Module 4

PRODUCTION FUNCTION

In this module we will study basic concepts related to industries or production units. Broadly we will examine production function, factors of production, select production related concepts and producers equilibrium.

Factors of Production

By factors of production, we mean those goods and services which is required for production. Factors of production is inevitable for producers to produce their commodities. In economics, there are four major factors of production. ie., Land, Labour, capital and organisation.

Land : Land is an important factor of production. It consists of not only the surface but also all natural resources such as rivers, ocean, climate, mines, fisheries forests, mountains etc.

Labour : Labour refers to all physical and mental works undertaken by someone for money. Works of labourers of a factory, services of professors, doctors, nurses etc. are considered as labour. Any activity performed for no monetary reward is not considered as labour. for example; hobbies, painting etc.

Capital : Capital refers to all manmade resources. It consists of all wealth other than land. It includes all machines, tools, raw materials, means of transportation such as roads, railways, canals etc. In modern times improving knowledge and skills of human resources is considered as capital (human capital). Now - modern firms are gaining more by investing money in human capital.

Organisation : The above three factors of productions are natural (land), human (labour) and material (capital) means of production. Production is possible only when all these factors are brought together and also get paid them their respective reward as follows.

Land ⟶ Rent
Labour ⟶ Wage
Capital ⟶ Interest.

An organisation refers to the services of an entrepreneur who bring all the factors of production together, control them and manages them.

These factors of productions are important for studying production theories, cost theories etc.

Production function

In production function, we study the use of factors of production and production. Factors of production can be considered as the inputs used for the production or output. In other words, the technical relationship between inputs and output is regarded as factors of production.

The production function represents how output changes with respect to changes in inputs. Algebraically, the production function may be expressed as follows.

$$Q = f(L, K, N, O, \bar{T})$$

where Q is the output, L is labour, K is capital, N is land, O is organiser, and T is the technology. We assume that technology is given/constant throughout the analysis.

In general, for the simplicity of studying production function models, economists consider labour and capital as the factors of production. Therefore, the production function can be presented as follows.

$$Q = f(L, K)$$

Select Features of Production (function)

Following are the select features of production/production function

1- Substitutability:

factors of production/inputs are substitutable based on requirement. (agriculture)

2- Complementary:

Inputs can be used together depending on requirements.

3- Production time.

The production process may be based on either short period of time or long period of time.

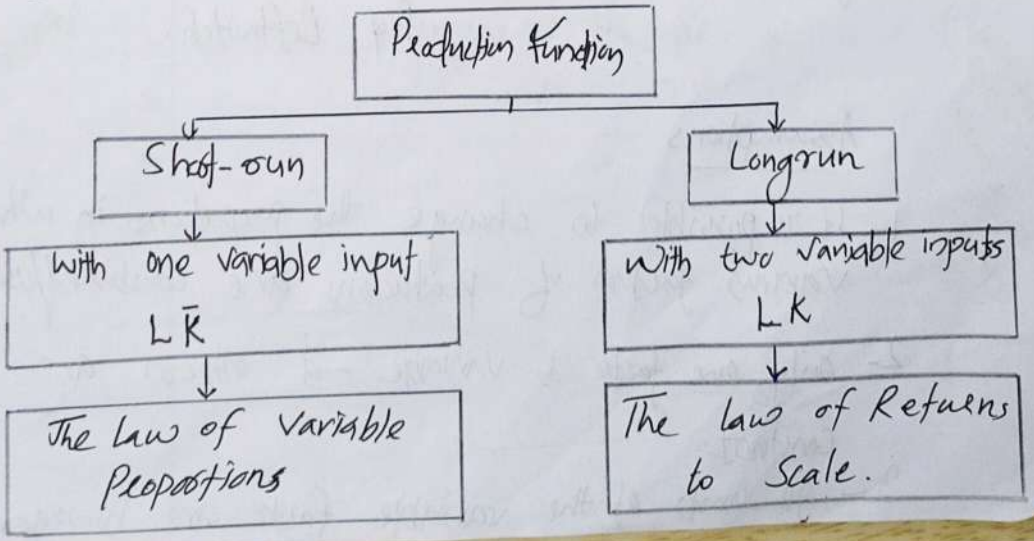
[upcoming discussion]

4- Specificity:

Production function is specific to products. For each specific product, inputs are specific. For another product, the production function may be different. It emphasises that none of the factors of production can be ignored.

Production function and Time

So far, we learned the concept of production function. We also learned that, depending on production requirement, the factors of production may be altered/changed. The possibility of changing a factor input is closely connected with the production time aspect. It is believed that firms can change (vary) only certain factor inputs in the Short-run. For example, to increase or decrease production, firms may change variable inputs like labour. But, in the long run, the firms may change not only factors such as labour, but also factors such as capital. In summary, production function and the association among its variables may be studied based on the possibility of varying along with considering time. A whole summary of the same is shown below.



Production with One Variable Input
[The Law of Variable Proportions]

The law of variable proportions is the case of Production function where we a Consider the possibility of changing one input. This law examines how output changes with respect to the changes in one factor input by keeping all other factors of production as constant. We Consider that to change the output the produces is only changing labour input.

Definition of the law of variable proportions.

"The law of variable Proportion states that if a variable quantity of one resource is applied to a fixed amount of other input, output per unit of variable input will increase, but beyond some point the resulting increases will be less and less, with total output reaching a maximum before it finally begins to decline"

Prof: Leftwitch.

Industrial Unit = 100 cars

Find 20 → Classes.

Assumptions

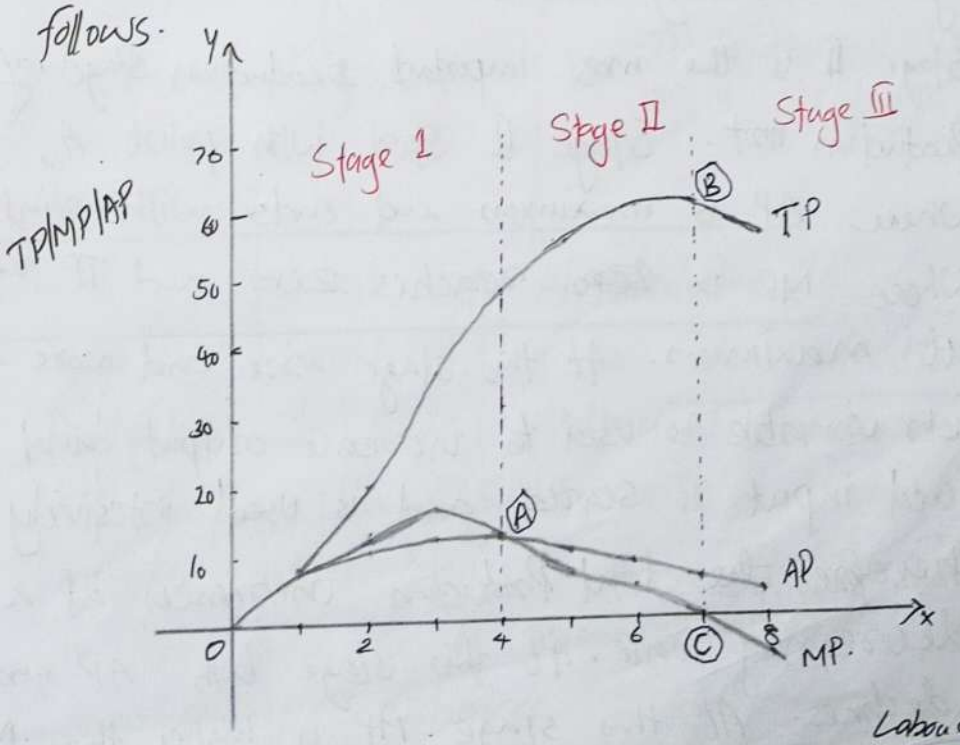
1- It is possible to change the proportions in which various factors of productions are combined/used.
2- Only one factor is variable and others are held constant.
3- All units of the variable factor are homogenous.

4- There is no change in technology.
5- It assumes a Short run Situation.
6- The output is measured in physical unit (eg:- quintals)
7- The price of the product is given and constant.

To illustrate the theory, we consider a firms which is using two factors of production. Labour (L) and capital (K), in which labour is variable and capital is a fixed input.

Capital (K)	Labour (L)	Total Product (TP)	Average Product (AP)	Margind Product (MP)	Stage.
5	1	8	8	8	Stage 1
5	2	20	10	12	
5	3	36	12	16	
5	4	48	12	12	Stage 2
5	5	55	11	7	
5	6	60	10	5	
5	7	60	8.6	0	Stage 3
5	8	56	7	-4	

Data in the above Schedule can be presented graphically as follows.



The above table and figure different stages of production. As the variable input changes the combination of TP, AP, and MP also changed. For a better understanding the Law of variable proportion can be studied in three stages as presented in the table and figure.

Stage I: Increasing Returns.

In stage I average product (AP) reaches at the maximum when 4 variable inputs are used. When average product (AP) is maximum it equals with the MP. It is also noted that the TP increases at a higher rate in the first stage. This is because of the large fixed factor at the beginning. When more variable factor are applied to a fixed factor, the use of fixed factor will be more intensive and therefore production increases substantially. At this stage MP is greater than AP.

Stage II: Decreasing Returns

Stage II is the most important production stage of a production unit. Stage II start with point A, where AP is maximum and ends with point C, where MP is reaches zero and TP reaches at maximum. At this stage more and more factor variable is used to increase output and the fixed input is scarce and is used intensively. Therefore the total Production increases at a decreasing rate. At this stage both AP and MP declines. Al this stage AP is highes than MP.

At stage II, production is feasible and profitable.

Stage III : Negative Marginal Returns

At this stage TP starts declining and MP becomes negative. At this stage, the variable input is too much in relation to the available capital. At this stage production will not take place.

Summary

Stages	TP	MP&AP or TP&MP
Stage I	Increases at an increasing rate	MP > AP
Stage II	Increases at a decreasing rate	AP > MP Both AP and MP - declining
Stage III	Decling	when MP = 0, TP at maximum. TP is declining when MP is negative.

Q → Which is the best stage of production?

@ → Stage II is considered as an ideal situation for production. In Stage I sufficient variable factor has not employed and still there is a possibility to increase output. Stage III is not preferred as there is too much use of variable factor/input and the total output is declining. The producers can produce until MP becomes zero.

The Law of Diminishing Returns

The law of diminishing returns is associated with the second stage of production discussed above. It implies that adding an additional factor of production results in small increase in output.

Production function with Two Variable Inputs
[The Law of Returns to Scale.]

In the long run, both variable and fixed are considered to be variable. To meet a long run change in demand the firm increases their production scale by using more labours, capital etc. The study of long run based production function is called the "Law of returns to scale"

Main assumptions

1- All factors of production (inputs) are variable, but enterprise is fixed.

2- Technological changes are absent.

3- There is perfect competition.

4- The product is measured in quantities.

- Under the law of returns to scale, we examine the change in proportion of inputs and change in the proportion of output. An example is provided below.

Scale of Production	Total Product	Marginal Product	phases.
1 Labour + 2 Capital	8	8	Increasing Returns to Scale
2 Labour + 4 Capital	17	9	
3 Labour + 6 capital	27	10	
4 Labour + 8 capital	38	11	Constant Returns to Scale
5 Labour + 10 capital	49	11	
6 Labour + 12 capital	59	10	Diminishing Returns to Scale.
7 Labour + 14 capital	68	9	
8 Labour + 16 capital	76	8	

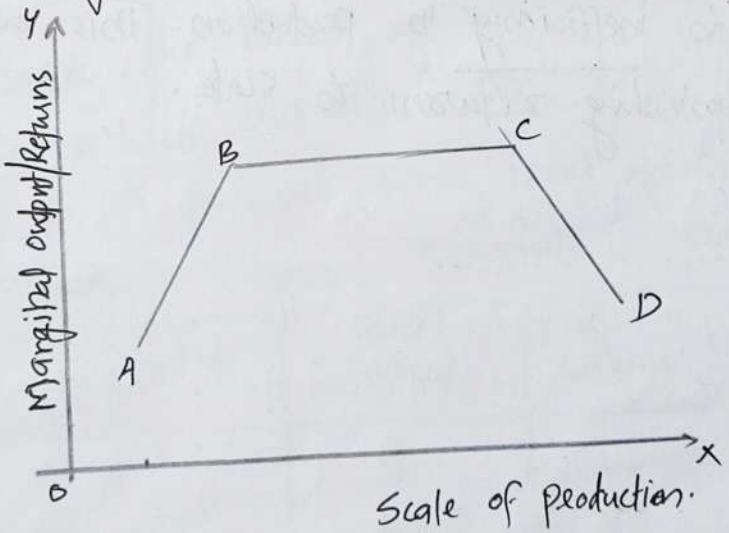
There are three types | phases of returns to scale.

Initially when the firm added more inputs, the return increased at a higher propotion than the proportionate increase in inputs. This is called "increasing returns - to scale". [when output is 8, 7, 27 in the table].

If the inputs are increased further the total output will increase at a constant rate [ie; Marginal product is constant]. This is called "constant returns to scale". [When output is 38 and 49 in the table]

An additional increase in the production will lead to a diminishing return. This is called as diminishing returns to scale.
[when output is 59, 68 or 76].

The concept of returns to scale can be presented - geaphically as follows.



A to B Increasing returns to scale
B to C Constant returns to scale
C to D Diminishing returns to scale.

Why there is increase in returns to scale initially followed by a decline in returns to scale ?.

→ Due to indivisibility of the factors of production. [Indivisibility of machines, finance etc.) When the business expands these indivisible factors will be used extensively. It will lead to increase in returns to scale.. It is also possible due to internal and external economies.

However, this initial advantage of increasing returns to scale do not continue indefinitely. After certain level, adding more inputs will returns a constant level of output. This is due to the possibility of factors of production which are perfectly divisible.

If more and more inputs are applied, which may lead to inefficiency in production. This will lead to diminishing returns to scale.

Concept of Isoquant and Isocost
A Simple Model of Producers Equilibrium

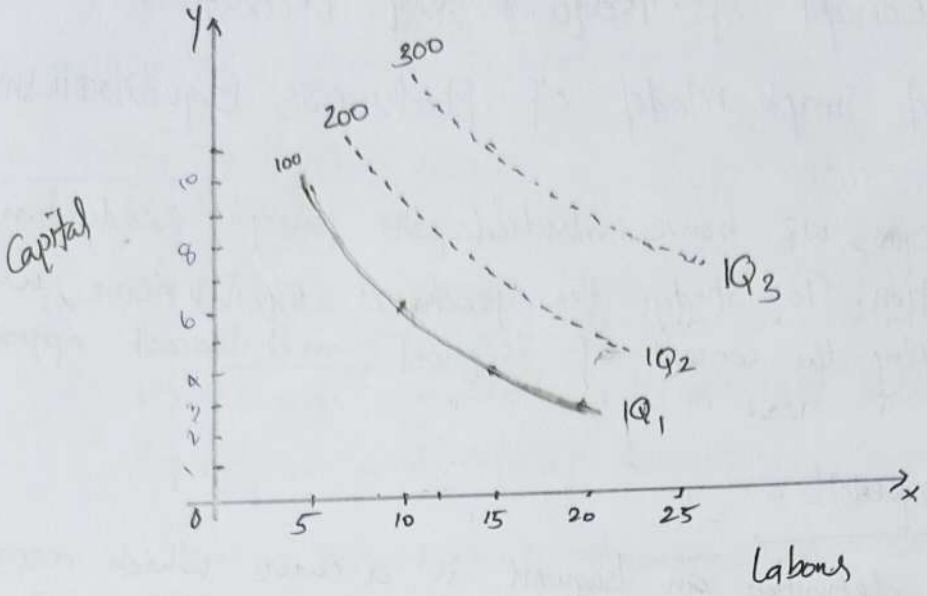
So far, we have studied the concept production function. To study the producers equilibrium, we employ the concept of Isoquant and Isocost approach.

Isoquant

By definition, an Isoquant is a curve which represents various combinations of inputs (labour and capital) that can be used for the production of same level of output. It is also known as a production indifference curve. On a Isoquant, at any point, the rate of production will be constant. Therefore it is also known as constant product curve.

To study the concept of Isoquant, we consider a firm which can use different combinations of inputs to produce same level of commodity. A hypothetical Isoquant schedule is provided below.

Combinations	Units of Capital	Units of Labour	Total output (in units)
A	9	5	100
B	6	10	100
C	4	15	100
D	3	20	100



The above figure presents the isoquant derived based on data from the Isoquant schedule.

The firm can produce 100 units of output on any point on IQ_1. It is to be noted that the firm can use different combinations of labour and capital to produce same 100 units. When the producer use more of labour, they will have to reduce the use of capital.

Isoquant Vs Indifference Curve

In Isoquant, we study about the possible productions of commodities using different factor inputs (two). Indifference curve measures the level of satisfaction from the consumption of different commodities (two).

The fundamental difference is that, in ICs, ~~are not~~ we do not measure satisfaction in a quantitative terms whereas IQs measures/represents a given quantity of output.

Properties of Isoquant

1- IQs are negetively inclined
2- IQs lying above and to the right of another IQ represents a higher output.
3- Two IQs can't touch each other.
4- IQ need not be parallel.
5- There can be many IQs for a producer.
6- Units of output shown on IQs are arbitrary.
7- No IQs can touch either axis.
8- Each IQs is convex to the origin.
9- The area of rational operation of IQs lie between ridge lines.

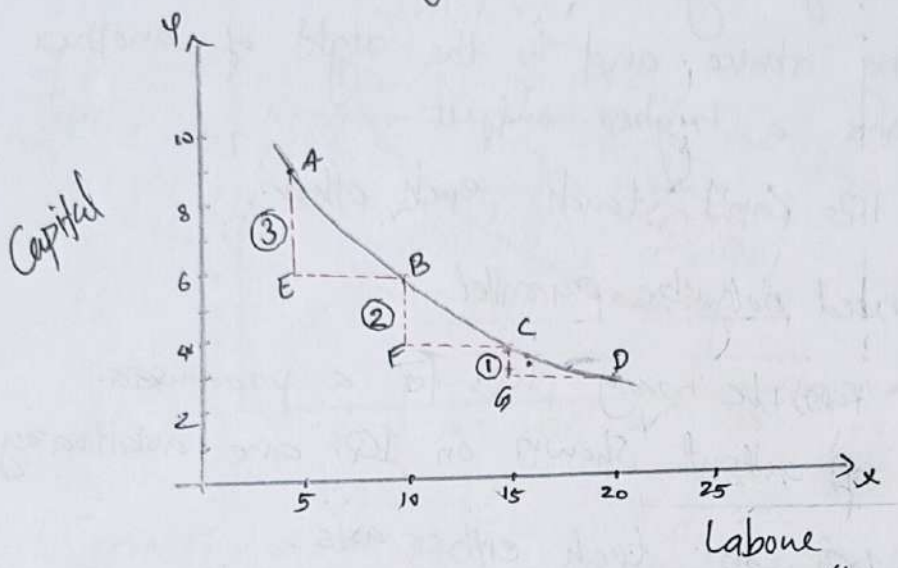
The Principle of Marginal Rate of Technical Substitution [MRTS]

The concept is based on the substitutability of factors of production to produce same level of output. ~~MRTS~~
In simple words, MRTS is ~~less~~ of reduction of one factor (capital) and an addition of the use of the second factor (labour), so that the producer can produce same level of output.

Using the previous IQ schedule MRTS can be derived as follows.

Combination	Labour	capital	MRTS	output
A	5	9	-	100
B	10	6	3:5	100
C	15	4	2:5	100
D	20	3	1:5	100

A graphical illustration of MRTS based on the above IQ schedule is given below.



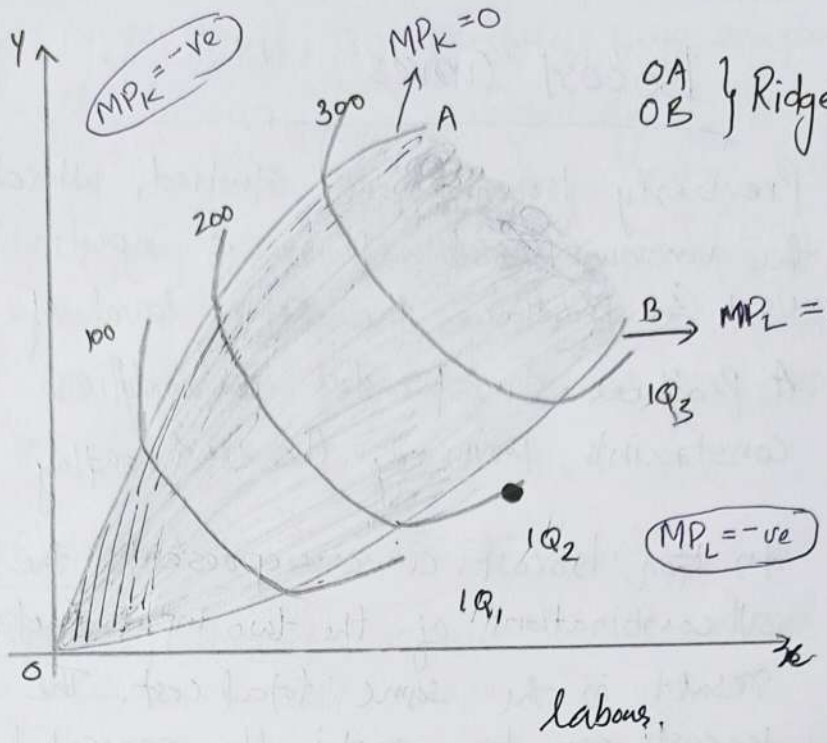
For each additional use of labour the substitution of capital declined.

$AE > BF > CG$

- It is diminishing marginal rate of technical substitution (DMRTS).

Concept of Ridge line & Iso quant.

As previously learned, Isoquant show the commodities/output produced using diffeant combinations of inputs. The concept of ridge line can be used to understand the economic efficient points on the IQ curves which the producers are likely to be preffered. The following figure summarises the ~~IQs and~~ feasible area of economic production using IQs and Ridge lines.



OA } Ridge lines.
OB }

In the above figure, IQ_1 represents the possibility of using different combinations of labour and capital to produce 100 units of commodities. IQ_2 and IQ_3 presents higher levels of output production using different combinations of labour and capital.

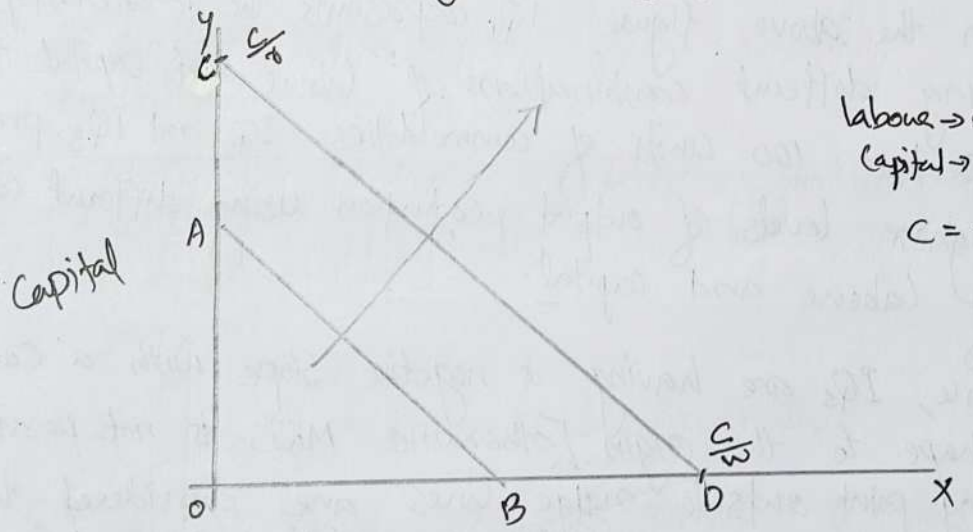
Since, IQs are having a negative slope with a convex shape to the origin. [otherwise MRTS is not possible]. any point outside ridge lines are considered to be inefficient. Ridge lines are constructed based on the locus of points of IQs, where the marginal product of factors are zero. Any point outside ridgeline will violate MRTS. Also any point outside ridgeline in an unnecessary use of factors (since the MP becomes negative). Any point outside ridge lines is therefor called as region of economic nonsense. meaning that it is technically inefficient.

Any points between ridge lines is called as the economic region of production.

Isocost Curves

Previously Isoquants are studied; which represents the various combinations of inputs that can be used to produce the same level of output.
A producer can produce commodities given the constraints, primarily the total outlay for production.

An ~~Isoq~~ Isocost curve represents the locus of all combinations of the two input factors which result in the same total cost. The concept of Isocost can be graphically presented as follows.



labour → wage (w)
capital → interest (r)
C = Cost.

Based on the above figure, Isocost curves AB and CD are different constraints of production for the producer.
On Isocost, By Spending available outlay (Total Cost of factors of production) the producer can employ different combinations of inputs (labour/capital).
We consider Isocost curve CD, if the producer use only labour by Spending entire cost (C)

on labours (wage - (w)), the producer can employ 'OD' levels of labour $\left(\frac{C}{w}\right)$. Similarly if the producer employs only capital by spending entire income, he can employ 'OC' levels of capital $\left(\frac{C}{r}\right)$. The producer can also spend entire cost and utilise different combinations ~~possi~~ of labour and capital possible on the Isocost curve.

Slope of Isocost Curve

The Slope of Isocost curve will be the the - following :-

$$\text{Slope of } CD = \frac{OC}{OD}$$

$$OC = \frac{C}{r}$$

$$OD = \frac{C}{w}$$

$$= \frac{\frac{C}{r}}{\frac{C}{w}}$$

$$= \frac{C}{r} \times \frac{w}{C}$$

$$= \frac{w}{r}$$

∴ The Slope of Isocost ~~curve~~ line is the ratio of prices of labours and capital.

A highes Isocost curve represents highes cost/production [i.e, Expansion of production].

→ What happen when prices of factors of production change?

Producer's Equilibrium

For a producer, there are two aspects when it comes to equilibrium analysis. On the side the producer has cost of production for to be paid for the factors of production, on the other hand, the producer earns profits out of production.

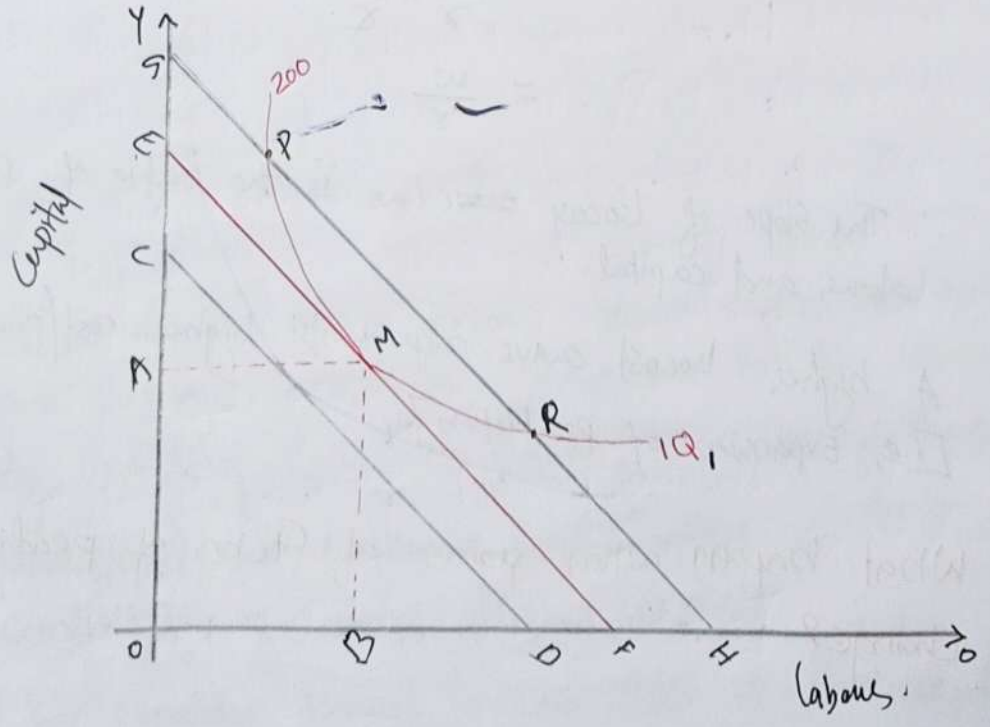
At equilibrium the producer may try to achieve any of the following.

i - Try to minimise cost for a given output.

ii - Try to maximise output for a given cost.

We will analyse these cases seperately.

To study the producer's equilibrium, we consider both Isoquant (output level) and Isocost (cost level/constraint).

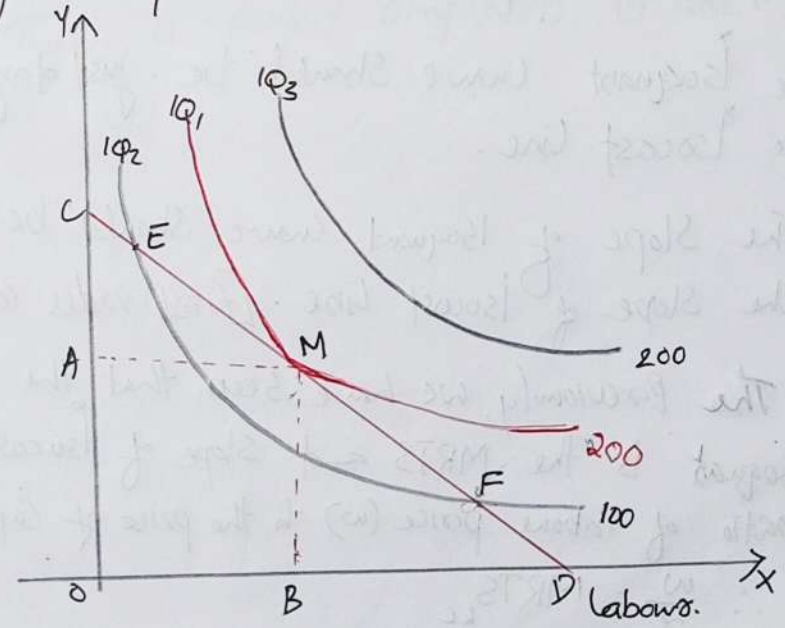


In the above figure, producer reach at equilibrium at point M, where he utilises OA and OB units of capital and labour, respectively, to produce 200 units of output.

The producer minimises costs at the Isocost line EF. Points such as and P and R are not a good choice through it lies on the same IQ. Because, production at point P and R is possible at a higher cost [since Isocost line GH > Isocost line EF].

Here the producer is minimising costs for a given output.

In a similar manner, the output maximisation for a given input (cost) may be presented as follows.



In the above figure, producer reach at equilibrium at point M on the Isocost curve IQ_1. He utilises OB and OA quantities of labour and capital, respectively, to produce 200 units of output.

In the above figure at, point M, the producer Maximises output on Isocost line CD. Any point on the Isocost line on a lower Isoquant curve such as points E and F is not suggested. Because using the same inputs (costs) the producer could have produced more quantity of output.

> Here the producer is maximising output for a given input/cost.

Equilibrium Conditions

In summary, the following conditions are necessary for producer's equilibrium.

→ The Isoquant curve should be just tangent on the Isocost line.

↳ The Slope of Isoquant curve should be equal to the Slope of Isocost line. [first order condition]

Previously, we have seen that the slope of Isoquant is the MRTS and slope of Isocost is the ratio of labour price (w) to the price of capital (r)

$$\therefore \frac{w}{r} = MRTS_{LC}$$

↳ At the point of tangency between Isocost and isoquant, the isoquant curve must be convex to the origin. [Second order condition]

Assumptions of the producers Equilibrium analysis

[To be read at the beginning]

The analysis of producers equilibrium is based on the following examples.

1- There are two factors, labour and capital.
2- All units of labour and capital are homogeneous.
3- The price of units of labour (w) and capital (r) are given and constant.
4- The total outlay is given.
5- The firm produces a single output.
6- Price of product is given and constant.
7- The firm aims at profit maximisation.
8- There is perfect competition in the market.