



**VIT**  
Vellore Institute of Technology  
Established by the Government of Tamil Nadu in 1984

**Continuous Assessment Test - 1**

Department of Mathematics, School of Advanced Sciences

Course Name: Calculus

Course Code: BMAT101L

Slot: C2 + TC2

Fall Semester 2022-23

VIT - Vellore

Total Marks: 50

Time: 90 minutes

Answer all the questions.

1. Consider the function  $f(x) = x^3 - 3x^2 + 2$  where  $x \in \mathbb{R}$ .

- (a) Find the critical points of  $f$ . [2]
- (b) Further, use the Second Derivative Test for Local Extrema to find the points of local maxima and minima of  $f$ . [2]
- (c) find the points as well as the value of the absolute extrema of  $f$  over  $[-2, 5]$ . [4]
- (d) Determine the intervals where  $f$  is concave up, and where  $f$  is concave down. [2]

2. Consider the function given by  $f(x) = 2 - x^2 + x$ .

- (a) Verify Lagrange's Mean Value Theorem for  $f$  over  $[-1, 2]$ . [5]
- (b) Using Lagrange's Mean Value Theorem find a critical point of  $f$  in the interval  $[-1, 2]$ . Is this a point of local maxima, or local minima, or neither? Justify. Is this a point of absolute maxima, or absolute minima, or neither? Justify. [5]

3. Consider the region  $\mathcal{R}$  bounded above by the given two curves, and bounded below by the  $X$ -axis:

$$f(x) = x,$$

$$g(x) = 6x - x^2$$

- (a) Find the area of region  $\mathcal{R}$ . [4]  
 (b) Assume that this planar region  $\mathcal{R}$  is rotated about the Y-axis. Find the volume of the generated solid. *5/4π* [6]

4. (a) Consider the function given by:

$$f(x, y) = \frac{xy^2 - 1}{y - 1}, \text{ if } y \neq 1$$

Does  $\lim_{(x,y) \rightarrow (1,1)} f(x, y)$  exist? If so, find the limit. [5]

- (b) Consider the function given by  $u = x^2 - y + xy^2$ , where  $y = \frac{x}{2}$ . Using Chain Rule for Partial Derivatives, find the total differential of  $u$  with respect to  $x$ . [5]

5. (a) Assuming that  $y$  is a differentiable function of  $x$ , consider the following relation:

$$xe^y + \sin xy + y - \ln 2 = 0$$

Find  $\frac{dy}{dx}$  at the point  $(0, \ln 2)$ . *-(2 + \ln 2)* [3]

- (b) Consider the given functions  $v$  and  $w$  as functions of variables  $x$  and  $y$ , as given below.

$$v = xy - 2,$$

$$w = xy(xy - 4).$$

- i. Find  $J\left(\frac{v, w}{x, y}\right)$ . [2]  
 ii. Hence determine if  $v$  and  $w$  are functionally dependent. [2]  
 iii. If so, find the relation between  $v$  and  $w$ . [3]

$$\hookrightarrow w = v^2 - 4$$