BEEE203L CIRCUIT THEORY

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MODULE – VII

TWO PORT NETWORKS

Contents

- Open circuit impedance parameters
- Short circuit admittance parameters
- Transmission parameters
- Hybrid parameters
- Relationship between parameter sets
- Interconnection of two port networks

Two-port network

- A pair of terminals through which a current may enter or leave a network is known as a port
- A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero
- Two-terminal devices or elements (such as resistors, capacitors, and inductors) result in one-port networks
- A two-port network has two pairs of terminals, one pair at the input known as input port and one pair at the output known as output port as shown in Fig

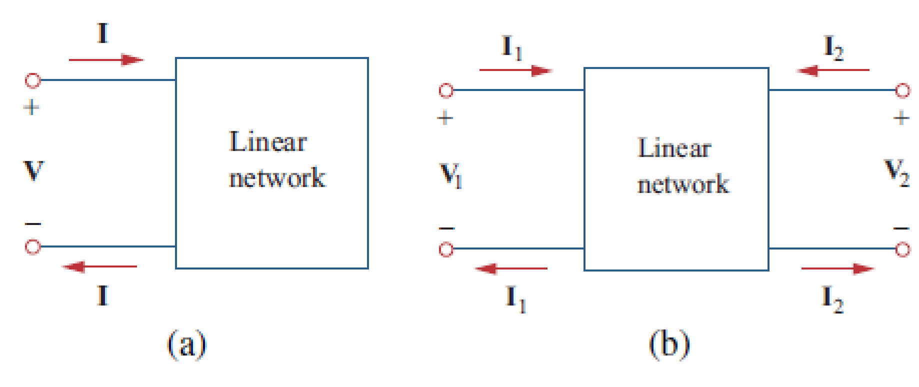


Figure: (a) One-port network (b) two-port network

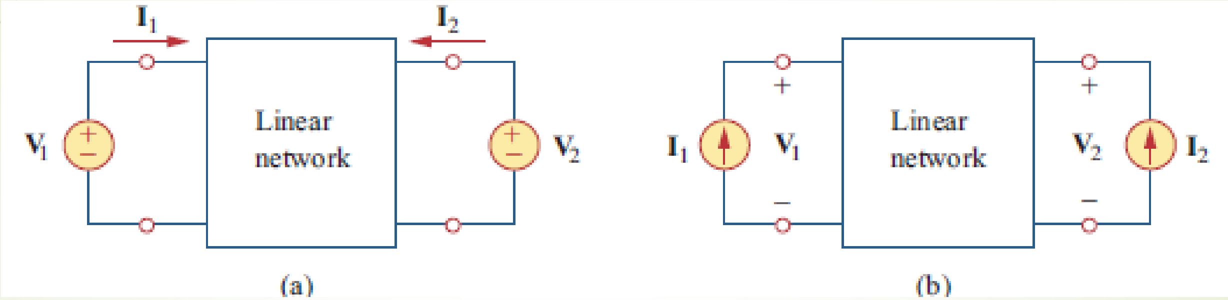
Parameters

- The two-port networks are useful in communications, control systems, power systems, and electronics
- To characterize a two-port network requires that we relate the terminal quantities V_1, V_2, I_1 and I_2
- The various terms that relate these voltages and currents are called ***parameters***
- Two of these variables can be expressed in terms of the other two variables. Thus, there will be two dependent variables and two independent variables.
- The number of possible combinations generated by four variables taken two at a time i.e., six.
- There are six possible sets of equations describing a two-port network.

Two port parameters

Parameter	*Variables*		*Equation*
	Express	*In terms of*	
Open-Circuit Impedance	V_1, V_2	I_1, I_2	$V_1 = Z_{11} I_1 + Z_{12} I_2$ $V_2 = Z_{21} I_1 + Z_{22} I_2$
Short-Circuit Admittance	I_1, I_2	V_1, V_2	$I_1 = Y_{11} V_1 + Y_{12} V_2$ $I_2 = Y_{21} V_1 + Y_{22} V_2$
Transmission	V_1, I_1	V_2, I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
Inverse Transmission	V_2, I_2	V_1, I_1	$V_2 = A' V_1 - B' I_1$ $I_2 = C' V_1 - D' I_1$
Hybrid	V_1, I_2	I_1, V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$
Inverse Hybrid	I_1, V_2	V_1, I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$

OPEN-CIRCUIT IMPEDANCE PARAMETERS (Z PARAMETERS)



- A two-port network may be voltage-driven as or current-driven, the terminal voltages can be related to the terminal currents as

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

- The values of the parameters can be evaluated by setting $I_1 = 0$ (input port open-circuited) or $I_2 = 0$ (output port open-circuited)

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}, \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

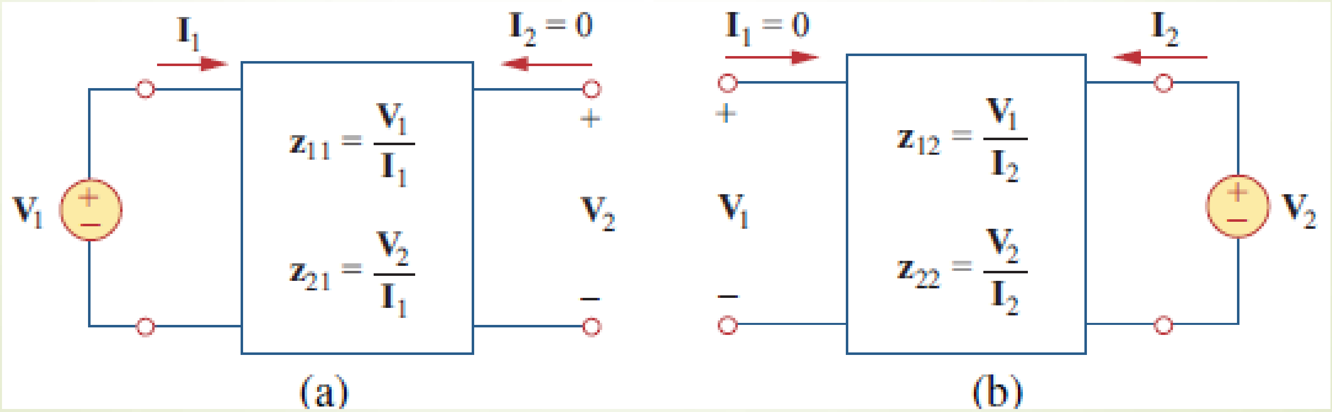
$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}, \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

Since the z parameters are obtained by open-circuiting the input or output port, they are also called the open-circuit impedance parameters

- Z_{11} is Open-circuit input impedance
- Z_{12} is Open-circuit transfer impedance from port 1 to port 2
- Z_{21} is Open-circuit transfer impedance from port 2 to port 1
- Z_{22} is Open-circuit output impedance

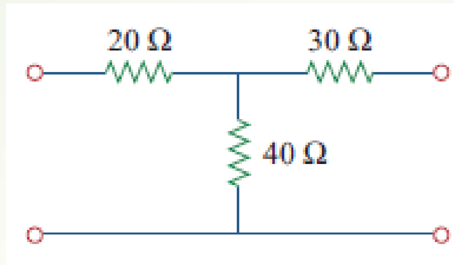
- Z_{11} and Z_{22} are called *driving-point impedances*
- Z_{12} and Z_{21} are called *transfer impedances*



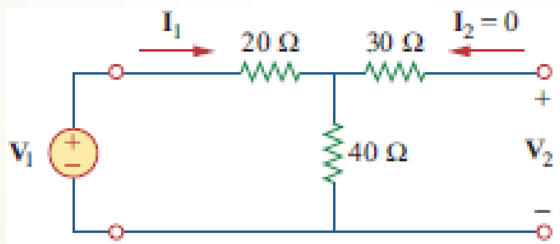
When the two-port network is linear and has no dependent sources the Z_{12}=Z_{21} and the two-port is said to be reciprocal

Problems

P.7.1 Determine the Z parameters for the circuit



Solution : To determine Z_{11} and Z_{21}, apply a voltage source V_1 to the input port and leave the output port open



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

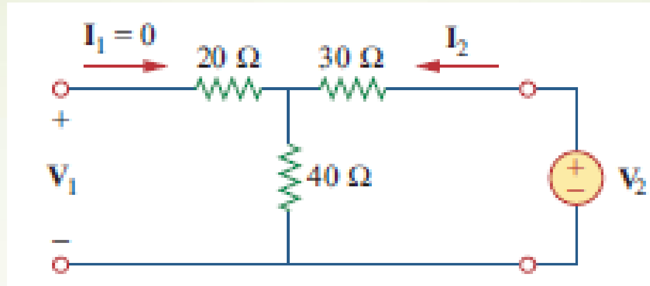
$V1 = (20 + 40)^* \, I_1 = 60^* I_1;$

$V2 = 40^* \, I_1$

Therefore,

$$z_{11} = \frac{V_1}{I_1} = 60\ \Omega, \quad z_{21} = \frac{V_2}{I_1} = 40\ \Omega$$

To determine Z_{12} and Z_{22}, apply a voltage source V_1 to the input port and leave the output port open



$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$V_1 = 40 * I_2$;

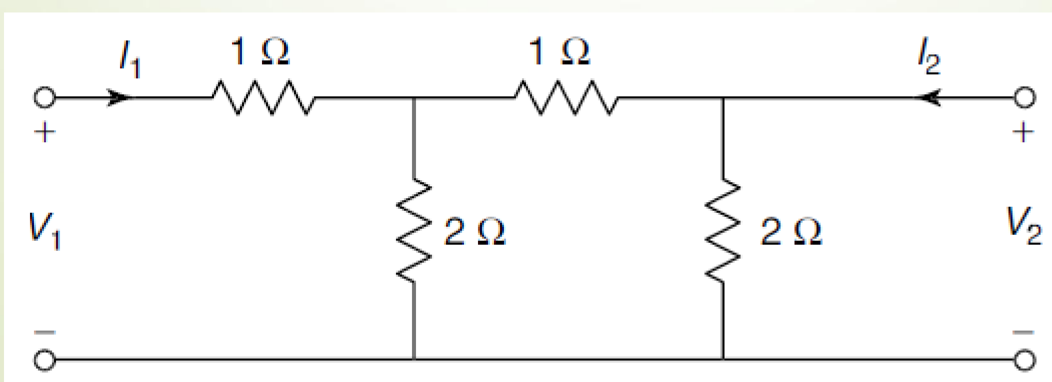
$V_2 = (30+40) * I_2 = 70 * I_2$

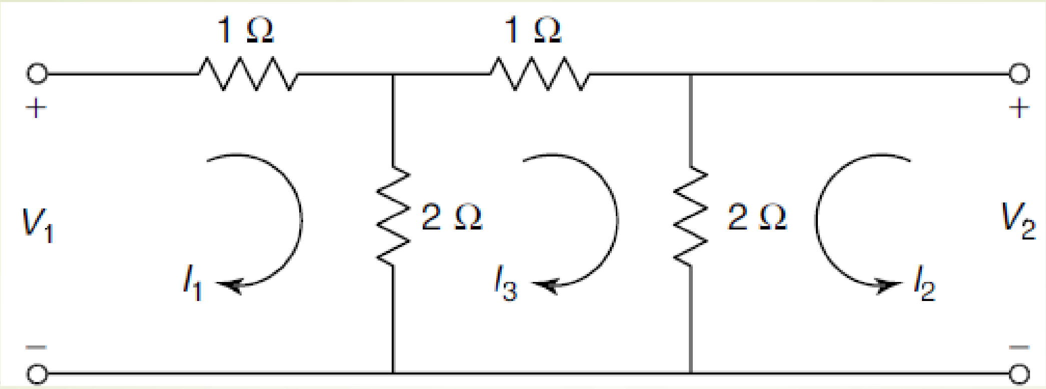
Therefore,

$$z_{12} = \frac{V_1}{I_2} = 40\ \Omega, \quad z_{22} = \frac{V_2}{I_2} = 70\ \Omega$$

$$[\mathbf{z}] = \begin{bmatrix} 60\ \Omega & 40\ \Omega \\ 40\ \Omega & 70\ \Omega \end{bmatrix}$$

P.7.2 Find Z-parameters for the network shown in Fig.





The network is redrawn as shown in Fig. 13.10.

- Applying KVL to Mesh 1, $V_1 = 3I_1 - 2I_3$ ...(1)
- Applying KVL to Mesh 2, $V_2 = 2I_2 + 2I_3$ ...(2)
- Applying KVL to Mesh 3, $-2I_1 + 2I_2 + 5I_3 = 0$

or $I_3 = \frac{2}{5}I_1 - \frac{2}{5}I_2$ ...(3)

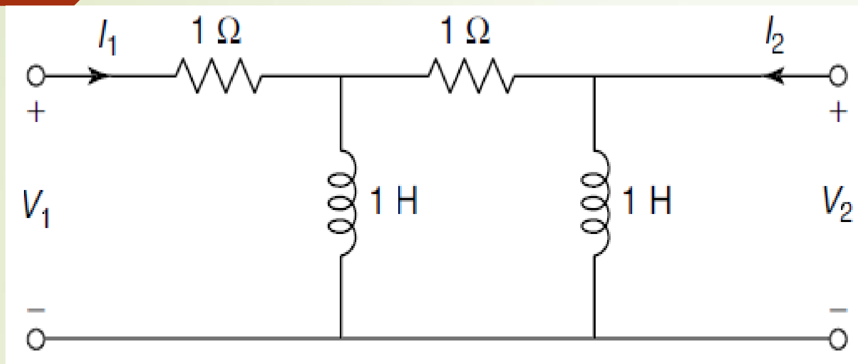
(3) In (1) gives $V_1 = 3I_1 - \frac{4}{5}I_1 + \frac{4}{5}I_2$ $= \frac{11}{5}I_1 + \frac{4}{5}I_2$

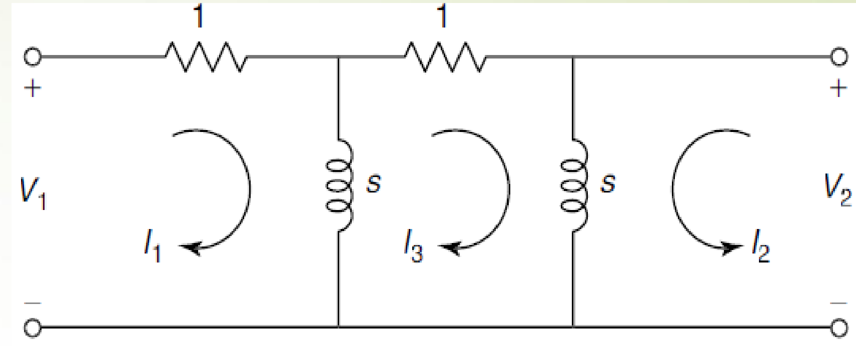
(3) In (2) gives $V_2 = 2I_2 + \frac{4}{5}I_1 - \frac{4}{5}I_2$ $= \frac{4}{5}I_1 + \frac{6}{5}I_2$

So Z parameters are: $$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{6}{5} \end{bmatrix}$$

P.7.3 Find the Z-parameters for the network shown in Fig

Solution: The transformed network is shown in Fig 2





Applying KVL to Mesh 1, $V_1 = (s+1)I_1 - sI_3$ ...(1)

Applying KVL to Mesh 2, $V_2 = sI_2 + sI_3$ ...(2)

Applying KVL to Mesh 3, $-sI_1 + sI_2 + (2s+1)I_3 = 0$

or $I_3 = \frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2$ ...(3)

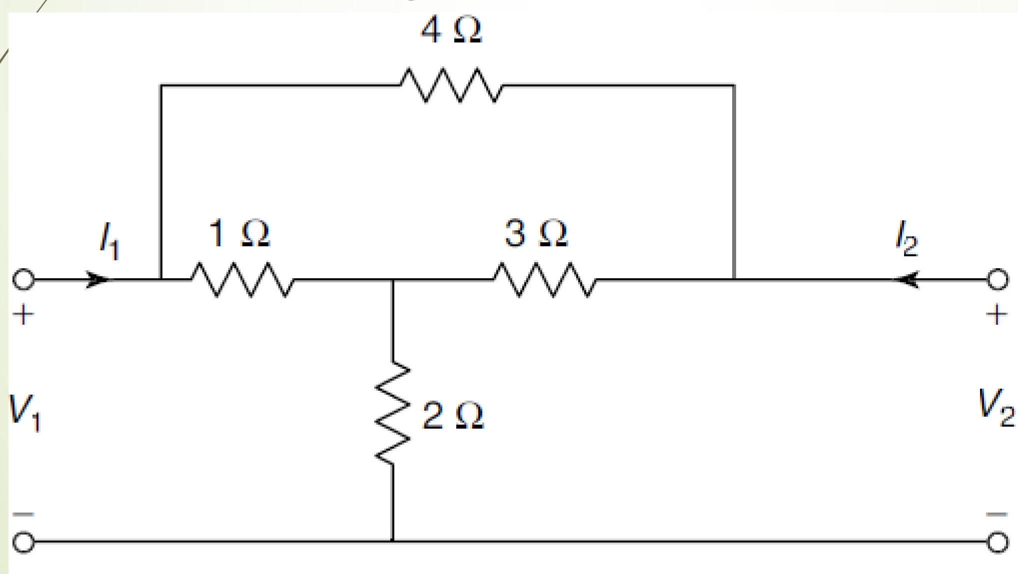
(3) In (1) gives $V_1 = (s+1)I_1 - s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right) = \left(\frac{s^2+3s+1}{2s+1}\right)I_1 + \left(\frac{s^2}{2s+1}\right)I$

(3) In (2) gives $V_2 = sI_2 + s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right) = \left(\frac{s^2}{2s+1}\right)I_1 + \left(\frac{s^2+s}{2s+1}\right)I_2$

So Z parameters are:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s^2+3s+1}{2s+1} & \dfrac{s^2}{2s+1} \\ \dfrac{s^2}{2s+1} & \dfrac{s^2+s}{2s+1} \end{bmatrix}$$

P.7.4 Find the open-circuit impedance parameters for the network shown in Fig.



$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{23}{8} & \dfrac{19}{8} \\ \dfrac{19}{8} & \dfrac{31}{8} \end{bmatrix}$$

P.7.5 Find I1 and I2 in the figure