Department of Mathematics
School of Advanced Sciences
BMAT 101P – Calculus (MATLAB)
Experiment 3–B
Evaluating Volume under surfaces

In this experiment, we consider a continuous function f such that $f(x,y) \geq 0$ for all (x,y) in a region R in the xy – plane, then the volume of the solid region that lies above R and below the graph of f is defined as the double integral $V = \iint_R f(x,y)dA$, where R is the region bounded by the curves $y = \phi_1(x)$ and $y = \phi_2(x)$ between $x = a$ and $x = b$.

In this case, the inner integration is with respect to y and outer integration is with respect to x. Hence

$$V = \iint_R f(x,y)dA = \int_{x=a}^{b} [\int_{y=\phi_1(x)}^{\phi_2(x)} f(x,y)dy]dx$$

MATLAB Syntax

`int(int(f, y, phi1, phi2),x,a,b)` where y is the inner variable, x is the outer variable.

When R is a region bounded by the curves $x = \psi_1(y)$ and $x = \psi_2(y)$ between $y = c$ and $y = d$, i.e., the inner integration is with respect to x and outer integration is with respect to y. Then $V = \iint_R f(x,y)dA = \int_{y=c}^{d} [\int_{x=\psi_1(y)}^{\psi_2(y)} f(x,y)dx]dy$

MATLAB Syntax

`int(int(f, x, psi1, psi2),y,c,d)` where x is the inner variable, y is the outer variable.

Supporting files required:

To visualize the surfaces two additional m-files viz., `viewSolid.m, viewSolidone.m` are required. These files are to be included in the current working directory before execution. Students are advised to upload these files (`viewSolid.m and viewSolidone.m`) to their MATLAB drive. These supporting files **should not be edited**.

Download `viewSolid.m` from the following link:
https://drive.google.com/file/d/1qEsq7VCgrmI60GI-C0bMY6kl8yRWBznk/view?usp=sharing

Download `viewSolidone.m` from the following link:
https://drive.google.com/file/d/1H1cJOfJArmUujQNVxeSeuGfBJejtbGgJ/view?usp=sharing

Syntax for visualization of the surfaces:

```
viewSolid(z,0+0*x+0*y,f,y,phi1,phi2,x,a,b)

viewSolidone (z,0+0*x+0*y,f,x,psi1,psi2,y,c,d)
```

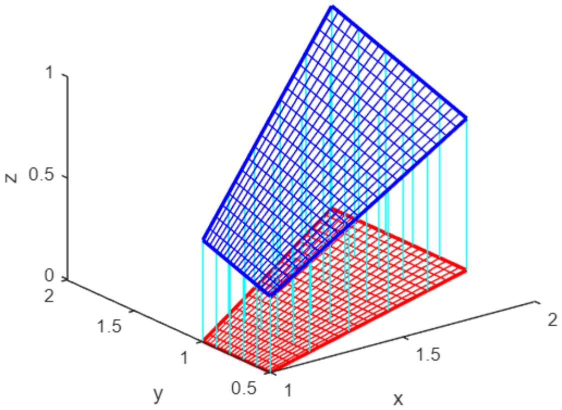
Example. 1 To find $\int_{1}^{2}\int_{x/2}^{x} \frac{x+y}{4}\,dydx$.

```
syms x y z
int(int((x+y)/4,y,x/2,x),x,1,2)
viewSolid(z,0+0*x+0*y,(x+y)/4,y,x/2,x,x,1,2)
```

Output

```
ans =
49/96
```

In this figure the required volume is above the plane z=0 (shown in red) and above the surface $z=\frac{x+y}{4}$ (shown in green) .



Example. 2 To find the volume of the prism whose base is the triangle in the $xy-$plane bounded by the $x-$axis and the lines $y=x$ and $x=1$ and whose top lies in the plane $z=f(x,y)=3-x-y$. The limits of integration here are $y=0$ to 1 while $x=y$ to 1.

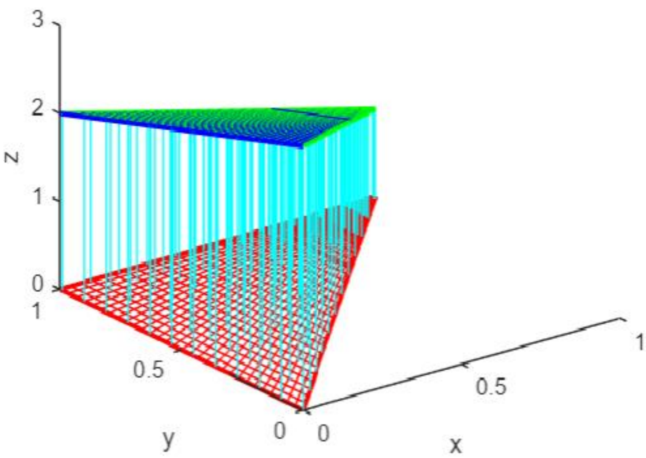
Hence $\iint_R (3-x-y)dA=\int_0^1\int_y^1(3-x-y)dxdy$

```
syms x y z
int(int(3-x-y,x,y,1),y,0,1)
viewSolidone(z,0+0*x+0*y,3-x-y,x,y,1,y,0,1)
```

Output:

```
ans =
1
```

In this figure the triangular region on the *xy* plane is shown in red, while the plane surface *z*=3–*x*–*y* above the *xy* plane is shown in green.



Example 3 Evaluate the integral $\int_0^2 \int_{x^2}^{2x} (4x+2)dydx$ by changing the order of integration.

As per the given limits of integration $x = 0$ to 2 while $y = x^2$ to $2x$.

MATLAB Code:

```
syms x y z
int(int((4*x+2),y,x^2,2*x),x,0,2)
viewSolid(z,0+0*x+0*y, 4*x+2,y,x^2,2*x,x,0,2)
```

Output

```
ans =

8
```

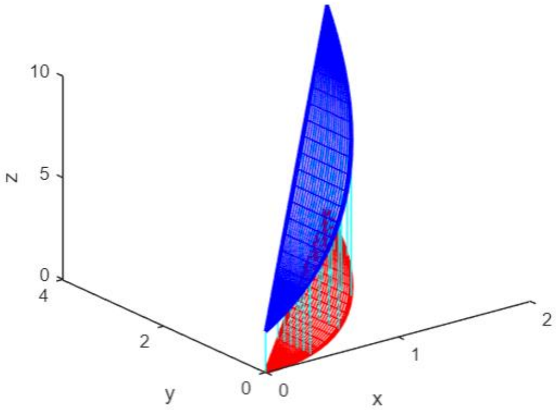
By changing the order of integration, the limits are

$y = 0$ to 4 while $x = \frac{y}{2}$ to $\sqrt{y}$.

```
int(int(4*x+2,x,y/2,sqrt(y)),y,0,4)

ans =

8
```



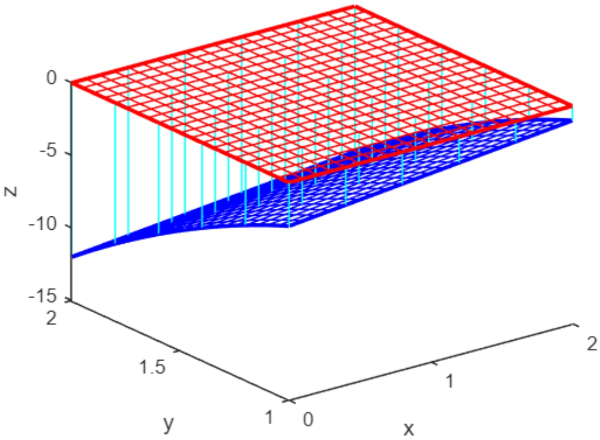
Example 4: Evaluate $\iint_R (x-3y^2)dA$ where $R = \{(x,y) \mid 0 \le x \le 2, 1 \le y \le 2\}$

```
clc
clear all
syms x y z
viewSolid(z, 0+0*x+0*y, x-3*y^2,y,1+0*x, 2+0*x,x,0,2)
int(int(x-3*y^2,y,1,2),x,0,2)
```

Output:

```
>> ans
     -12
```

In this figure the required volume is below the plane $z = 0$ (shown in red) and above the surface $z = x - 3y^2$ (shown in blue). The reason why the answer is negative is that the surface $z = x - 3y^2$ is below $z = 0$ for the given domain of integration.



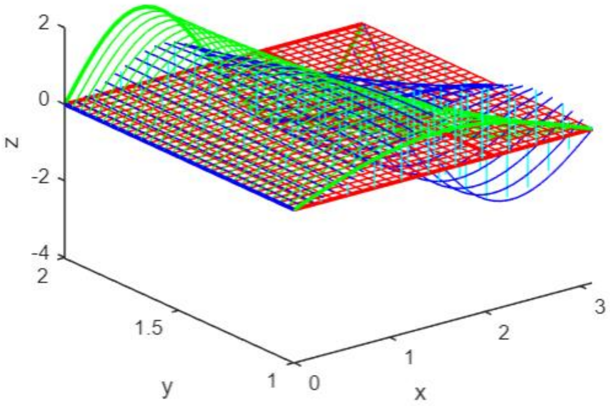
Example 5: Evaluate $\iint_R y\sin(xy)\,dA$ where $R=[1,2]\times[0,\pi]$

MATLAB Code:

```
syms x y z
viewSolidone(z, 0+0*x+0*y, y*sin(x*y),x,1+0*y, 2+0*y,y,0,pi)
int(int(y*sin(x*y),x,1,2),y,0,pi)
```

Output:

```
>>  ans
    0
```



For a function $f(x,y)$ that takes on both positive and negative values $\iint_R f(x,y)dA$ is a difference of volumes V_1-V_2, V_1 is the volume above *R* and below the graph of *f* and V_2 is the volume below R and above the graph. The integral in this example is 0 means $V_1=V_2$

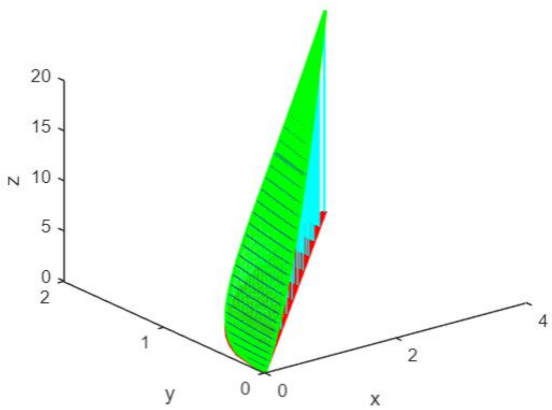
Example 6: Find the volume of the solid that lies under the paraboloid $z=x^2+y^2$ and above the region *D* in the *xy*-plane bounded by the lines $y=2x$ and the parabola $y=x^2$

MATLAB Code:

```
syms x y z
viewSolidone(z, 0+0*x+0*y, x^2+y^2,x,y/2,sqrt(y),y,0,4)
int(int(x^2+y^2,x,y/2,sqrt(y)),y,0,4)
```

Output:

```
>>  ans
    216/35
```



Exercise:

1. Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2+2y^2+z=16$, the planes $x=2$ *and* $y=2$, and the three coordinate planes.
2. Evaluate $\iint_R \sin x\cos y\,dA$ where $R=[0,\pi/2]\times[0,\pi/2]$