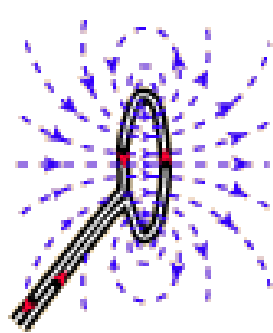


# **Magnetic Circuits**

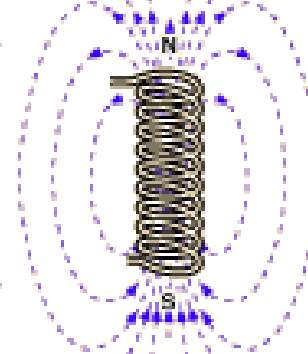
# SOURCES



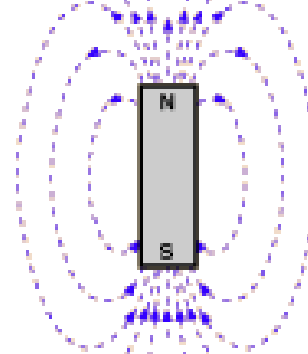
Current  
in wire



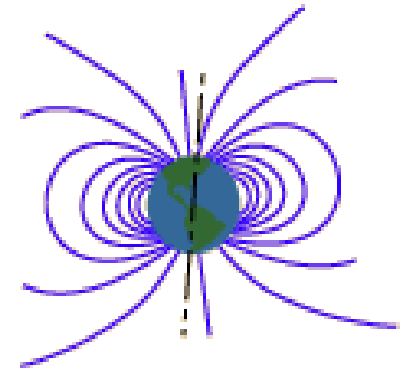
Loop of  
wire



Solenoid



Bar Magnet



The Earth

Magnetic Field Sources

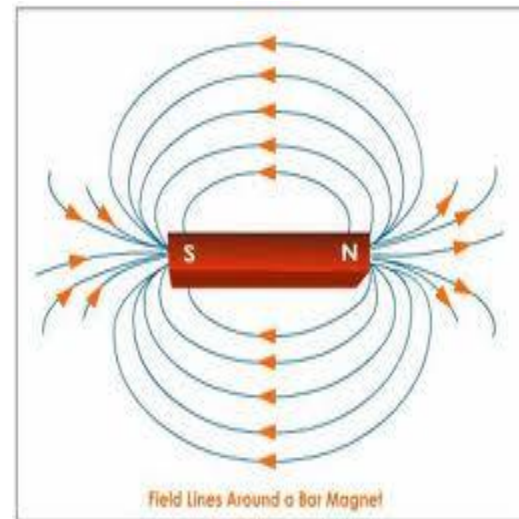
# Introduction

## Magnetic lines of force:

Closed path radiating from north pole, passes through the surrounding, terminates at south pole and is from south to north pole within the body of the magnet.

## Properties:

- Each line forms a closed loop and never intersect each other.
- Lines are like stretched elastic cords, always trying to shorten themselves.
- Lines of force which are parallel and in the same direction repel each other.



# Magnetic Field

The magnetic field lines around a long wire which carries an electric current form concentric circles around the wire. The direction of the magnetic field is perpendicular to the wire and is in the direction the fingers of your right hand would curl if you wrapped them around the wire with your thumb in the direction of the current.

## Magnetic Materials

### Properties:

- Points in the direction of geometric north and south pole when suspended freely and attracts iron fillings.

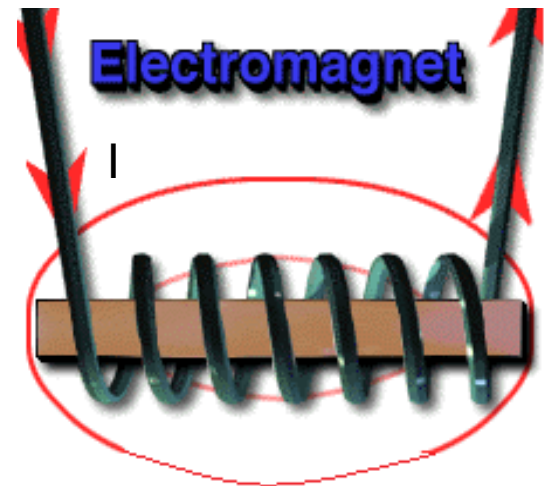
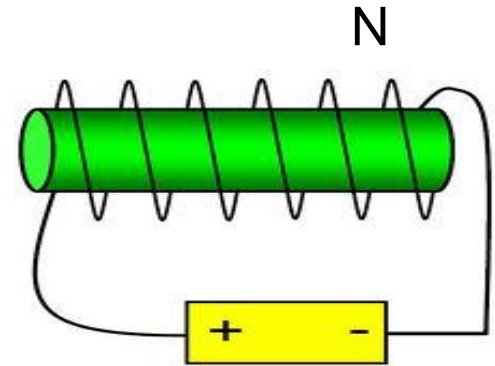
### Classification :

- Natural Magnets
- Temporary magnets (exhibits these properties when subjected to external force)
- Non-magnetic materials.

# Electromagnets:

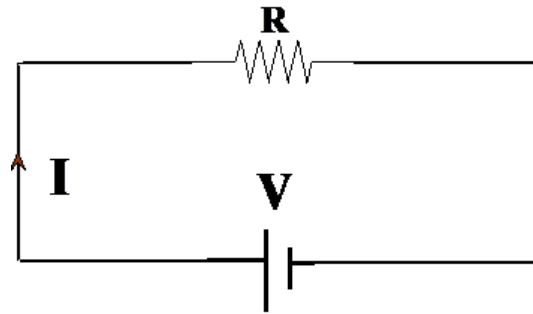
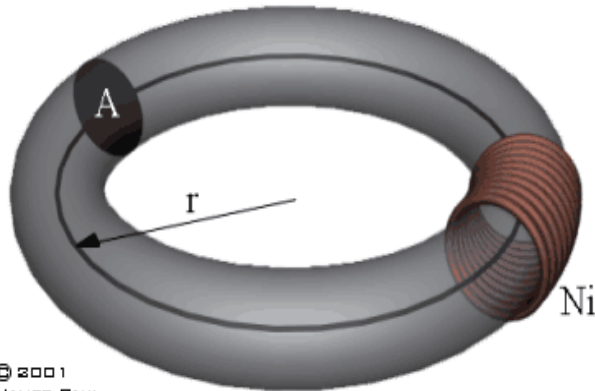
**Principle:** An electric current flowing in a conductor creates a magnetic field around it.

- Strength of the field is proportional to the amount of current in the coil.
- The field disappears when the current is turned off.
- A simple electromagnet consists of a coil of insulated wire wrapped around an iron core.
- Widely used as components of motors, generators, relays etc.



# Magnetic circuit

The complete closed path followed by any group of magnetic lines of flux



Equivalent electrical circuit

## Basic Definitions

### Magneto Motive Force, MMF ( $F$ )

- Force which drives the magnetic lines of force through a magnetic circuit
- $MMF, F = \Phi S$ , where 'Φ' is the magnetic flux and 'S' is the Reluctance of the magnetic path.  
*Analogy: EMF,  $V=IR$*
- Also, For Electromagnets:  
 $MMF = NI$  (No. of turns\*Current),  
where N is the number of turns of the coil and I is the current flowing in the coil
- Unit:  $AT$  (Ampere Turns)

## Magnetic flux ( $\Phi$ ):

- Number of magnetic lines of force created in a magnetic circuit.

Analogy: Electric Current, I

- Unit : *Weber (Wb)*

## Reluctance [S]

- Opposition of a magnetic circuit to the setting up of magnetic flux in it.

$$\text{Flux} = \phi = BA; F = mmf = Hl; B = \mu H$$

$$\frac{\phi}{F} = \frac{BA}{Hl} = \frac{\mu_0 \mu_r A}{l}; \text{ Hence } \phi = \left( \frac{\mu_0 \mu_r A}{l} \right) F$$

$$\phi = \frac{F}{\left( \frac{l}{\mu_0 \mu_r A} \right)} = \frac{F}{S}; \text{ where } S = \left( \frac{l}{\mu_0 \mu_r A} \right)$$

- $S = F/\phi$
- Unit:  $AT/Wb$

Analogy: Resistance

## Magnetic Flux Density ( $B$ ):

- No. of magnetic lines of force created in a magnetic circuit per unit area normal to the direction of flux lines
- $B = \Phi/A$
- Unit : *Weber/m<sup>2</sup> (Tesla)*

Analogy: Current Density

## Magnetic Field Strength ( $H$ )

- The magneto motive force per meter length of the magnetic circuit
- $H = (NI) / l$  Unit : *AT / meter*

Analogy: Electric field strength

## Permeability ( $\mu$ )

- A property of a magnetic material which indicates the ability of magnetic circuit to carry magnetic flux.
- $\mu = B / H$
- Unit: *Henry / meter*
- Permeability of free space or air or non magnetic material  $\mu_0 = 4 * \Pi * 10^{-7}$  *Henry/m*
- Relative permeability,  $\mu_r : \mu/\mu_0$

Analogy: Conductivity

# Magnetic circuit

## Analogy with Electric circuits

### Similarities:

Electric circuit		Magnetic circuit	
<b>Quantity</b>	<b>Unit</b>	<b>Quantity</b>	<b>Unit</b>
<b>EMF (<math>E=IR</math>)</b>	<b>Volt (V)</b>	<b>MMF (<math>F=\phi S</math>)</b>	<b>Ampere-turns</b>
<b>Current (I)</b>	<b>Ampere (A)</b>	<b>Flux (<math>\phi</math>)</b>	<b>Weber (Wb)</b>
<b>Current density (J)</b>	<b>A/ m<sup>2</sup></b>	<b>Flux density (B)</b>	<b>Wb / m<sup>2</sup> or Tesla</b>
<b>Resistance (R)</b>	<b>Ohm (<math>\Omega</math>)</b>	<b>Reluctance (S)</b>	<b>Ampere-turns/Wb</b>
<b>Electric field strength (E)</b>	<b>Volts/m</b>	<b>Magnetic field strength (H)</b>	<b>Ampere-turns/m</b>
<b>Conductivity (<math>\sigma</math>)</b> <b><math>\sigma=l/RA</math></b>	<b>Siemen/m</b>	<b>Permeability, <math>\mu</math></b> <b><math>\mu=l/SA</math></b>	<b>Henry/m</b>

'l' is the length and 'A' is the area of cross section of the conductor

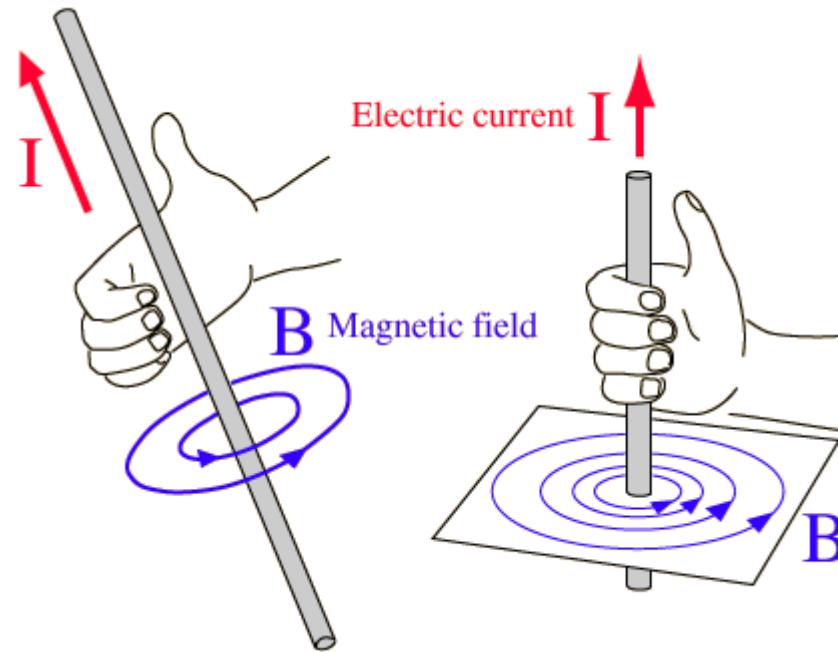
# Differences between electric and magnetic circuits

- In electrical circuit current actually flows.
- In magnetic circuit flux is created, and it is not a flow.

## Summary

- Current flowing in a conductor creates a magnetic field around it.
- The complete closed path followed by any group of magnetic lines of force is termed as magnetic circuit.
- The characteristics of magnetic circuits are analogous with that of electric circuits.

# MAGNETIC FIELD DUE TO ELECTRIC CURRENT



Right Hand Thumb Rule OR RH Screw rule

# FORCE ON A CURRENT CARRYING CONDUCTOR

**Fleming's left hand rule** (for [electric motors](#)) shows the direction of the [thrust](#) on a [conductor](#) carrying a [current](#) in a [magnetic field](#). The left hand is held with the [thumb](#), [index finger](#) and [middle finger](#) mutually at [right angles](#).

The **F**irst finger represents the direction of the **F**ield.

The **S**econd finger represents the direction of the **C**urrent [conventional current, positive(+) to negative(-)].

The **T**humb represents the direction of the **T**hrust or resultant **M**otion.

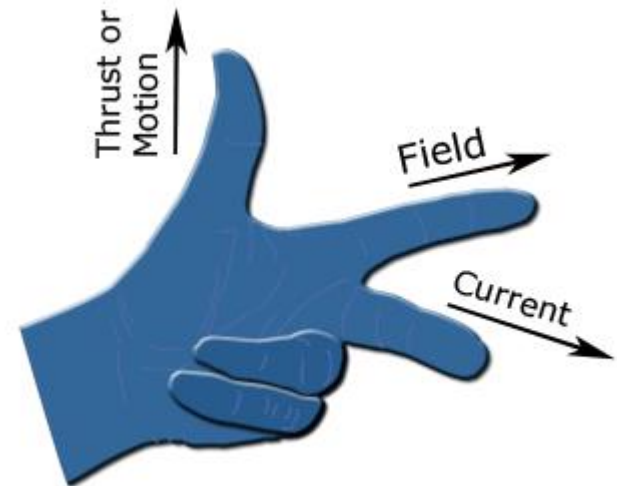
This can also be remembered using "FBI" and moving from thumb to second finger.

The thumb is the force **F**

The first finger is the magnetic field **B**

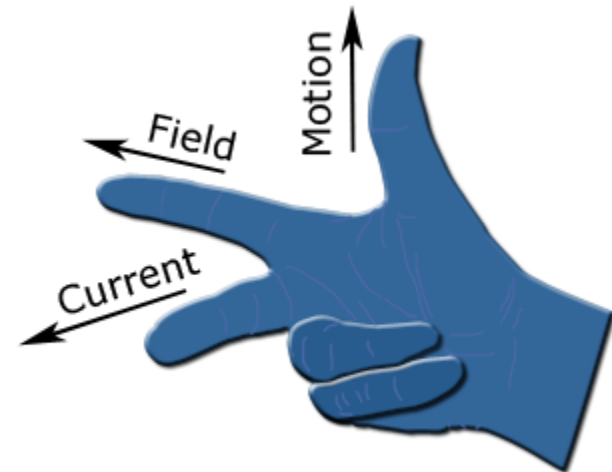
The second finger is the of current **I**

$$F = BIL$$



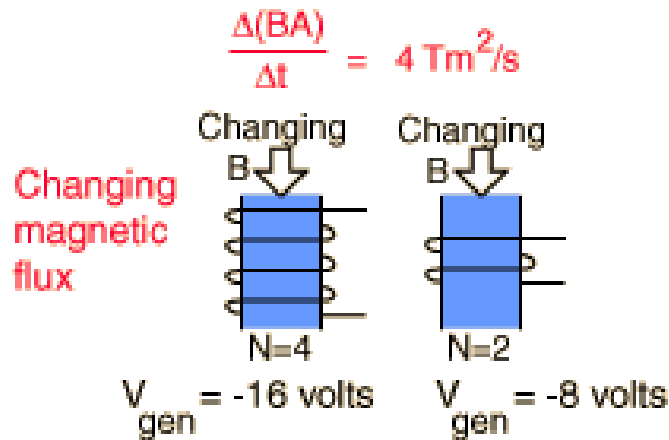
# Electromagnetic Induction

- **Fleming's right hand rule** (for generators) shows the direction of induced current flow when a conductor moves in a magnetic field.
- The right hand is held with the thumb, first finger and second finger mutually perpendicular to each other {at right angles}, as shown in the diagram .
- The **Thumb** represents the direction of **Motion** of the conductor.
- The **First** finger represents the direction of the **Field**.
- The **Second** finger represents the direction of the induced or generated **Current** (in the classical direction, from positive to negative).



# Faraday's Law

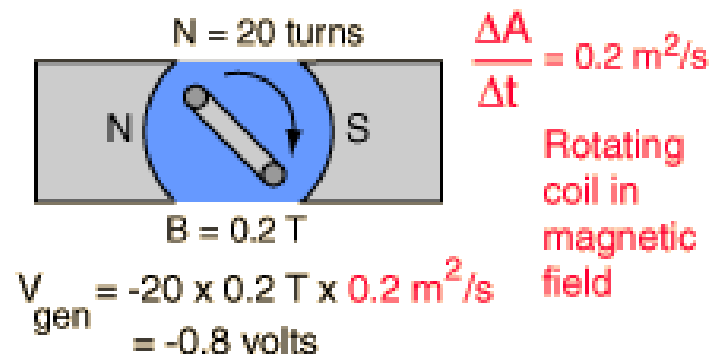
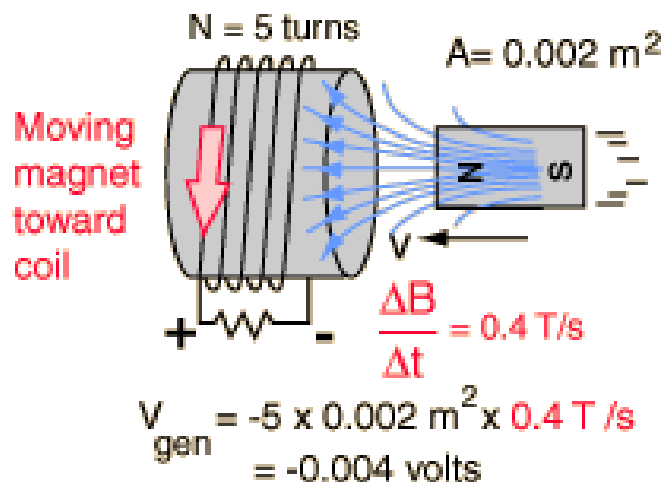
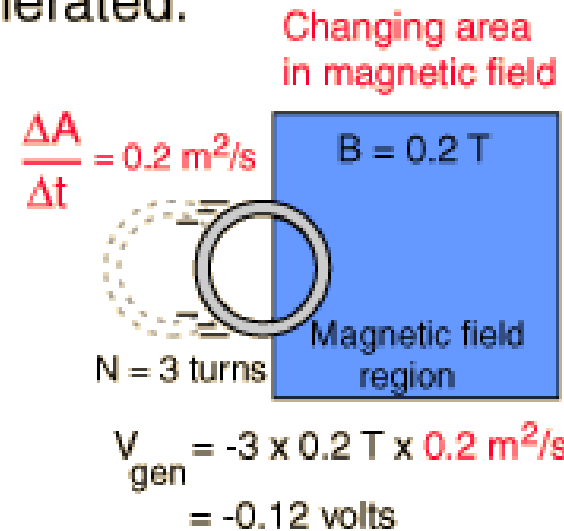
- Any change in the magnetic environment of a coil of wire will cause a voltage (emf) to be "induced" in the coil. No matter how the change is produced, the voltage will be generated. The change could be produced by changing the magnetic field strength, moving a magnet toward or away from the coil, moving the coil into or out of the magnetic field, rotating the coil relative to the magnet, etc.



Faraday's Law summarizes the ways voltage can be generated.

Voltage generated =  $-N \frac{\Delta(BA)}{\Delta t}$

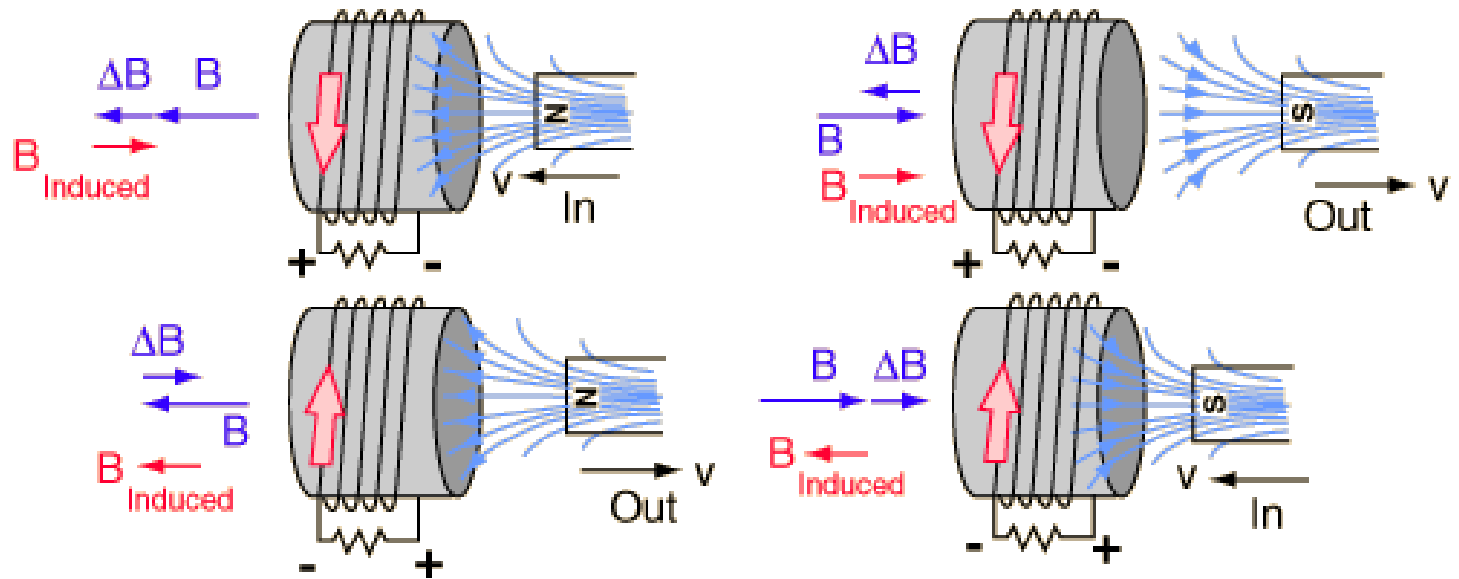
Faraday's Law



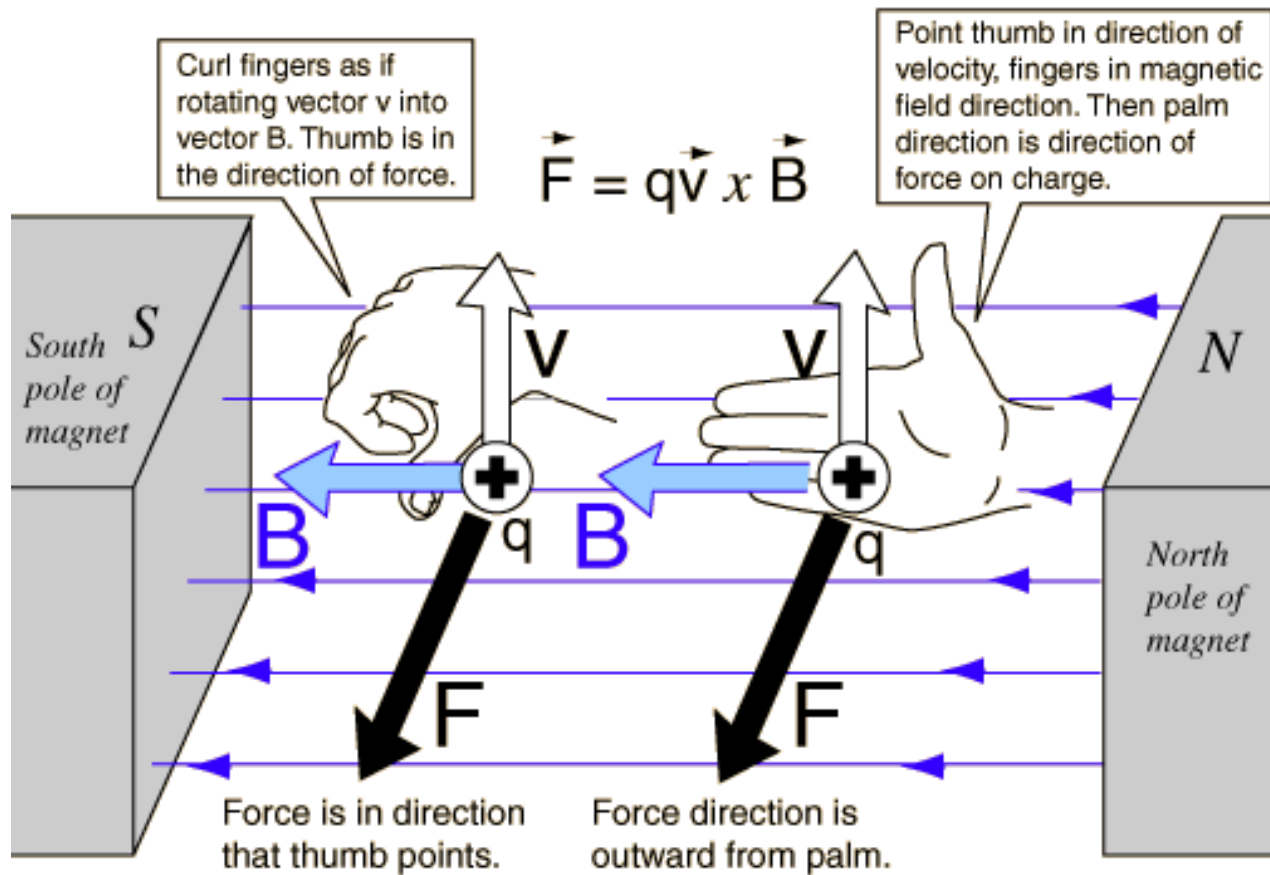
# Lenz's Law

- When an emf is generated by a change in magnetic flux according to [Faraday's Law](#), the polarity of the induced emf is such that it produces a current whose magnetic field opposes the change which produces it. The induced magnetic field inside any loop of wire always acts to keep the magnetic flux in the loop constant. For examples, if the B field is increasing, the induced field acts in opposition to it. If it is decreasing, the induced field acts in the direction of the applied field to try to keep it constant.

# Lenz's Law



# Lorentz Force



# Lorentz Force Law

- Both the electric field and magnetic field can be defined from the Lorentz force law:
- The electric force is straightforward, being in the direction of the electric field if the charge  $q$  is positive, but the direction of the magnetic part of the force is given by the right hand rule.

# SUMMARY

1. A magnetic field is described by using lines of flux . Such lines form closed loops, they do not cross and when in parallel repel each other.
2. Magnetic fields have North and South Poles. Like Poles repel and Unlike poles attract one another.
3. The relative direction of the field, force and current are given by the **Fleming's** Left Hand rule.
4. When the Magnetic flux linking a circuit is varied, an e.m.f. is induced in the circuit. This is known as Faraday's Law.
5. The induced e.m.f. opposes the change of condition. This is known as Lenz's Law.
6. The relative directions of the field , motion and induced e.mf. Are given by the **Fleming's** Right Hand rule.

# FORMULAE

Force on a conductor  $F = BIL$  ( newtons)

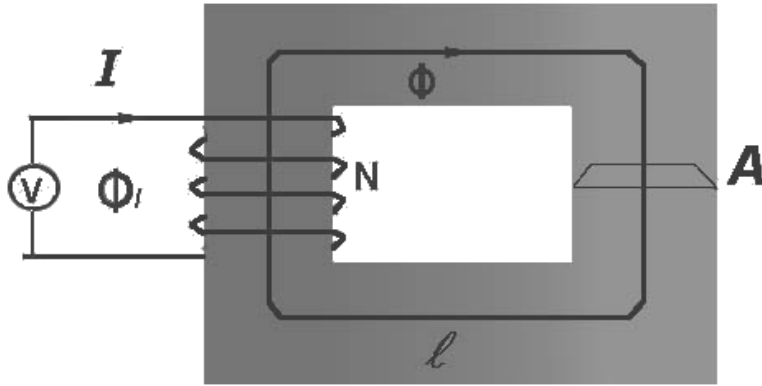
Flux  $\Phi = B A$  ( webers)

Flux density  $B = \Phi / A$  (teslas)

Induced e.m.f  $E = BLu$  (volts)

$$e = d \Phi / dt \quad \text{or} \\ = d/dt( N \Phi)$$

# Magnetic Field Relationship



$$\oint H \cdot dl = NI \quad \text{where} \quad \begin{array}{l} H = \text{Field intensity (A/m)} \\ N = \text{No. of winding turn} \end{array}$$

$$\mu = \text{permeability} = \mu_r \mu_0$$

$$H = \frac{B}{\mu}$$

$B$  = Flux density (Web<sup>2</sup> / m )

$\mu_r$  = Relative permeability

$$\mu_0 = 4\pi \times 10^{-7} \text{ (H / m)}$$

$$\phi = \int_s B \cdot ds$$

$$= B \cdot A$$

Assume

$\Phi_\ell$  : leakage = 0

$\Phi$  : flux linkage

A : cross section area

$$H\ell = \text{mmf}$$

mmf = magnetomotive force

$$\frac{B}{\mu} \ell = \text{mmf}$$

$$B = \frac{\phi}{A} \quad \frac{\phi}{A} \cdot \frac{\ell}{\mu} = \text{mmf}$$

$$\phi \cdot \frac{\ell}{\mu \cdot A} = \text{mmf}$$

$$\mathcal{R} = \frac{\ell}{\mu \cdot A}$$

Reluctance

$$\phi \cdot \mathcal{R} = mmf$$

Can be represented as:



$$\mathcal{R}_1 = \ell_1 / \mu A_1 \quad \mathcal{R}_2 = \ell_2 / \mu A_2$$

$$\mathcal{R}_3 = \ell_3 / \mu A_3$$

For reluctance in series :



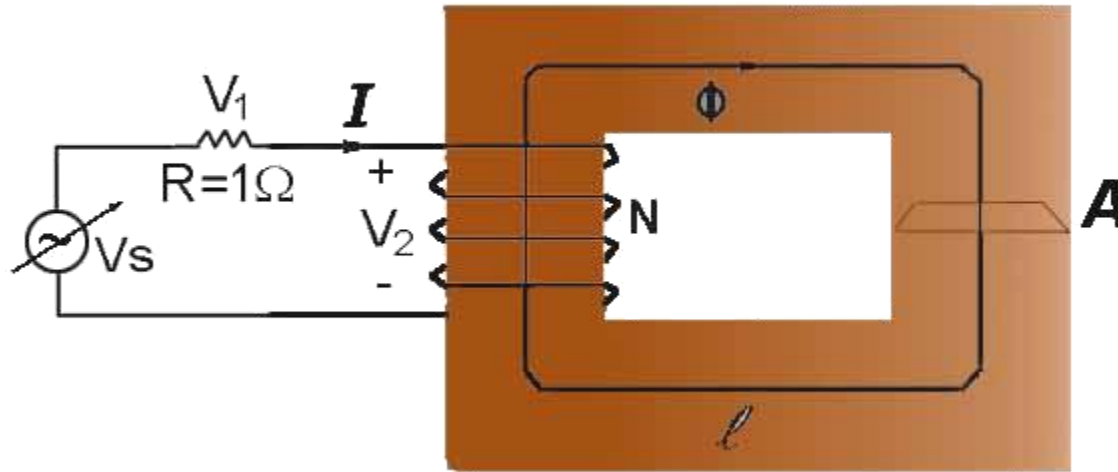
$$\mathcal{R}_{eq} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3$$

For reluctance in parallel :



$$\frac{1}{\mathcal{R}_{eq}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3}$$

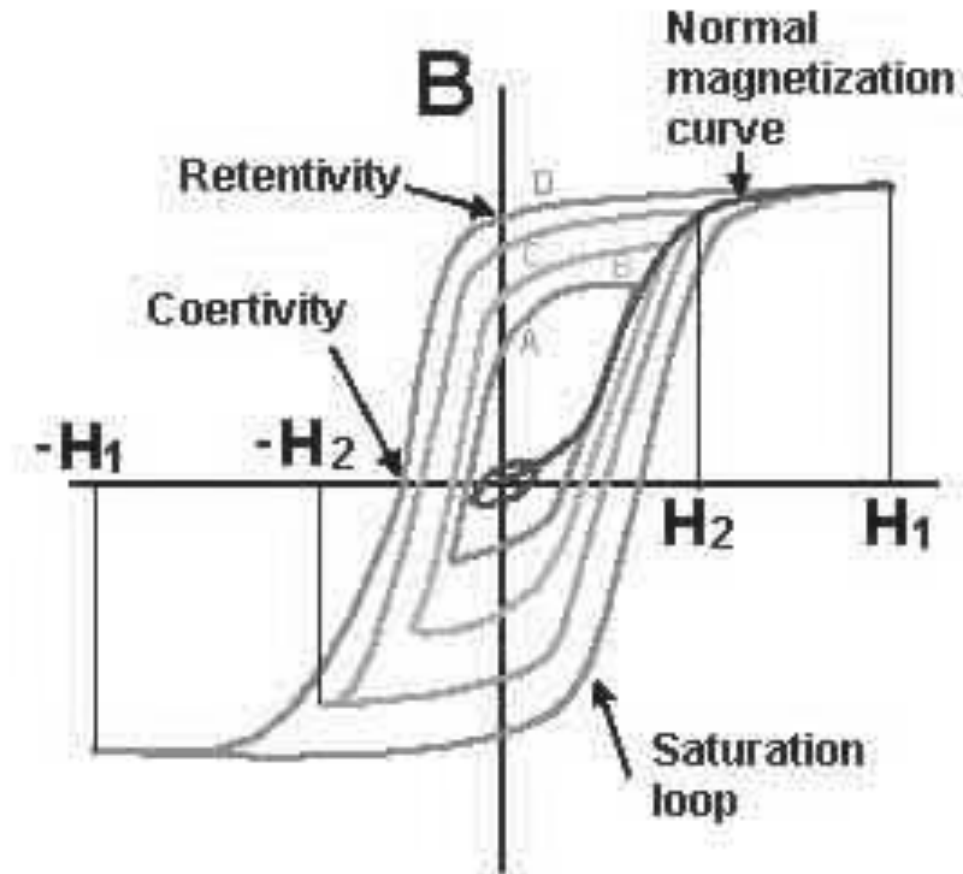
# Magnetization Curve



Measure  $v_1$  and  $v_2$ :

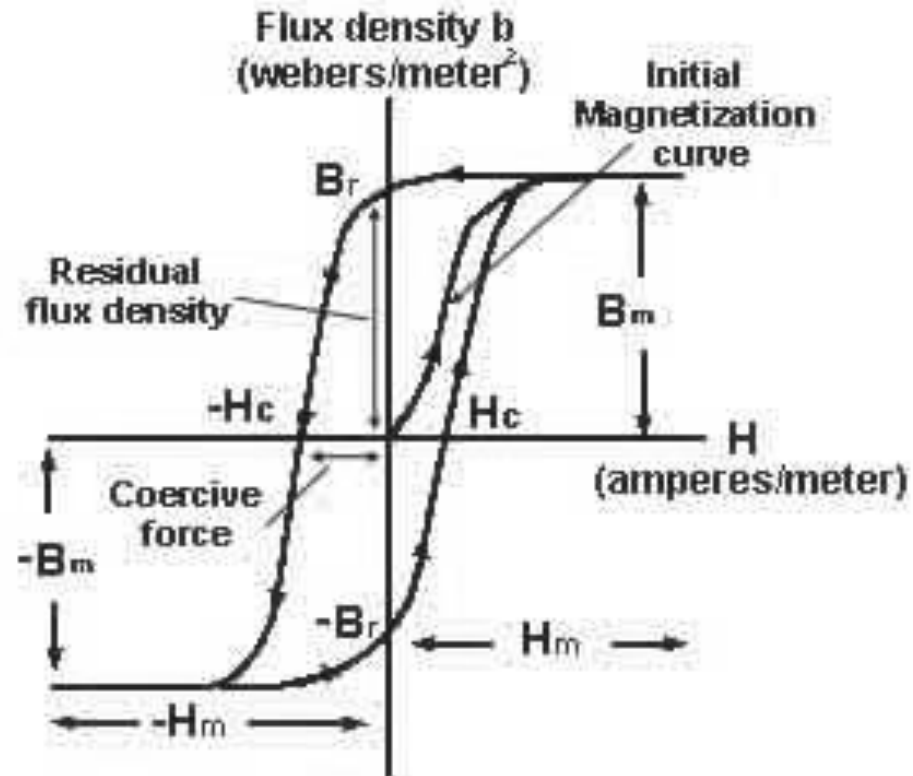
$$v_2 = \frac{d\phi}{dt}$$
$$\phi = \int_v^t v_2 \cdot dt = \int B \cdot ds$$

# Hysteresis



The phenomenon that causes  $B$  to lag behind the applied  $H$  for a ferromagnetic material is referred to as **Hysteresis**.

# Hysteresis Loop

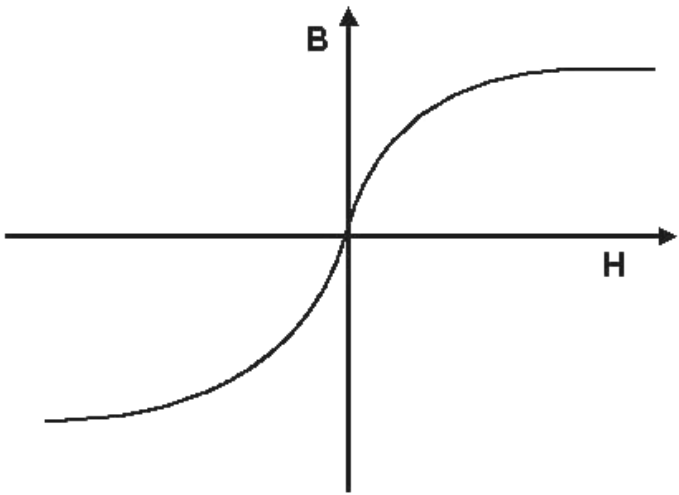


When  $H$  reaches zero, there is a residual flux density or **remanence  $B_r$** .

In order to reduce  $B$  to Zero, a negative field,  $H_c$  must be applied. This is called the **coercive force**.

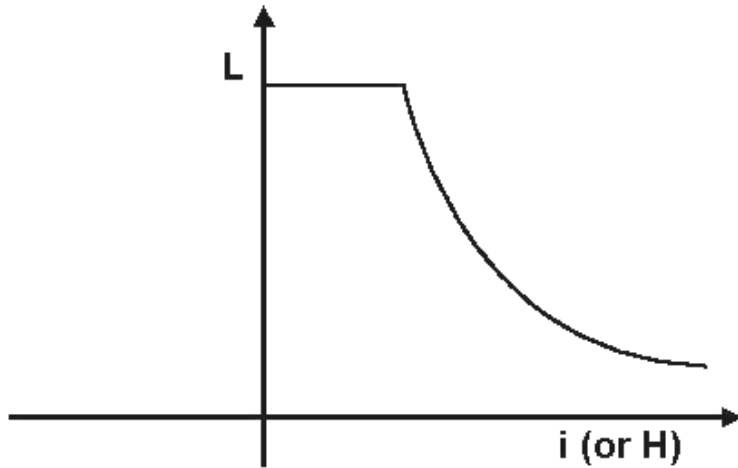
The phenomenon which causes  $B$  to Lag behind  $H$  is called **Hysteresis**.

## Normal magnetization curve



$$L = \mu \frac{N^2 A}{\ell} = \frac{N^2}{\mathfrak{R}}$$

$$\mathfrak{R} = \frac{\ell}{\mu A}$$



$$\mu = \frac{B}{H}$$

# INDUCTANCE

$$L = \frac{N\phi}{I} = \frac{\lambda}{I}$$

$$\lambda = N\phi = N \cdot B \cdot A$$

$$B = \mu \cdot H$$

$$H = \frac{NI}{\ell}$$

# Self Inductance

$$L = \frac{\lambda}{i} = \frac{N\phi}{i}$$

$$L = \frac{N^2}{\mathfrak{R}}$$

$$e = L \frac{di}{dt}$$

# Mutual Inductance

$$L_1 = \frac{\lambda_{11}}{i_1}$$

$$M = \frac{\lambda_{21}}{i_1} = \frac{N_2 K \phi_1}{i_1}$$

$$L_2 = \frac{\lambda_{22}}{i_2}$$

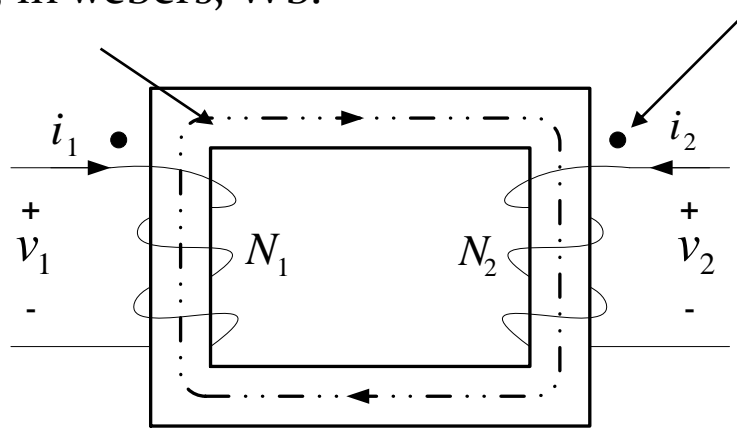
$$M = \frac{\lambda_{12}}{i_2} = \frac{N_1 K \phi_2}{i_2}$$

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

# Magnetic Flux

Magnetic flux,  $\phi$ , in webers, Wb.

Current entering "dots" produce fluxes that add.



$\phi_{11}$  = flux in coil 1 produced by current in coil 1

$\phi_{12}$  = flux in coil 1 produced by current in coil 2

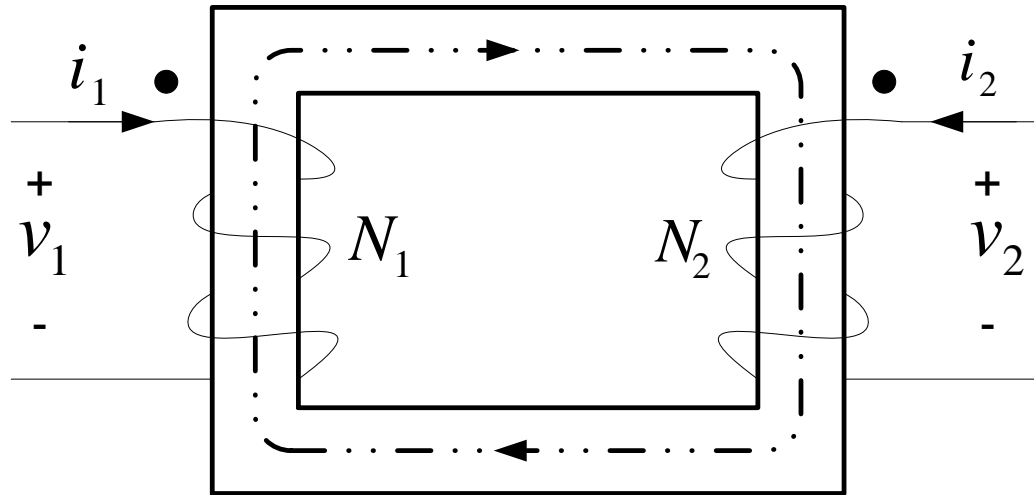
$\phi_{21}$  = flux in coil 2 produced by current in coil 1

$\phi_{22}$  = flux in coil 2 produced by current in coil 2

$\phi_1$  = total flux in coil 1 =  $\phi_{11} + \phi_{12}$

$\phi_2$  = total flux in coil 2 =  $\phi_{21} + \phi_{22}$

# Mutual Inductance



Faraday's Law

$$v_1(t) = N_1 \frac{d\phi}{dt} = N_1 \frac{d\phi_{11}}{dt} + N_1 \frac{d\phi_{12}}{dt}$$

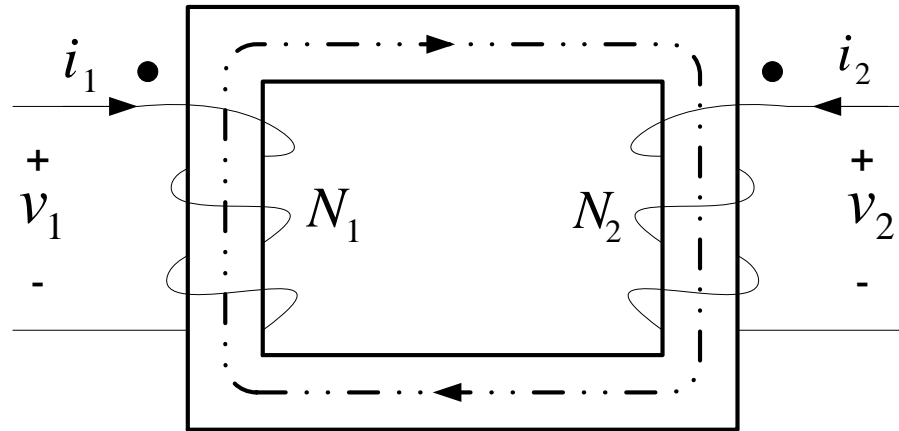
In linear range, flux is proportional to current

$$v_1(t) = L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

self-inductance

mutual inductance

# Mutual Inductance



$$v_1(t) = L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

$$v_2(t) = L_{21} \frac{di_1}{dt} + L_{22} \frac{di_2}{dt}$$

Linear media

$$L_{12} = L_{21} = M$$

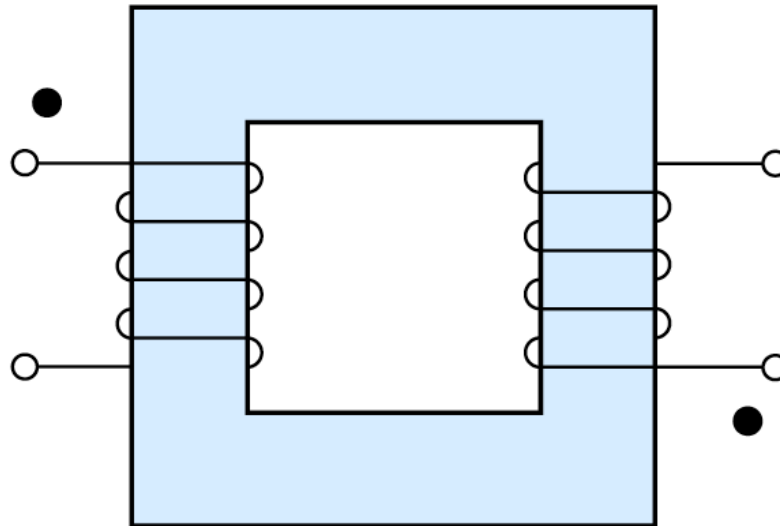
Let  $L_2 = L_{22}$   $L_1 = L_{11}$

$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

# Dot Convention

Aiding fluxes are produced by currents entering like marked (or) dotted terminals.

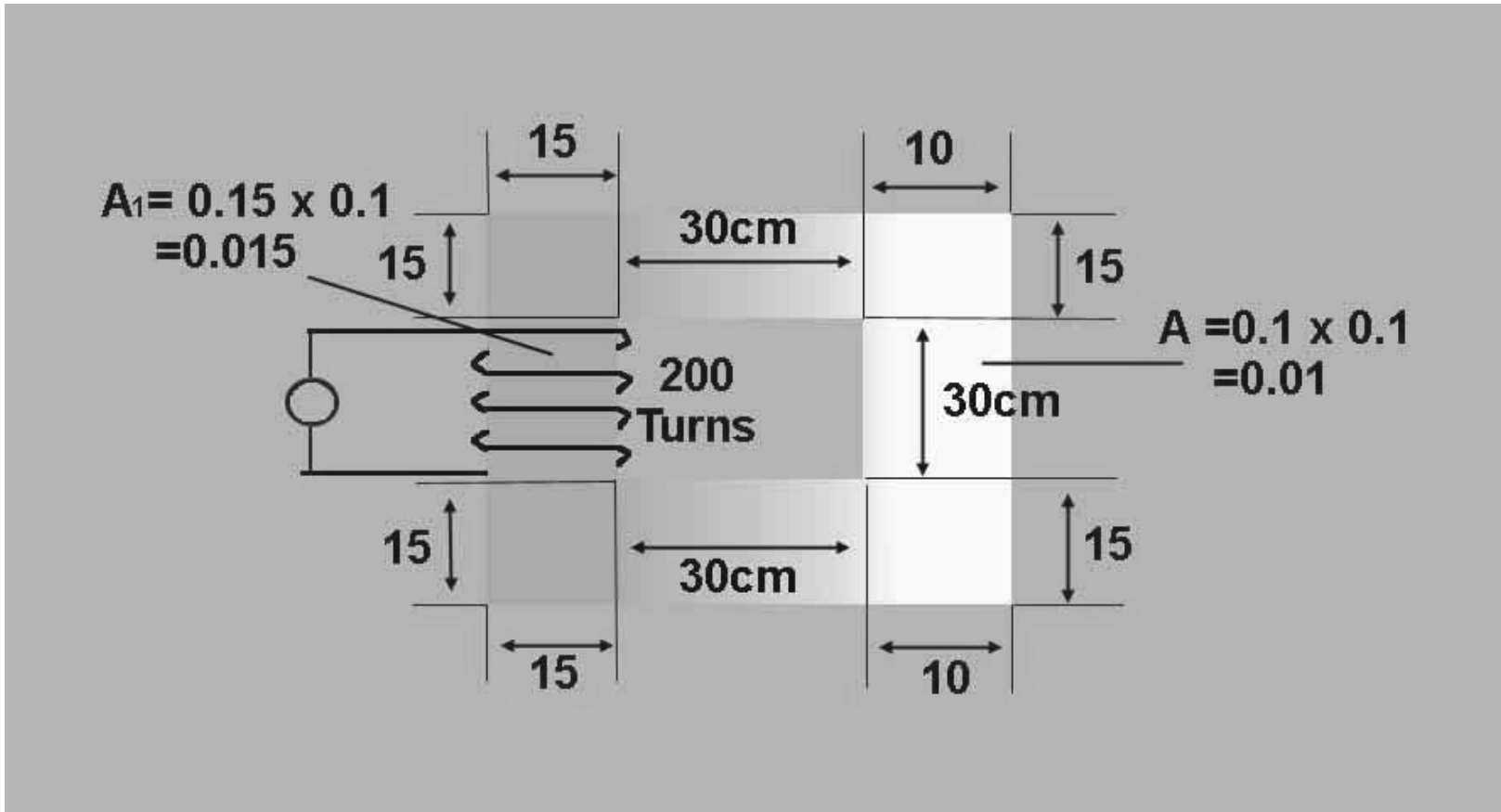


$$\begin{aligned}\lambda &= NA\mu H = NA\mu \frac{NI}{\ell} \\ &= \mu \frac{N^2 AI}{\ell}\end{aligned}$$

$$L = \frac{\lambda}{I} = \mu \frac{AN^2}{\ell}$$

$$\mathfrak{R} = \frac{\ell}{\mu A} \qquad L = \frac{N^2}{\frac{\ell}{\mu A}} = \frac{N^2}{\mathfrak{R}}$$

# Problem 1



Find the flux  $\phi$  and draw the magnetic circuit

## Solution

$$\ell_1 = 30 + 15 = 45 \text{ cm}$$

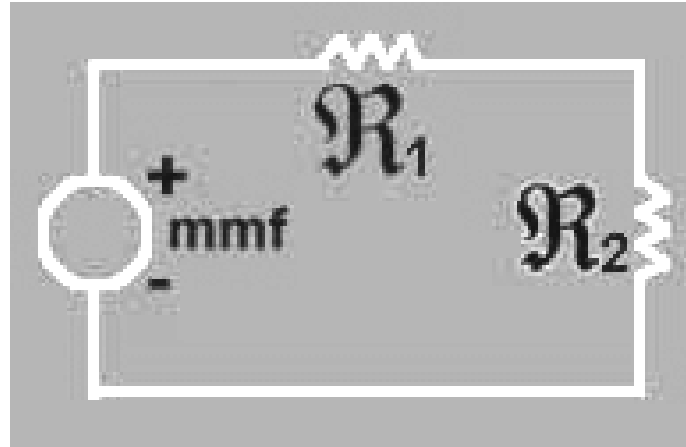
$$\ell_2 = 30 + 30 + 30 + 7.5 + 7.5$$

$$+ 7.5 + 7.5 + 5 + 5 = 130 \text{ cm}$$

$$\mathfrak{R}_1 = \frac{\ell_1}{\mu_r \mu_o A_1} = \frac{0.45}{(2500)(4\pi \times 10^{-7})(.01)} = 14300 \frac{A \cdot T}{Wb}$$

$$R_2 = \frac{\ell_2}{\mu A_2} = \frac{\ell_2}{\mu_r \mu_o A_2} = \frac{1.3}{(2500)(4\pi \times 10^{-7})(.015)} = 27600 \frac{AT}{Wb}$$

## Circuit diagram

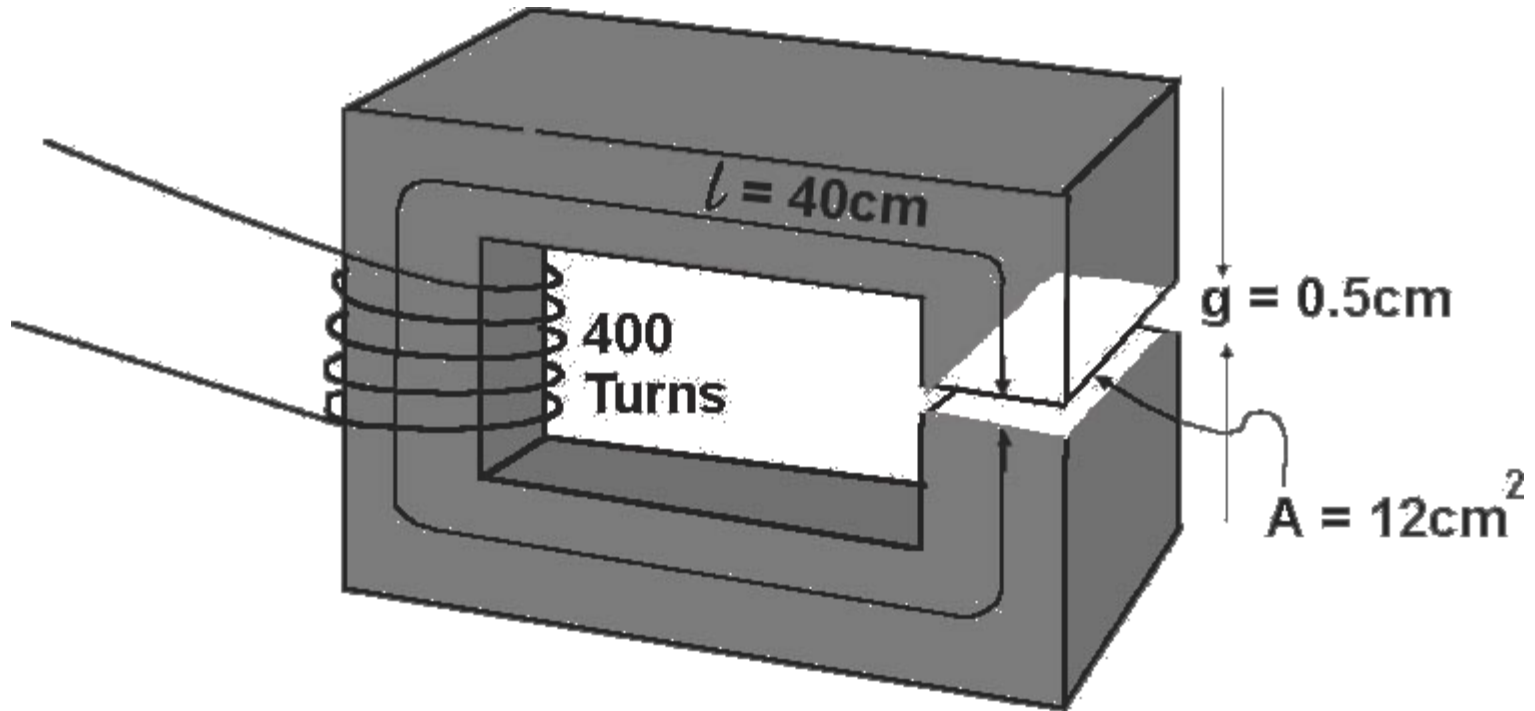


$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2$$

$$mmf = Ni = (200)(1.0) = 200 A \cdot T$$

$$\phi = \frac{mmf}{\mathcal{R}} = \frac{200}{14300 + 27600} = 0.0048 Wb$$

# Problem 2



Diagram

Assume that fringing in the air gap increases the effective cross-sectional area of the air gap by 5 percent.  $\mu_r = 4000$ .

Find:

- a) The total reluctance of the flux path.
- b) The current required to produce a flux density of  $0.5 \text{ Wb/m}^2$  in the air gap.

# Solutions

a) core

$$\begin{aligned}\mathcal{R}_c &= \frac{\ell_c}{\mu_r \mu_o A_c} \\ &= \frac{0.4}{(4000)(4\pi \times 10^{-7})(.0012)} \\ &= 66300 \frac{AT}{Wb}\end{aligned}$$

air

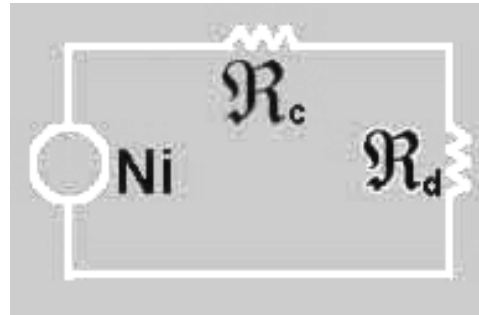
$$A_g = 1.05 \times A_c = 1.05 \times 12 = 12.6 \text{ cm}^2$$

$$\mathcal{R}_a = \frac{\ell_a}{\mu_o A_a} = \frac{.0005}{(4\pi \times 10^{-7})(0.00126)}$$

$$= 316000 \frac{AT}{Wb}$$

$$\mathcal{R}_{eq} = \mathcal{R}_a + \mathcal{R}_c = 382300 \frac{AT}{Wb}$$

## Circuit diagram



$$mmf = Ni = \phi \cdot \mathcal{R}_{eq}$$

$$Ni = BA \mathcal{R}_{eq}$$

$$i = \frac{BA \mathcal{R}_{eq}}{N} = \frac{(.5)(.00126)(382300)}{400}$$

$$= .602 A$$

# Problem

In the magnetic field system shown in figure 1,

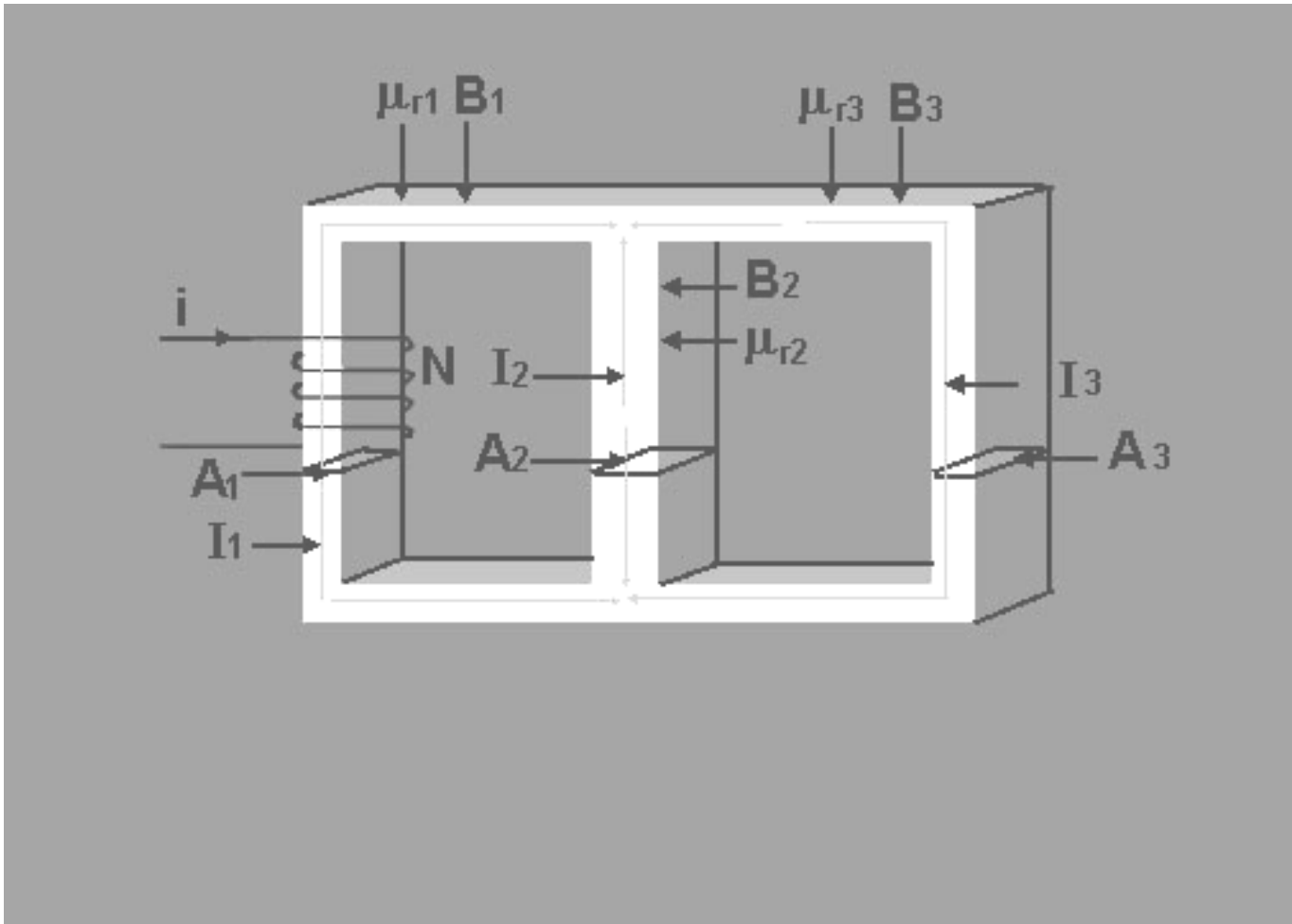
$$l_1 = l_3 = 300\text{mm} \quad l_2 = 100\text{mm}$$

$$N = 25$$

$$A_1 = A_3 = 200 \text{ mm}^2 \quad A_2 = 400\text{mm}^2$$

$$\mu_{r1} = \mu_{r3} = 2250 \quad \mu_{r2} = 1350$$

Determine the flux densities  $B_1$ ,  $B_2$ , and  $B_3$  in the three branches of the circuit when the coil current is 0.5 A.



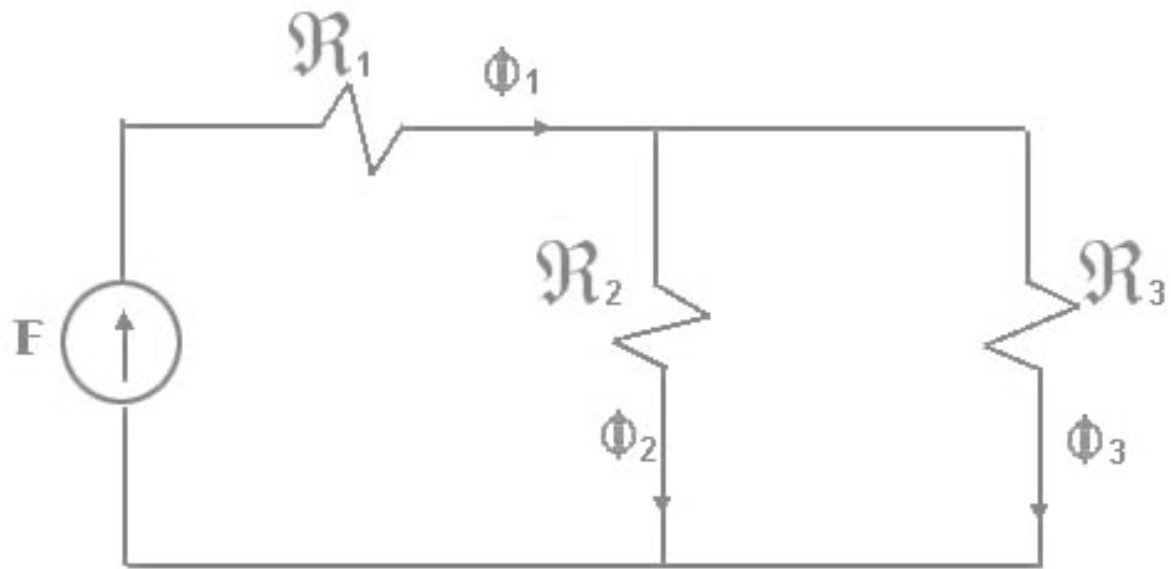
**Magnetic Circuit**

## Solution

$$\mathfrak{R}_1 = \mathfrak{R}_1 = \frac{300 \times 10^{-3}}{2250 \times 4\pi \times 10^{-7} \times 200 \times 10^{-6}}$$
$$= 0.531 \times 10^6 \text{ A/Wb}$$

$$\mathfrak{R}_2 = \frac{100 \times 10^{-3}}{1350 \times 4\pi \times 10^{-7} \times 400 \times 10^{-6}}$$
$$= 0.148 \times 10^6 \text{ A/Wb}$$

The eq. circuit for this system is shown in figure, and this can be solved by writing mmf equations for the two loops employing branch fluxes.



Equivalent magnetic circuit for the system of figure.

Thus

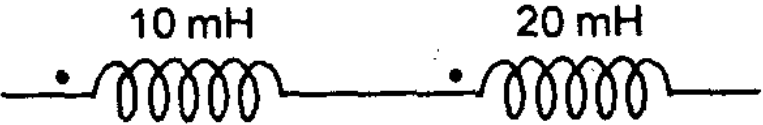
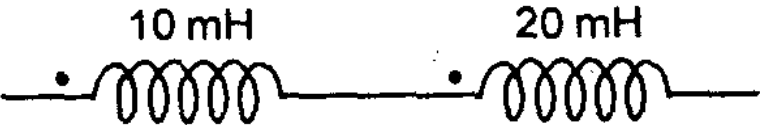
$$F = \mathfrak{R}_1\phi_1 + \mathfrak{R}_2\phi_2$$

$$0 = \mathfrak{R}_3\phi_3 - \mathfrak{R}_2\phi_2$$

These are analogous to equations of potential difference for dc circuit. Also,

$$\phi_1 = \phi_2 + \phi_3$$

**P.1** The resultant inductance of the inductors configuration shown in figure is 40 mH then find the mutual inductance of coils. Calculate the  $M$ .



**P.1** The resultant inductance of the inductors configuration shown in figure is 40 mH then find the mutual inductance of coils. Calculate the M



**Given**

$$L = L_1 + L_2 + 2M$$

$$L = 40$$

$$L_1 = 10$$

$$L_2 = 20$$

$$M = ?$$

⇒

$$40 = 10 + 20 + 2M$$

$$5 = M$$

$$M = 5 \text{ mH.}$$

**P.2** *The combined inductance of two coils connected in series is 0.6 H or 0.1 H depending on the relative directions of the currents in the coils. If one of the coils when isolated has a self-inductance of 0.2 H, calculate (a) mutual inductance and (b) coupling coefficient. (c) the two possible values of the induced emf in coil 2 when the current is increasing at 500 A/s in series combination.*

**P.2** The combined inductance of two coils connected in series is 0.6 H or 0.1 H depending on the relative directions of the currents in the coils. If one of the coils when isolated has a self-inductance of 0.2 H, calculate (a) mutual inductance and (b) coupling coefficient.

**Solution. (i)**

$$L = L_1 + L_2 + 2M \quad \text{or} \quad 0.6 = L_1 + L_2 + 2M$$

and

$$0.1 = L_1 + L_2 - 2M$$

**(a)** From (i) and (ii) we get,  $M = \mathbf{0.125 \text{ H}}$

Let  $L_1 = 0.2 \text{ H}$ , then substituting this value in (i) above, we get  $L_2 = 0.15 \text{ H}$

**(b)** Coupling coefficient  $k = M\sqrt{L_1L_2} = 0.125 / \sqrt{0.2 \times 0.15} = \mathbf{0.72}$

**P.3** Two similar coils have a coupling coefficient of 0.25. When they are connected in series cumulatively, the total inductance is 80 mH. Calculate the self inductance of each coil. Also calculate the total inductance when the coils are connected in series differentially.

**Solution.** If each coil has an inductance of  $L$  henry, then  $L_1 = L_2 = L$ ;  $M = k\sqrt{L_1L_2} = k\sqrt{L \times L} = kL$

When connected in series cumulatively, the total inductance of the coils is

$$= L_1 + L_2 + 2M = 2L + 2M = 2L + 2kL = 2L(1 + 0.25) = 2.5L$$

$$\therefore 2.5L = 80 \quad \text{or} \quad L = \mathbf{32 \text{ mH}}$$

When connected in series differentially, the total inductance of the coils is

$$= L_1 + L_2 - 2M = 2L - 2M = 2L - 2kL = 2L(1 - k) = 2L(1 - 0.25)$$

$$\therefore 2L \times 0.75 = 2 \times 32 \times 0.75 = \mathbf{48 \text{ mH.}}$$

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#### P.4

*Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise (a) in the same direction (b) in the opposite direction. The corresponding values of total inductances are for (a) 1.9 H and for (b) 0.7 H. Find the self-inductances of the two coils and the mutual inductance between them.*

**P.4** Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise (a) in the same direction (b) in the opposite direction. The corresponding values of total inductances are for (a) 1.9 H and for (b) 0.7 H. Find the self-inductances of the two coils and the mutual inductance between them.

**Solution. (a)**  $L = L_1 + L_2 + 2M$  or  $1.9 = L_1 + L_2 + 2M$  ...**(i)**

**(b)** Here  $L = L_1 + L_2 - 2M$  or  $0.7 = L_1 + L_2 - 2M$  ...**(ii)**

Subtracting **(ii)** from **(i)**, we get

$$1.2 = 4M \quad \therefore M = 0.3 \text{ H}$$

Putting this value in **(i)** above, we get  $L_1 + L_2 = 1.3 \text{ H}$  ...**(iii)**

We know that, in general,  $M = k\sqrt{L_1L_2}$

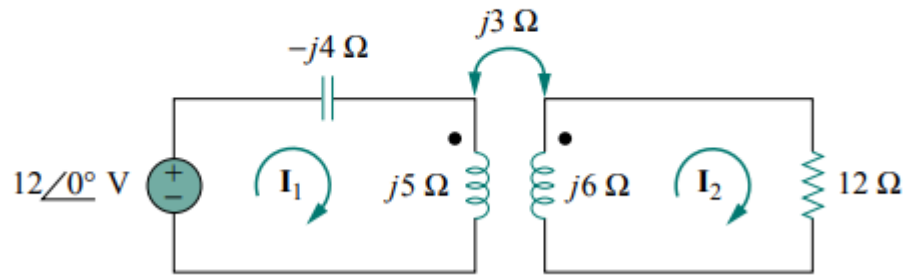
$$\therefore \sqrt{L_1L_2} = \frac{M}{k} = \frac{0.3}{0.5} = 0.6 \quad \therefore L_1L_2 = \mathbf{0.36}$$

From **(iii)**, we get  $(L_1 + L_2)^2 - 4L_1L_2 = (L_1 - L_2)^2$

$$\therefore (L_1 - L_2)^2 = 0.25 \quad \text{or} \quad L_1 - L_2 = 0.5 \quad \dots\mathbf{(iv)}$$

From **(iii)** and **(iv)**, we get  $L_1 = \mathbf{0.9 \text{ H}}$  and  $L_2 = \mathbf{0.4 \text{ H}}$

Calculate the phasor currents  $\mathbf{I}_1$  and  $\mathbf{I}_2$  in the circuit of Fig. 13.9.



For coil 1, KVL gives

$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$$

or

$$j\mathbf{I}_1 - j3\mathbf{I}_2 = 12$$

For coil 2, KVL gives

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_1 = \frac{(12 + j6)\mathbf{I}_2}{j3} = (2 - j4)\mathbf{I}_2$$

Substituting this in Eq. (13.1.1), we get

$$(j2 + 4 - j3)\mathbf{I}_2 = (4 - j)\mathbf{I}_2 = 12$$

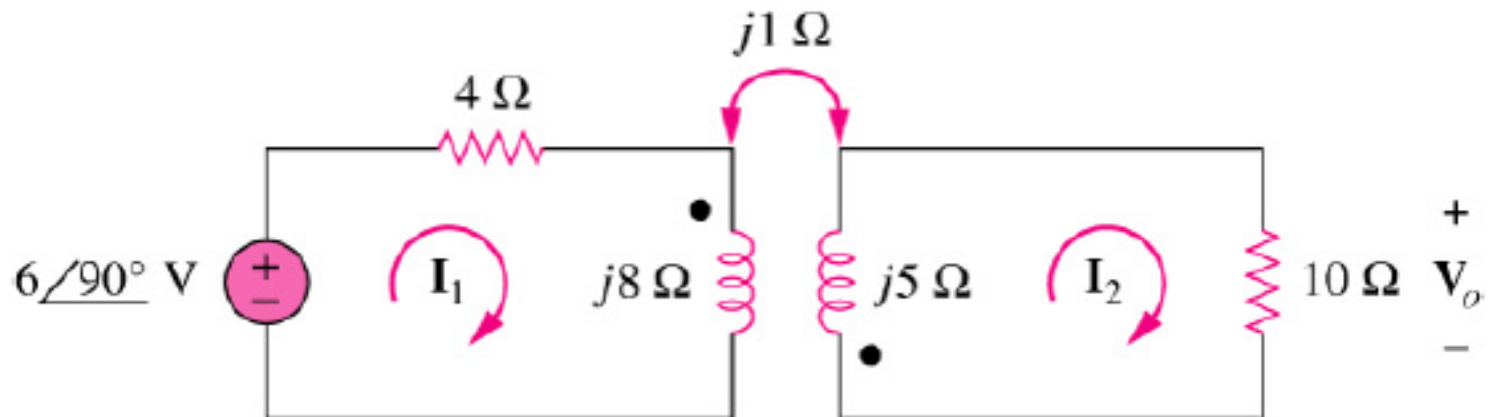
or

$$\mathbf{I}_2 = \frac{12}{4 - j} = 2.91 \angle 14.04^\circ \text{ A}$$

From Eqs. (13.1.2) and (13.1.3),

$$\begin{aligned}\mathbf{I}_1 &= (2 - j4)\mathbf{I}_2 = (4.472 \angle -63.43^\circ)(2.91 \angle 14.04^\circ) \\ &= 13.01 \angle -49.39^\circ \text{ A}\end{aligned}$$

**P.P. 13.1: Determine the voltage  $V_o$  in the given circuit.**



Induced mutual voltages

For mesh 1,  $j6 = 4(1 + j2)I_1 + jI_2$  (1)

For mesh 2,  $0 = jI_1 + (10 + j5)I_2$  (2)

For the matrix form 
$$\begin{bmatrix} j6 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j8 & j \\ j & 10 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$\Delta = j100, \Delta_2 = 6, I_2 = \Delta_2/\Delta = 6/j100$

$V_o = 10I_2 = 60/j100 = \underline{\underline{0.6\angle-90^\circ \text{ V}}}$