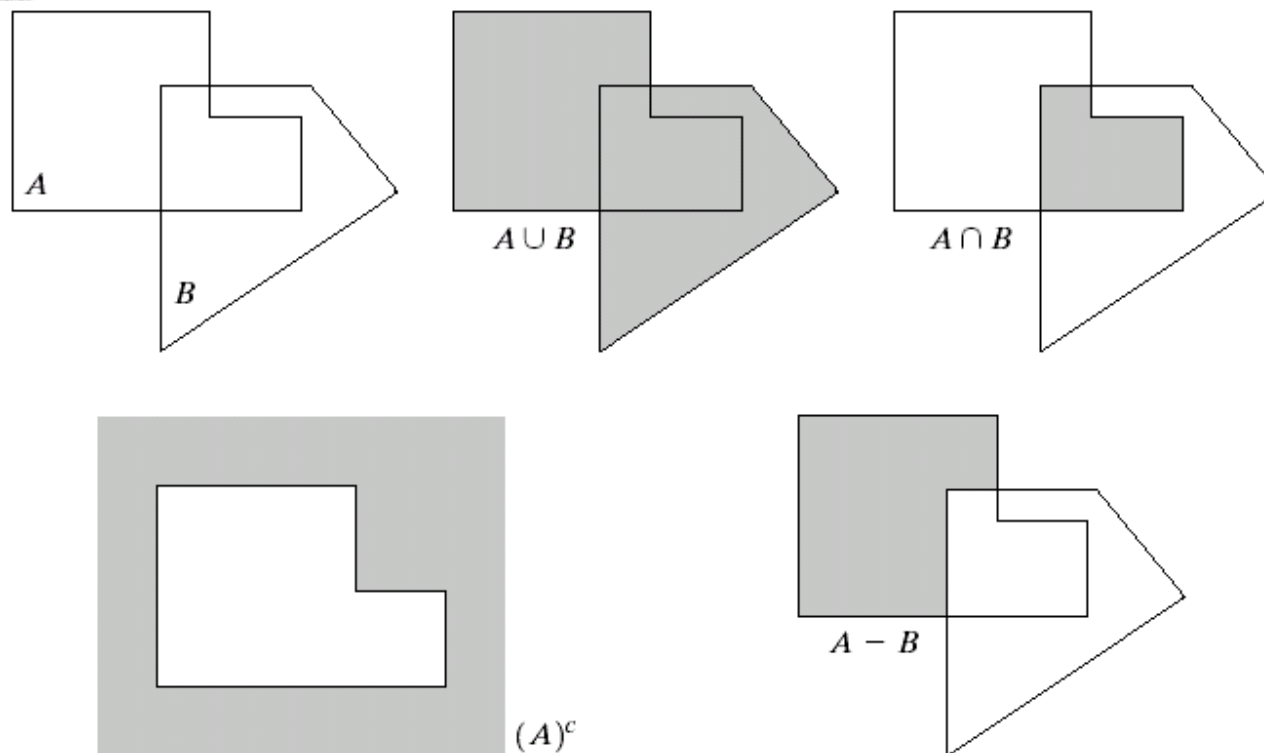


Chapter 9

Morphological Image Processing

Some Basic Concepts from Set Theory

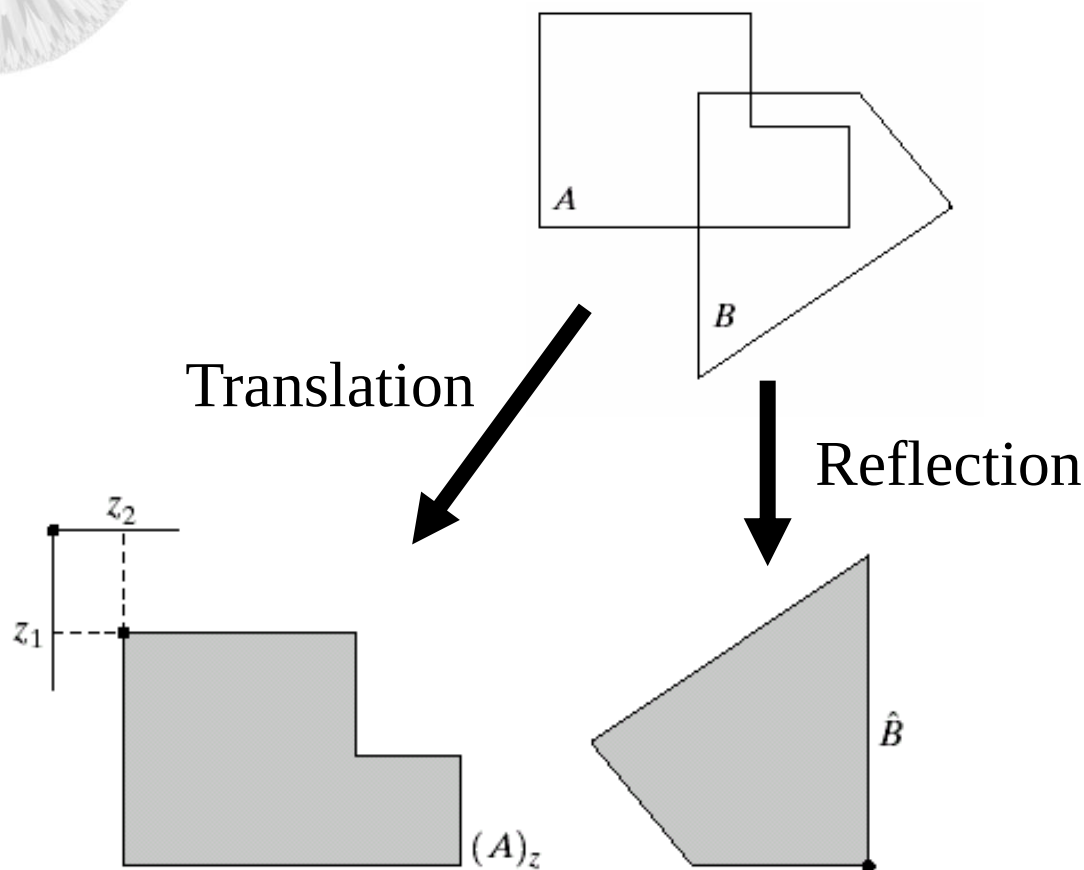


a	b	c
d	e	

FIGURE 9.1

(a) Two sets A and B . (b) The union of A and B . (c) The intersection of A and B . (d) The complement of A . (e) The difference between A and B .

Some Basic Concepts from Set Theory



a b

FIGURE 9.2

(a) Translation of A by z .
(b) Reflection of B . The sets A and B are from Fig. 9.1.

Logic Operations Involving Binary Images

TABLE 9.1

The three basic
logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

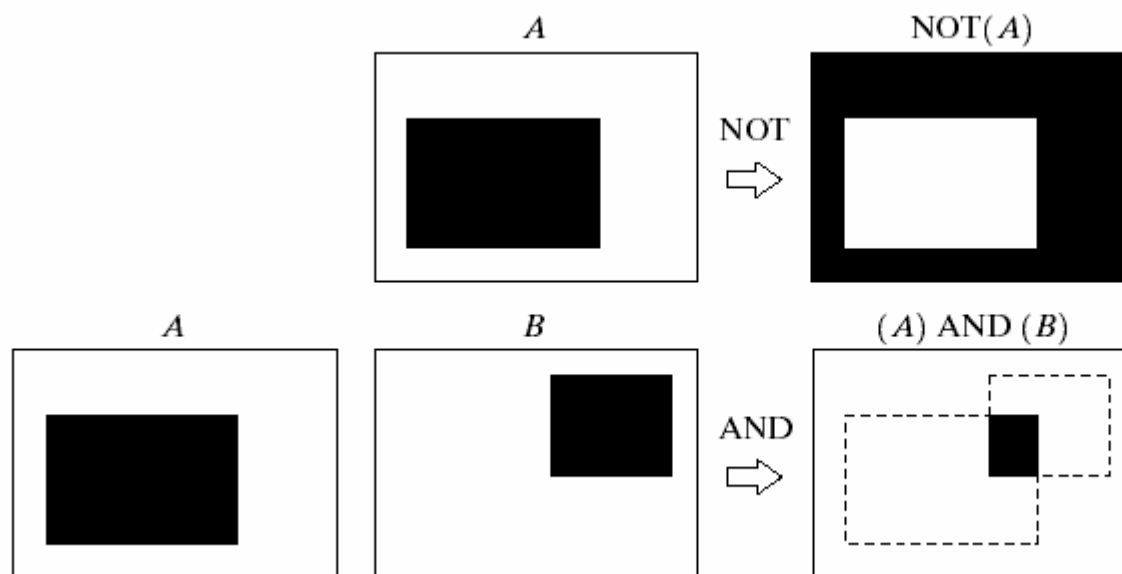
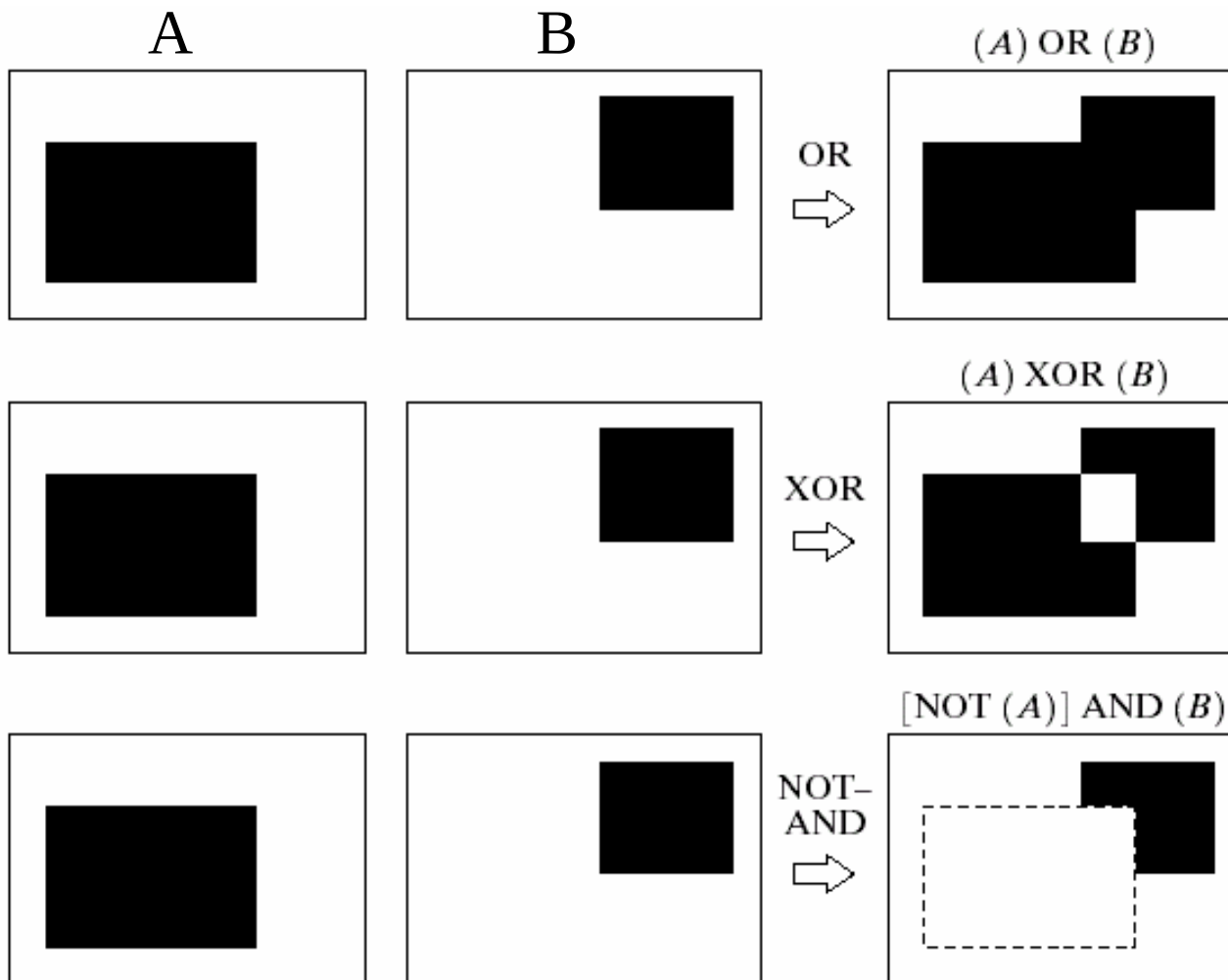
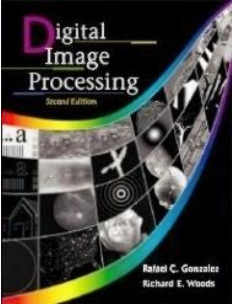


FIGURE 9.3 Some
logic operations
between binary
images. Black
represents binary
1s and white
binary 0s in this
example.

Logic Operations Involving Binary Images





Dilation and Erosion

Dilation

- With A and B as sets in Z^2 , the dilation of A by B is defined as

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \phi\}$$

where $\hat{B}$: the reflection of B about its origin and shifting this reflection by z

- The dilation of A by B is the set of all displacements, z , such that $\hat{B}$ and A **overlap by at least one element**. Thus,

$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

- Set B is referred to as the **structuring element** in dilation.

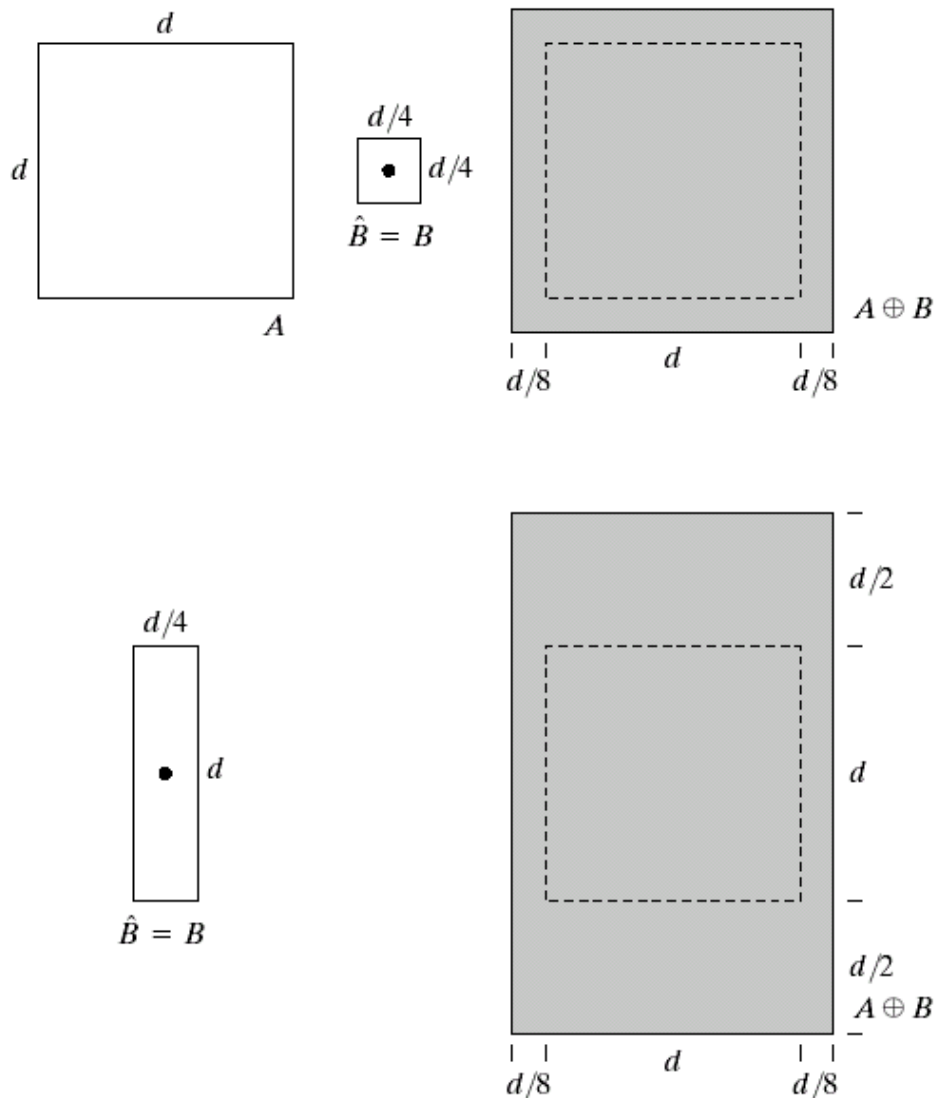
Dilation and Erosion

Dilation

a b c
d e

FIGURE 9.4

- (a) Set A .
- (b) Square structuring element (dot is the center).
- (c) Dilation of A by B , shown shaded.
- (d) Elongated structuring element.
- (e) Dilation of A using this element.



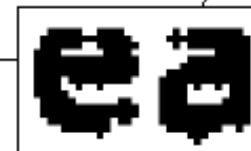
Dilation and Erosion

Dilation Example

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



0	1	0
1	1	1
0	1	0

Structuring element

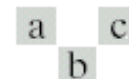
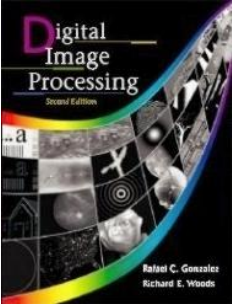


FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.



Dilation and Erosion

Erosion

- For sets A and B in Z^2 , the erosion of A by B is defined as

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

where $\hat{B}$: the reflection of B about its origin and shifting this reflection by z

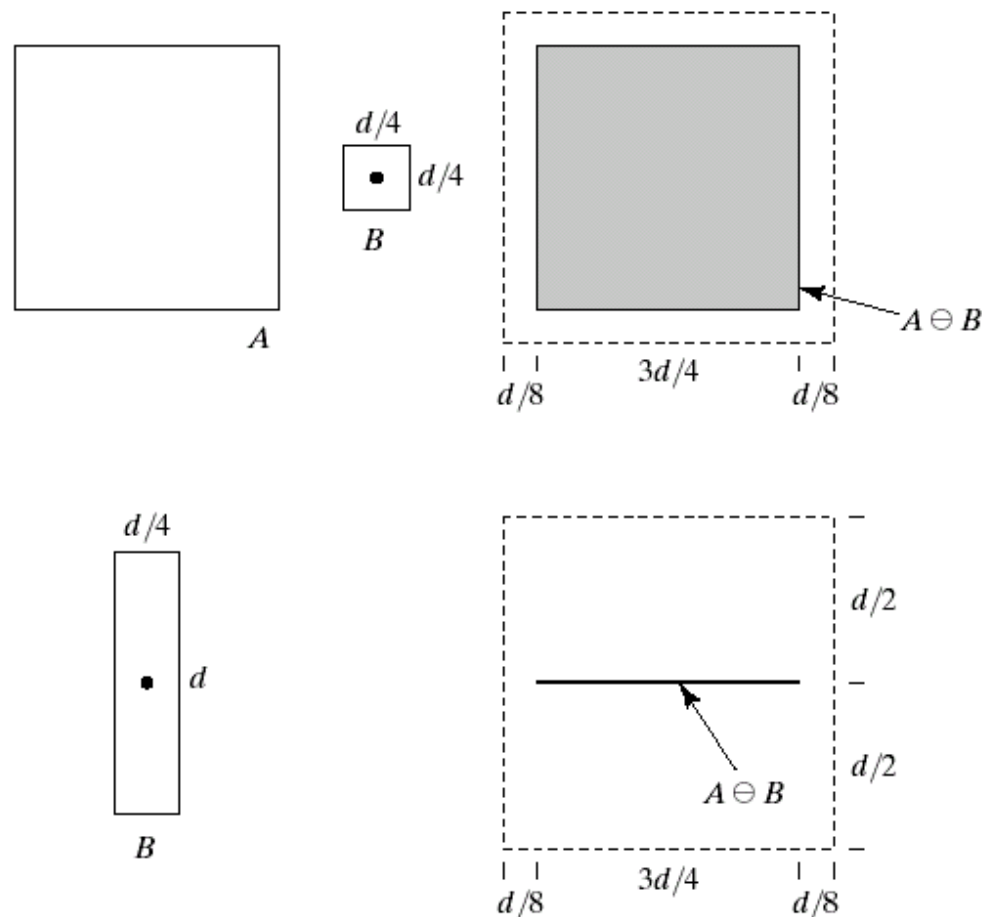
- The erosion of A by B is the set of all points z , such that B , translated by z is contained in A .

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$\begin{aligned}(A \ominus B)^c &= \{z \mid (B)_z \subseteq A\}^c \\ &= \{z \mid (B)_z \subseteq A^c = \phi\}^c \\ &= \{z \mid (B)_z \subseteq A^c \neq \phi\} = A^c \oplus \hat{B}\end{aligned}$$

Dilation and Erosion

Erosion



a	b	c
d		e

FIGURE 9.6 (a) Set A . (b) Square structuring element. (c) Erosion of A by B , shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

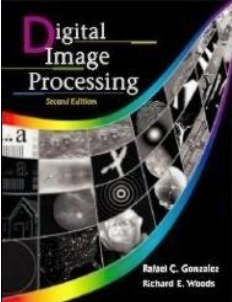
Dilation and Erosion

Erosion Example



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.



Opening and Closing

- **Opening:**

- smooth the contour of an object, break narrow isthmuses, and eliminate thin protrusions.
- The opening A by B is the erosion of A by B , followed by a dilation of the result by B

$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \bigcup \{ (B)_z \mid (B)_z \subseteq A \}$$

- **Closing:**

- smooth sections of contours but it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.

$$A \bullet B = (A \oplus B) \ominus B$$

Opening and Closing

Opening

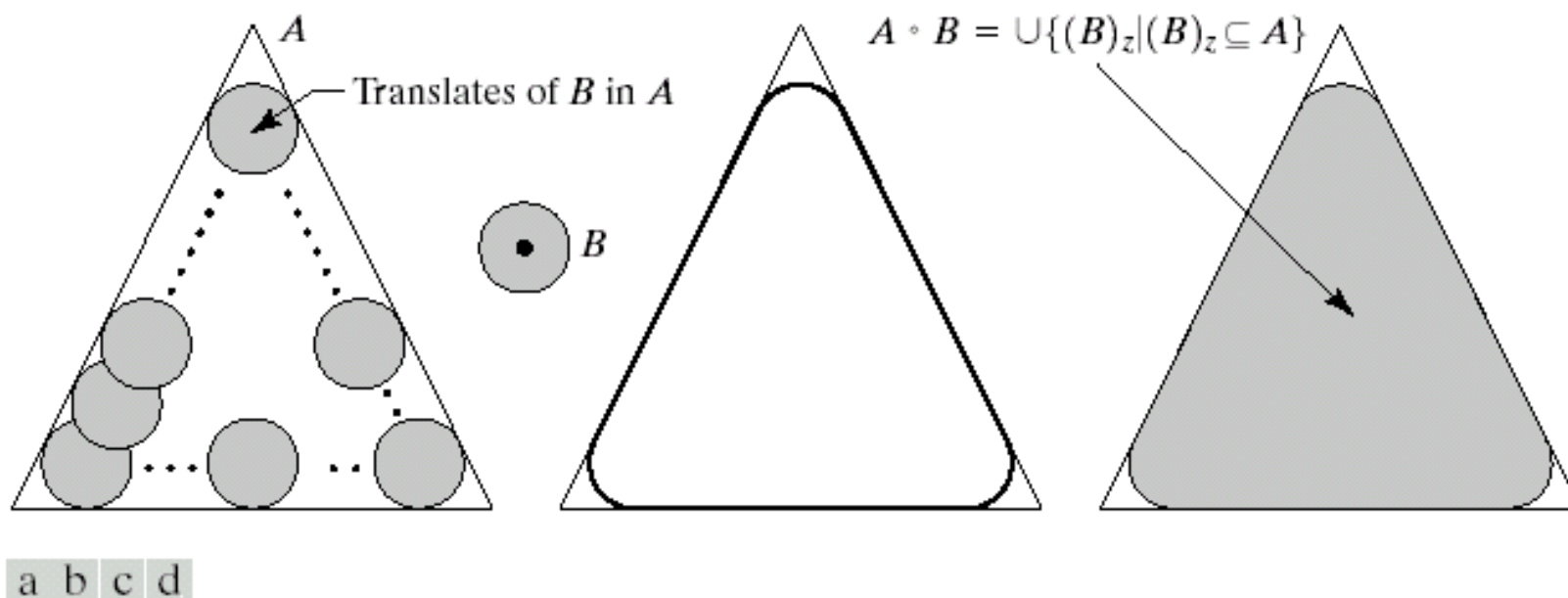


FIGURE 9.8 (a) Structuring element B “rolling” along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Opening and Closing

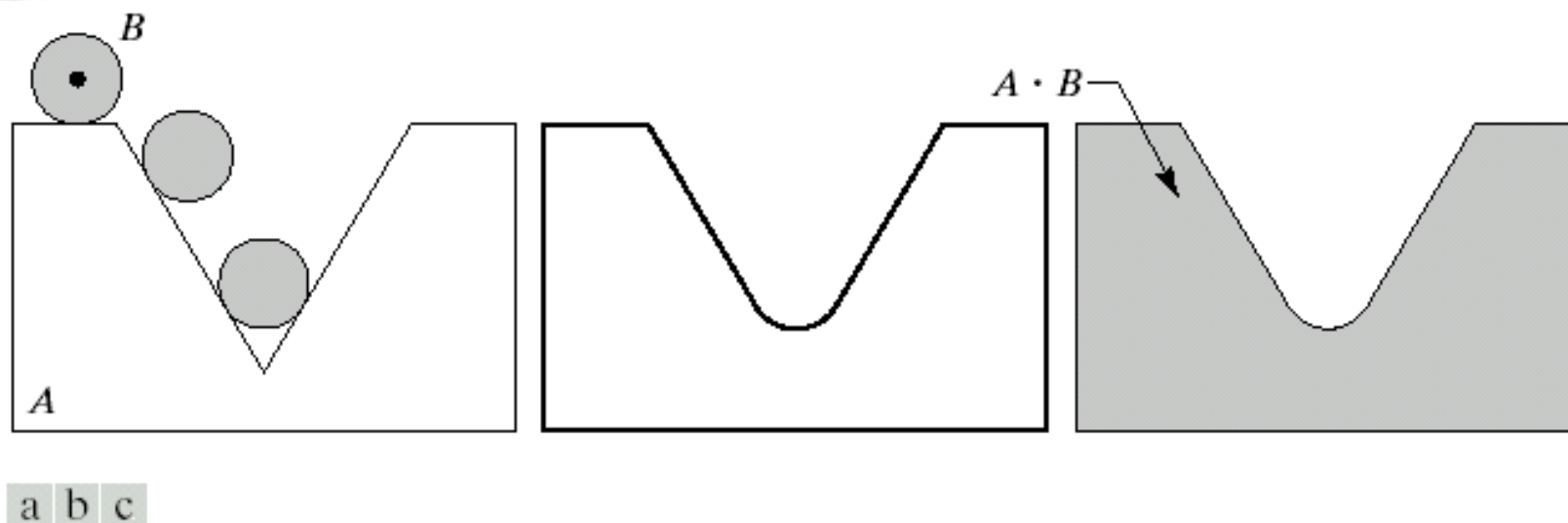


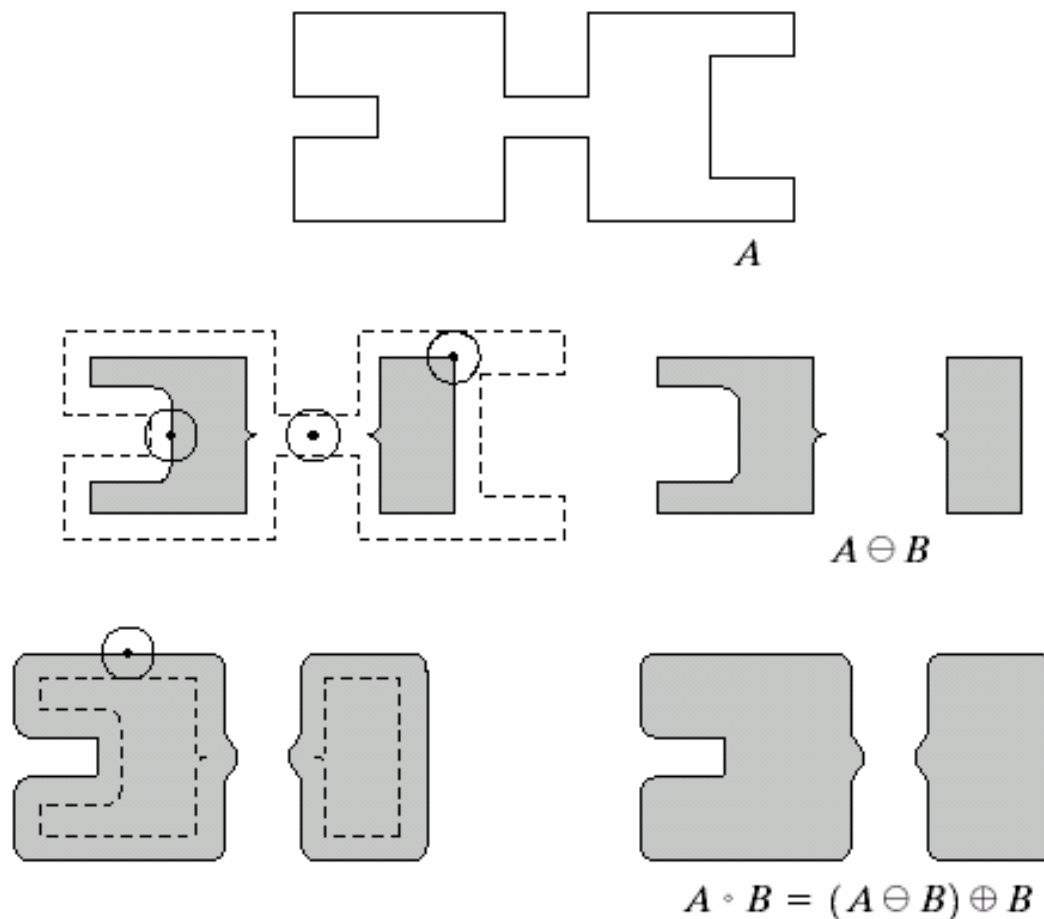
FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A . (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

Opening and Closing Opening Example

a
b c
d e
f g
h i

FIGURE 9.10

Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



Opening and Closing Closing Example

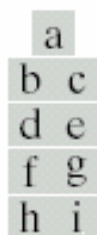
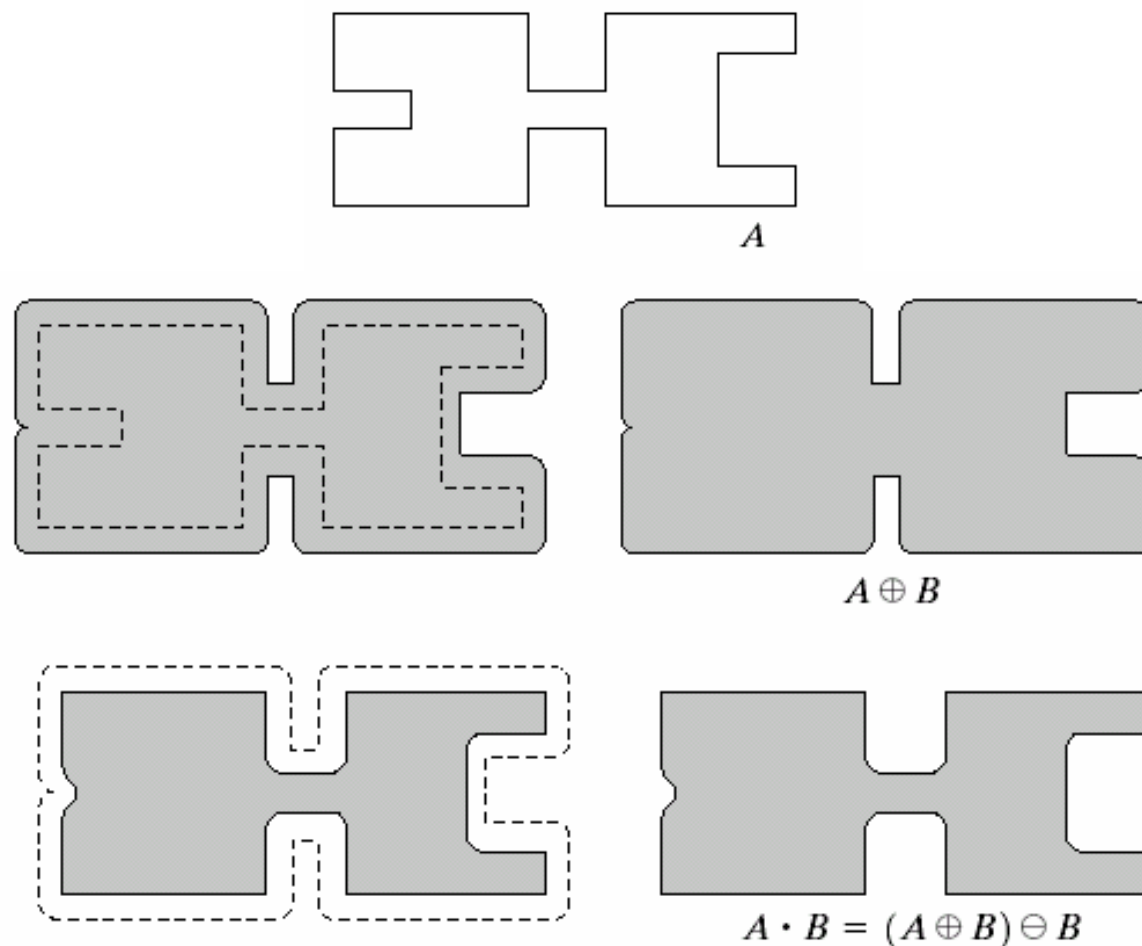
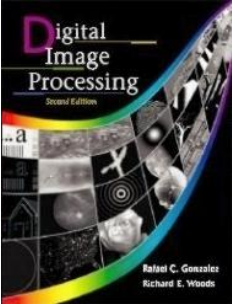


FIGURE 9.10

Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

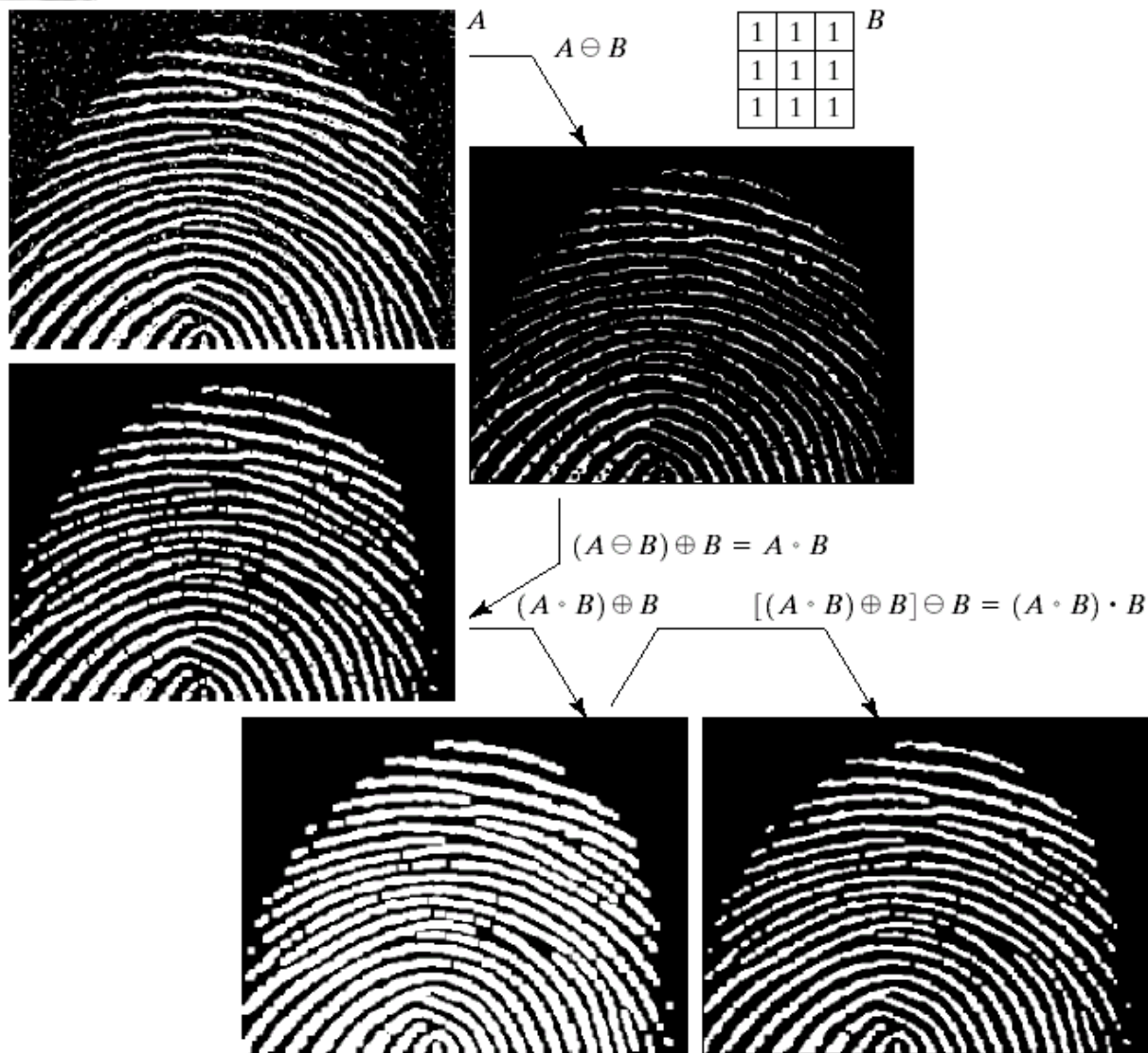




Opening and Closing

- The opening operation satisfies the following properties:
 - $A \circ B$ is a subset (subimage) of A .
 - If C is a subset of D , then $C \circ B$ is a subset of $D \circ B$.
 - $(A \circ B) \circ B = A \circ B$
- The closing operation satisfies the following properties:
 - A is a subset (subimage) of $A \bullet B$.
 - If C is a subset of D , then $C \bullet B$ is a subset of $D \bullet B$.
 - $(A \bullet B) \bullet B = A \bullet B$

Opening and Closing



a b
d c
e f

FIGURE 9.11

(a) Noisy image.
(c) Eroded image.
(d) Opening of A .
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

The Hit-or-Miss Transformation

$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)] \quad B = (X, W - X)$$

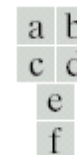
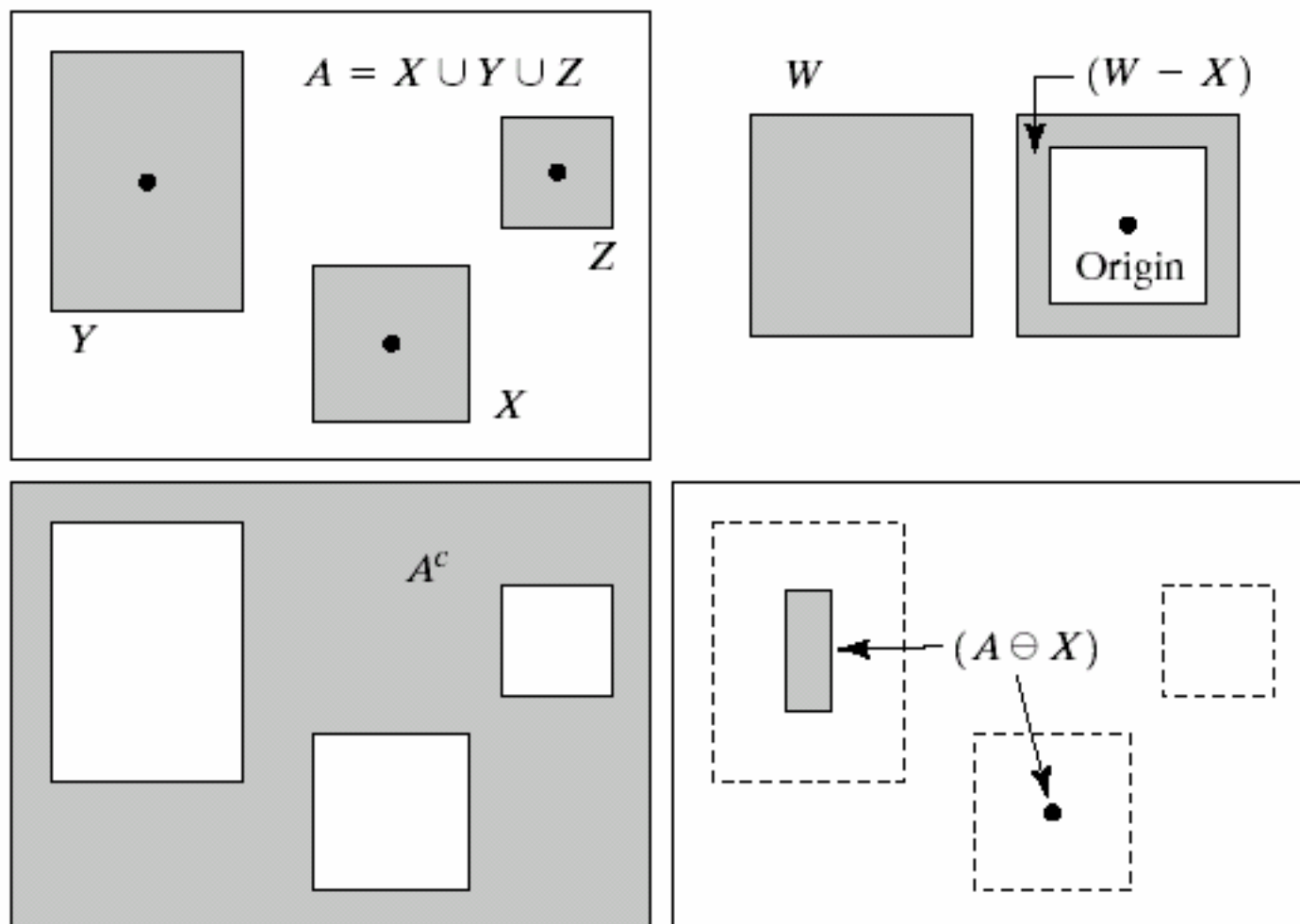
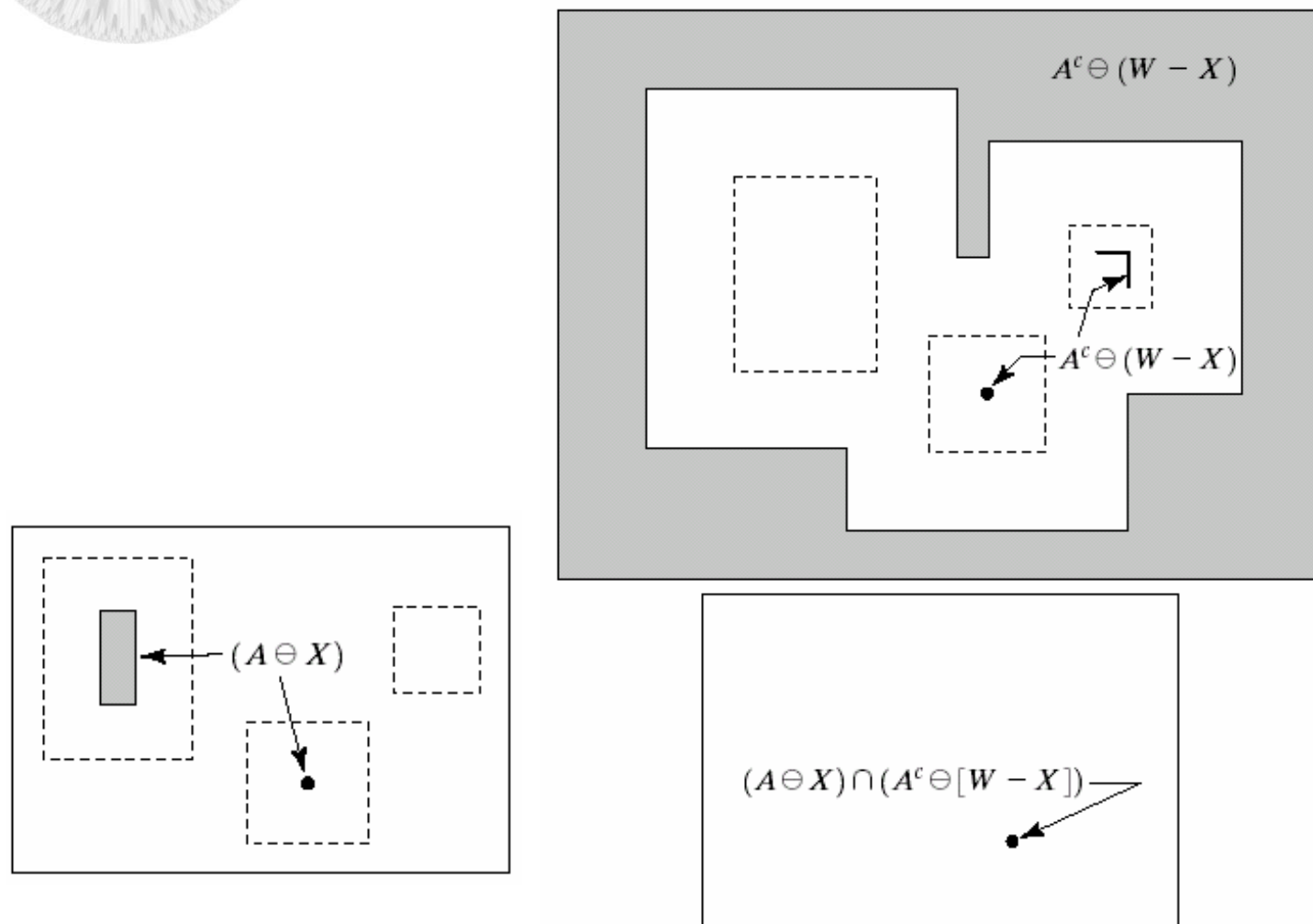


FIGURE 9.12

(a) Set A. (b) A window, W , and the local background of X with respect to W , $(W - X)$. (c) Complement of A. (d) Erosion of A by X . (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X , as desired.

The Hit-or-Miss Transformation

$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)] \quad B = (X, W - X)$$



a b
c d
e
f

FIGURE 9.12

(a) Set A. (b) A window, W , and the local background of X with respect to W , $(W - X)$. (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

Some Basic Morphological Algorithms Boundary Extraction

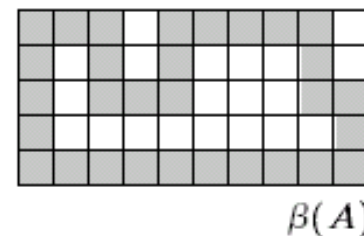
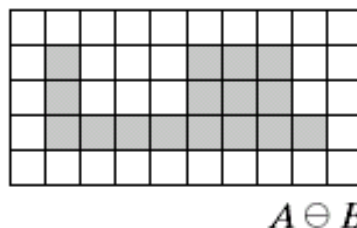
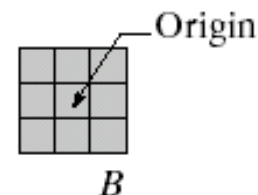
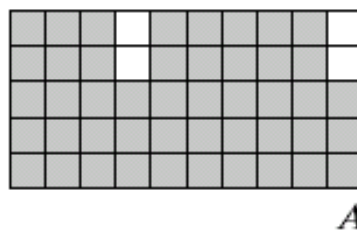
- The boundary of a set A ,

$$\beta(A) = A - (A \ominus B)$$

where B is a suitable structuring element.



FIGURE 9.13 (a) Set A . (b) Structuring element B . (c) A eroded by B . (d) Boundary, given by the set difference between A and its erosion.



Some Basic Morphological Algorithms

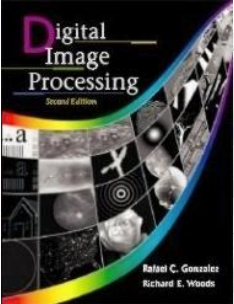
Boundary Extraction Example



a b

FIGURE 9.14

(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).



Some Basic Morphological Algorithms Region Filling

- The following procedure can fill the region:

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

where $X_0 = p$, and B is the symmetric structuring element shown in Fig. 9.15.

- Note that p is the initial point we should assign.

Some Basic Morphological Algorithms

Region Filling

a	b	c
d	e	f
g	h	i

FIGURE 9.15

Region filling.

(a) Set A .

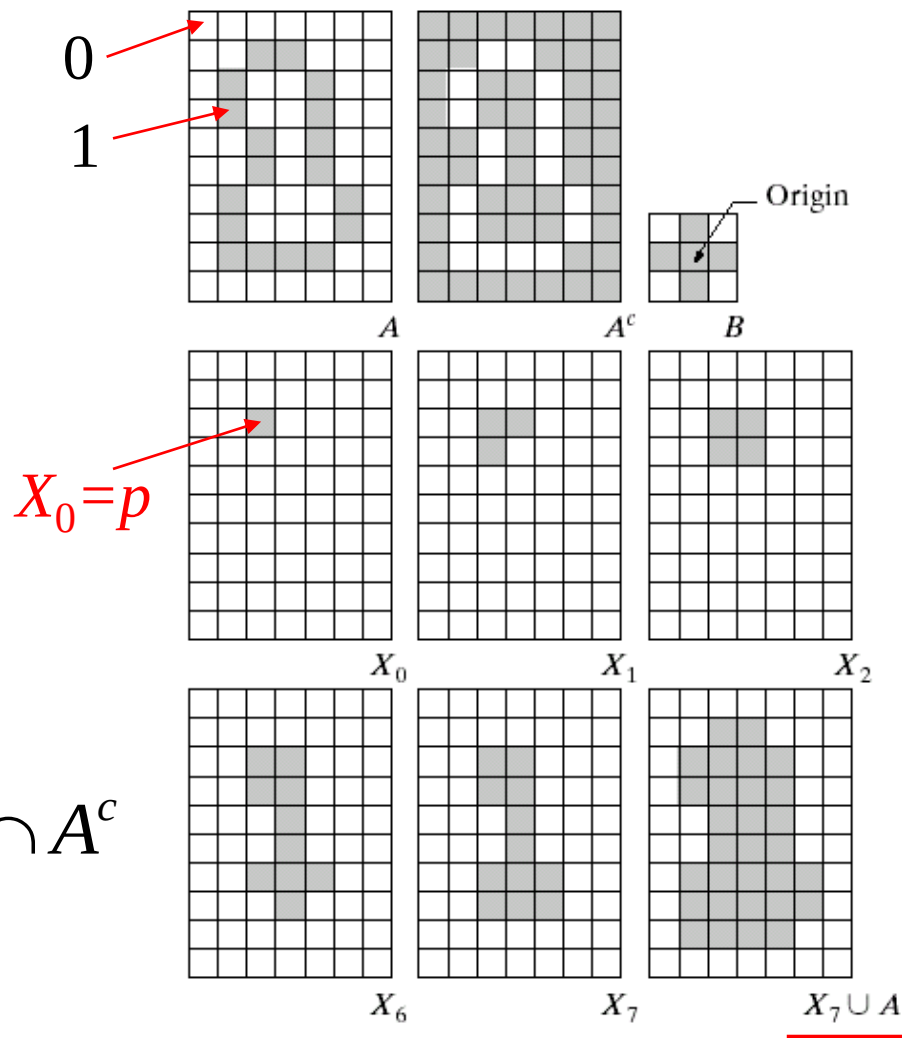
(b) Complement of A .

(c) Structuring element B .

(d) Initial point inside the boundary.

(e)–(h) Various steps of Eq. (9.5-2).

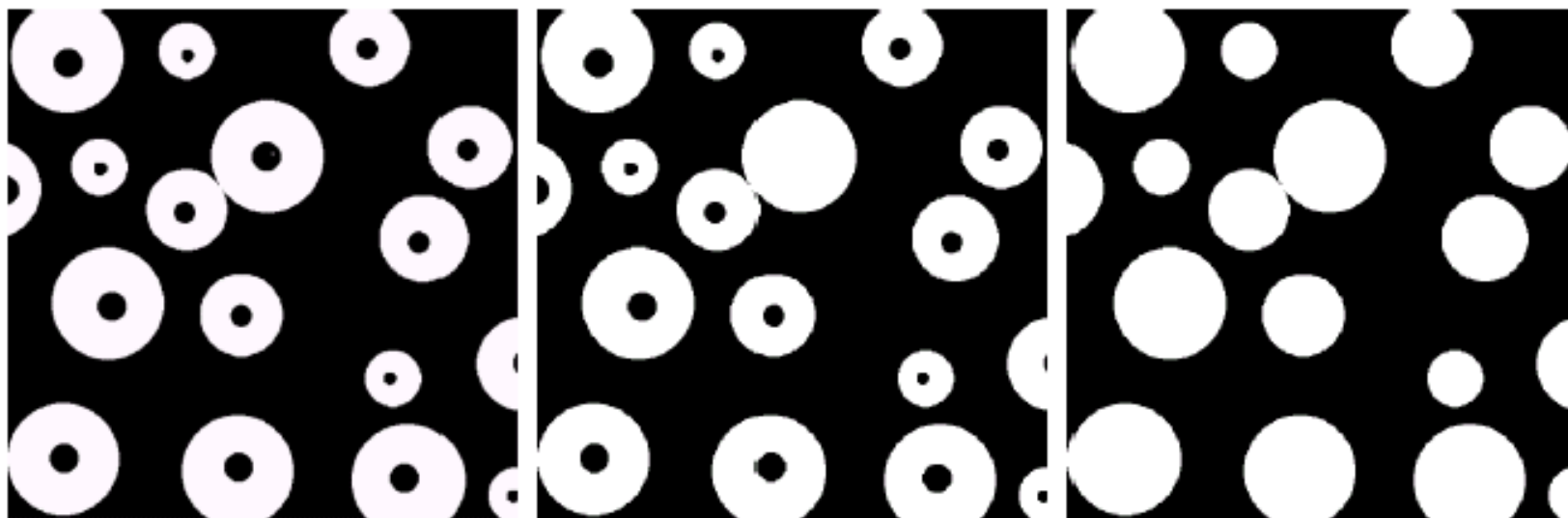
(i) Final result [union of (a) and (h)].



$$X_k = (X_{k-1} \oplus B) \cap A^c$$

Some Basic Morphological Algorithms

Region Filling



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Some Basic Morphological Algorithms

Extraction of Connected Components

$$X_k = (X_{k-1} \oplus B) \cap A$$

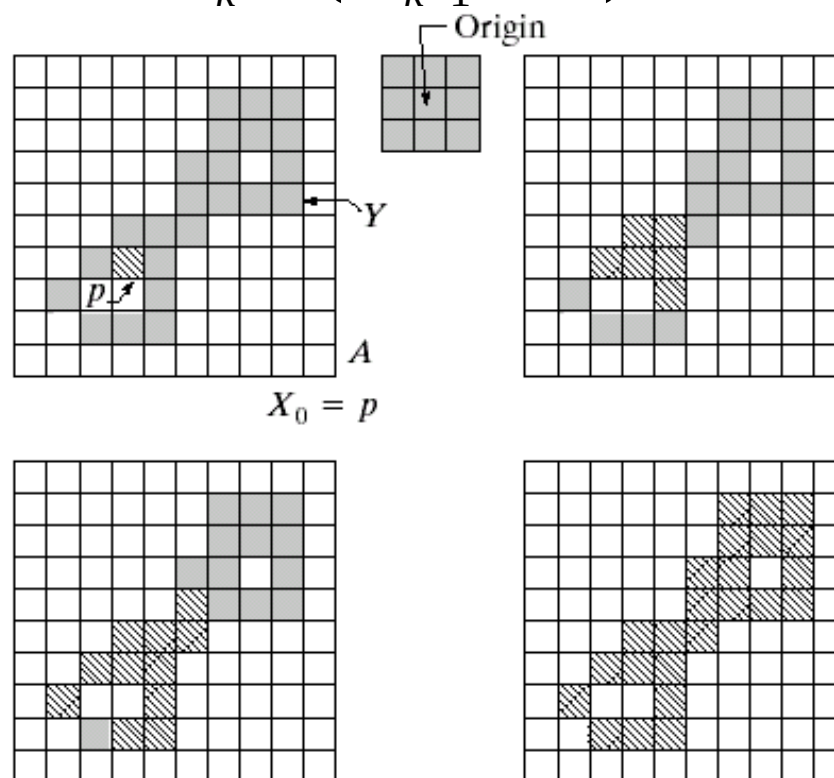


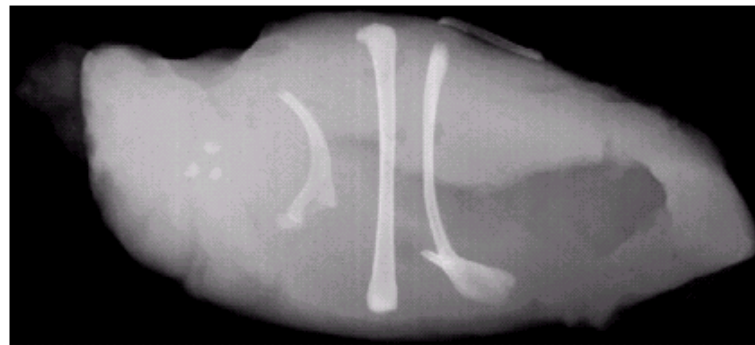
FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

Some Basic Morphological Algorithms Extraction of Connected Components

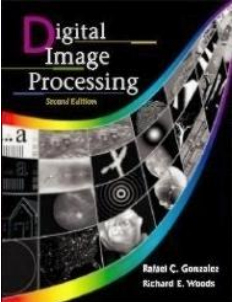
a
b
c d

FIGURE 9.18

(a) X-ray image of chicken filet with bone fragments.
(b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's.
(d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85



Some Basic Morphological Algorithms

Thinning

- The thinning of a set A by a structuring element B can be defined in terms of the hit-or-miss transform:

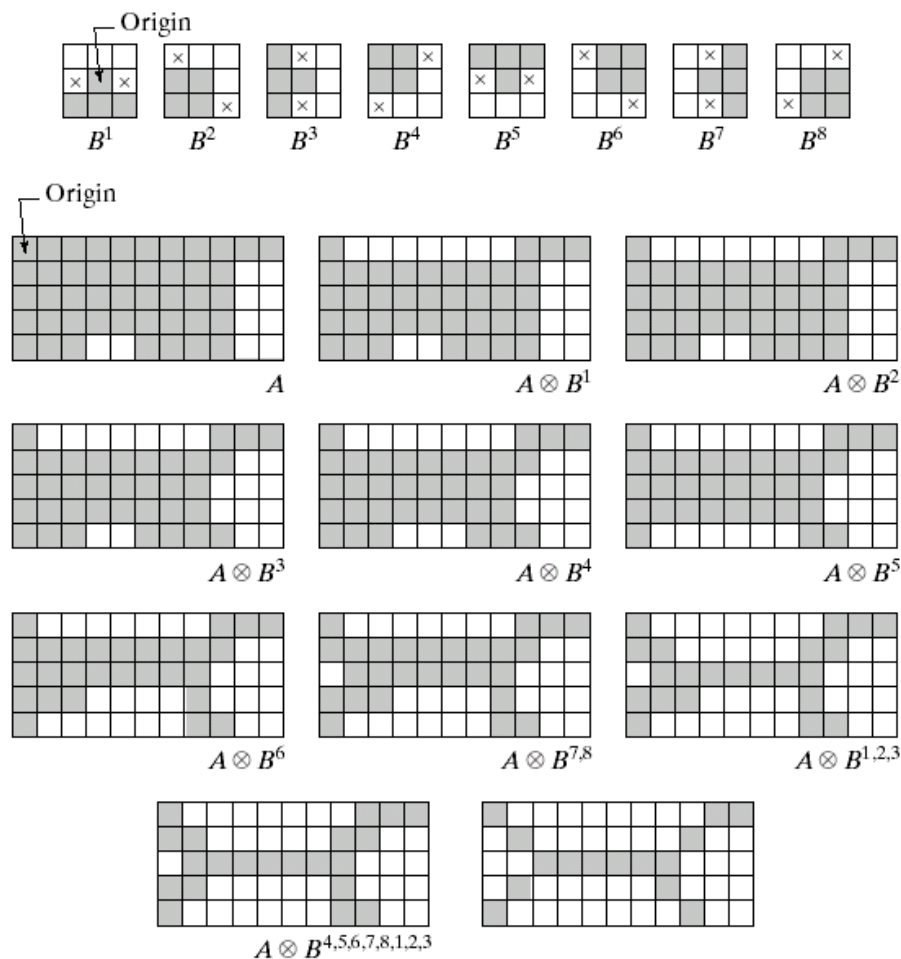
$$\begin{aligned} A \otimes B &= A - (A * B) \\ &= A \cap (A * B)^c \end{aligned}$$

- A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\begin{aligned} \{B\} &= \{B^1, B^2, B^3, \dots, B^n\} \\ A \otimes \{B\} &= (((...((A \otimes B^1) \otimes B^2)...) \otimes B^n) \end{aligned}$$

Some Basic Morphological Algorithms

Thinning



a
b c d
e f g
h i j
k l

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A . (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m -connectivity.