

Engineering Mechanics

Faculty : Dr. Prashik Gaikwad
School of Mechanical Engineering

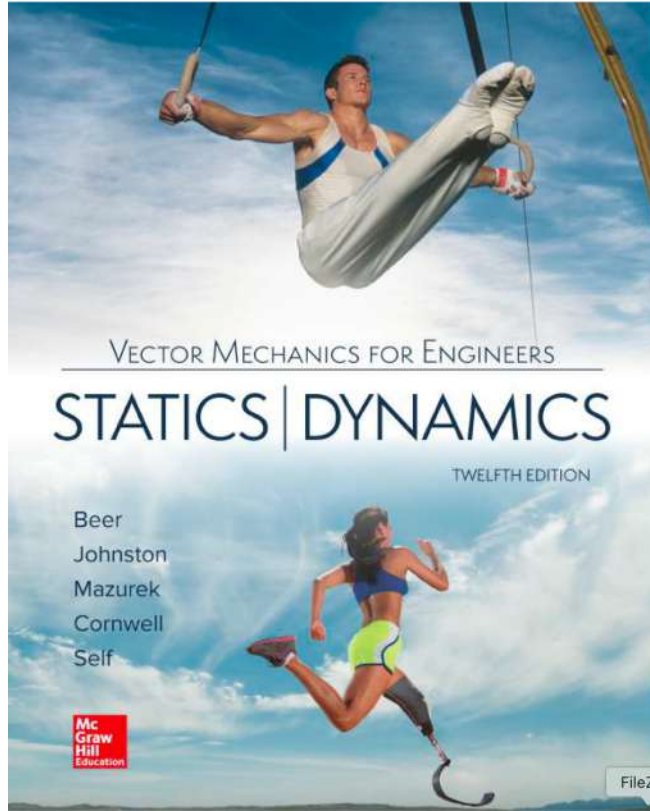
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Cabin: CBMR 304-B

BMEE201L	Engineering Mechanics	L	T	P	C
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Pre-requisite	NIL	Syllabus version			
		1.0			
Course Objectives:					
1. To enable students to apply fundamental laws and basic concepts of rigid body mechanics to solve problems of bodies under rest or in motion.					
2. To enable the students to apply conditions of static equilibrium to analyse physical systems.					
3. To compute the properties of areas and bodies.					
Course Outcome:					
Upon successful completion of the course the students will be able to					
1. Compute the resultant and analyse equilibrium (without and with friction) of system of forces acting on particles and rigid bodies in plane and space.					
2. Predict the support-reactions and the internal forces of the members of trusses and frames.					
3. Apply transfer theorems to determine properties of various sections.					
4. Calculate motion parameters of particles and rigid bodies.					
Module:1	Statics of Particles	5 hours			
Fundamental concepts and principles - Resolution of a force -Resultant of forces in a plane- Equilibrium of a particle in a plane; Addition of concurrent forces in space- Equilibrium of a particle in space.					
Module:2	Statics of Rigid Bodies	7 hours			
Equivalent systems of forces- Principle of Transmissibility - Moment of a force about a point and an axis- Couples and force-couple systems- Equilibrium of rigid bodies in two and three dimensions- Types of beams, supports and reactions; Principle of virtual work – System of connected rigid bodies.					
Module:3	Analysis of Structures	5 hours			
Analysis of plane trusses - Method of joints and method of sections- Frames					
Module:4	Friction	5 hours			
The laws of dry friction – Coefficients of Friction- Angles of Friction- Types of Friction Problems - Wedges and Ladder friction- Belt friction.					
Module:5	Properties of Surfaces and Solids	7 hours			
First moments of areas and lines- Centroids of composite areas and lines- - Theorems of Pappus-Guldinus- Second moment of area- Parallel axis theorem- Rectangular and Polar Moments of inertia of composite areas- Radius of Gyration- Product of Inertia- Principal Axes and Principal Moments of Inertia- Mass moments of inertia of thin plates.					
Module:6	Dynamics of Particles	8 hours			
Kinematics of Particles: Displacement, Velocity and Acceleration – Rectilinear motion – Curvilinear motion – Tangential and Normal components – Radial and Transverse components.					
Kinetics of Particles: Newton's Second Law- Energy and Momentum Methods-Principle of Work and Energy-Principle of Impulse and Momentum- Direct Central Impact					
Module:7	Dynamics of Rigid Bodies	8 hours			
Kinematics of rigid bodies: Translation and fixed-axis rotation- General plane motion: velocity- Instantaneous centre of rotation- General plane motion: acceleration.					
Kinetics of rigid bodies:Equations of motion -Angular momentum- Plane motion of a rigid body- Principle of work and energy for rigid bodies- Principle of impulse and momentum for rigid bodies.					
	Total Lecture hours:	45 hours			

Textbook for this course



Beer, Johnston, Cornwell, David Mazurek,
and Sanghi, Vector Mechanics for
Engineers: Statics and Dynamics, 12th
Edition, McGraw-Companies

Module 2

Statics of Rigid Bodies

Module Overview

- Equivalent system of forces
- Principle of Transmissibility
- Moment of force about a point
- Couple and force-couple systems
- Equilibrium of rigid bodies in two and three dimensions
- Types of beams, support and reactions

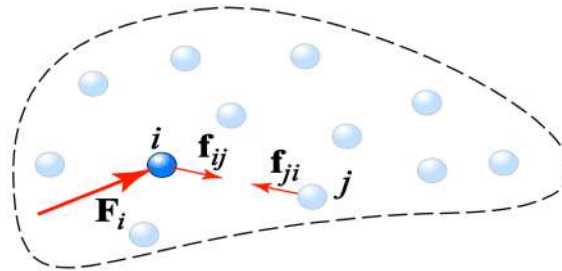
Rigid bodies

- Particle is said to be in equilibrium if
 - Resultant force acting on it is zero
- So this condition is **both necessary and sufficient**.
- Rigid body : System of infinitely many particles with **fixed distance** between them.



$\mathbf{F}_i$ = resultant external force on particle i
 $\mathbf{f}_{ij}$ = internal force on particle i by particle j
 $\mathbf{f}_{ji}$ = internal force on particle j by particle i

Note that $\mathbf{f}_{ji} = -\mathbf{f}_{ij}$ by Newton's third law and therefore the internal forces will not appear in the equilibrium equations.



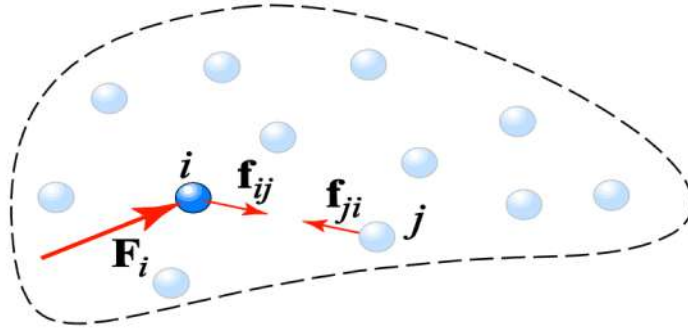
Rigid bodies

$\mathbf{F}_i$ = resultant external force on particle i

$\mathbf{f}_{ij}$ = internal force on particle i by particle j

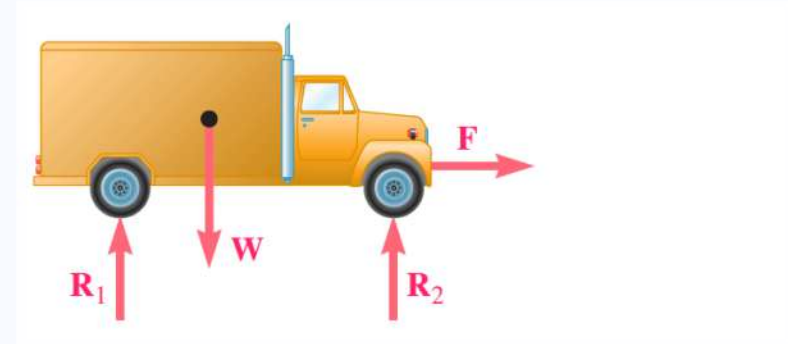
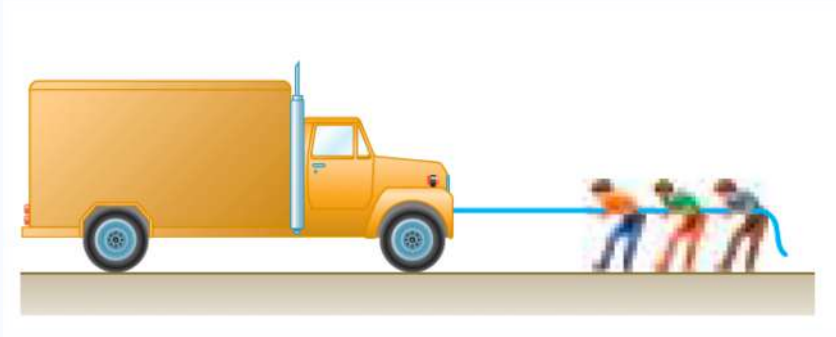
$\mathbf{f}_{ji}$ = internal force on particle j by particle i

Note that $\mathbf{f}_{ji} = -\mathbf{f}_{ij}$ by Newton's third law and therefore the internal forces will not appear in the equilibrium equations.



- External forces are exerted by other bodies on the rigid body under consideration. They are entirely responsible for the external behavior of the rigid body
- Internal forces hold together the particles forming the rigid body. If the rigid body is structurally composed of several parts, the forces holding the component parts together are also defined as internal forces

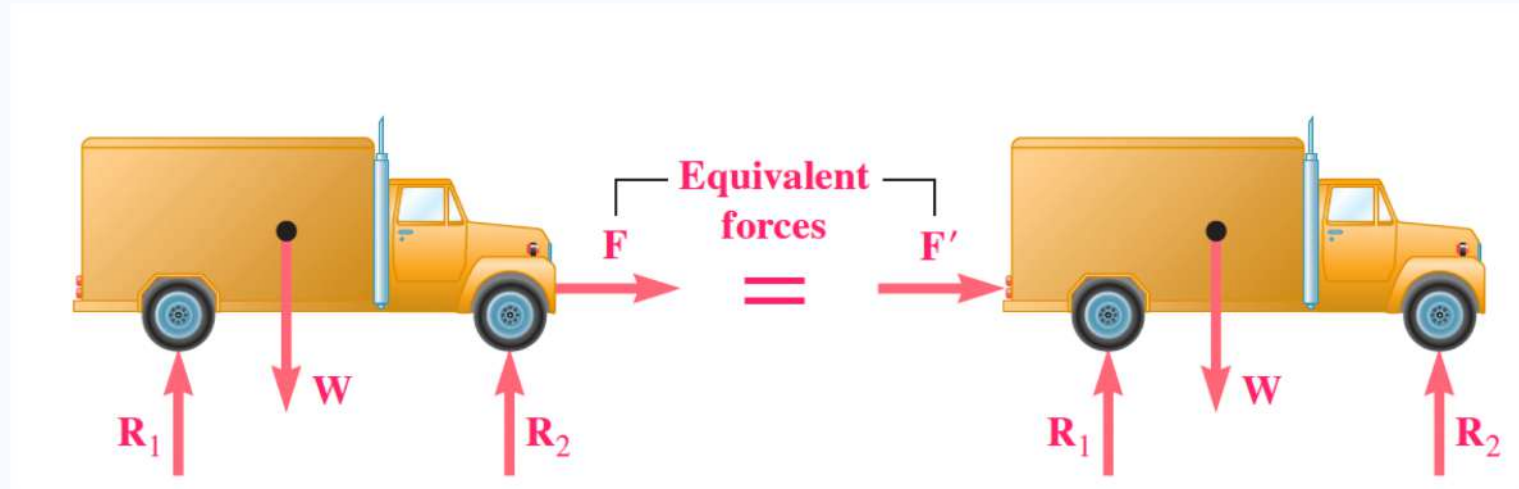
Example of external forces



1. Consider weight of the truck (W) acting downward and point of application – center of gravity
2. The weight W tends to make the truck move vertically downward. In fact, it would actually cause the truck to move downward, i.e., to fall, if it were not for the presence of the ground.
3. The ground opposes the downward motion of the truck by means of the reactions **R_1 and R_2** .
4. The people pulling on the rope exert the force **F** .

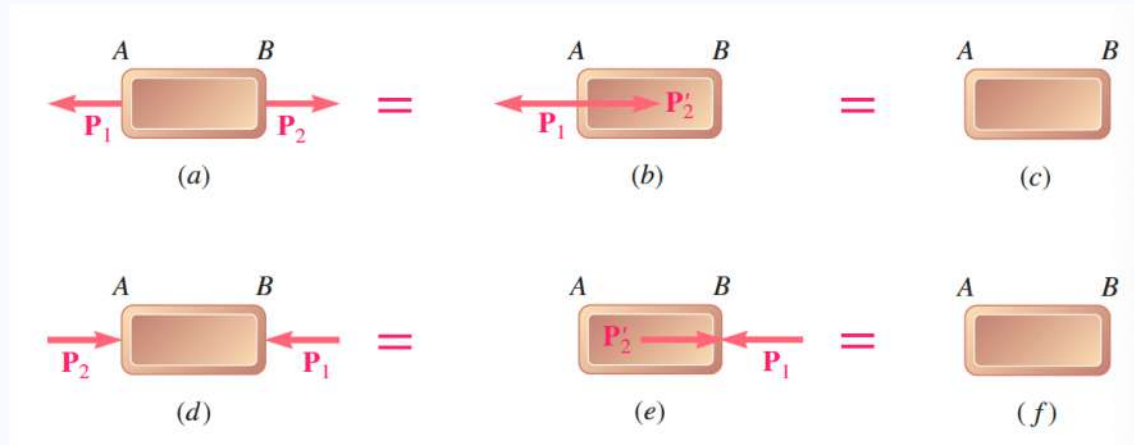
Principle of Transmissibility

- The principle of transmissibility states that the **conditions of equilibrium** or motion of a rigid body **remain unchanged** if a force F acting at a given point of the rigid body is replaced by a force F' of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action.



Limitation of Principle of Transmissibility

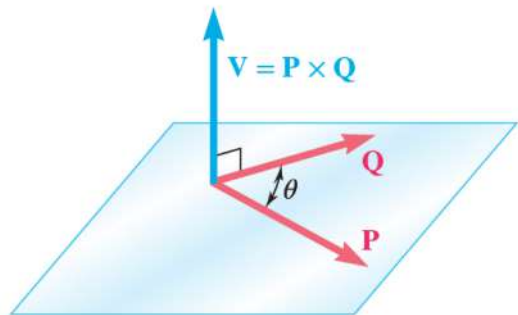
- The principle of transmissibility have limitations.



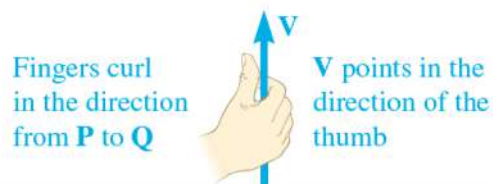
From mechanics point of view, these both bodies are in equilibrium

Thus, although we can use the principle of transmissibility to determine the conditions of motion or equilibrium of rigid bodies and to compute the external forces acting on these bodies, it should be avoided, or at least used with care, in determining internal forces and deformations.

Vector Products



(a)



The vector product of two vectors $\mathbf{P}$ and $\mathbf{Q}$ is defined as the vector $\mathbf{V}$ that satisfies the following conditions.

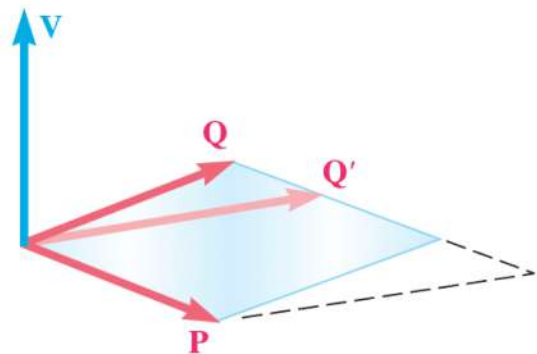
1. The line of action of $\mathbf{V}$ is perpendicular to the plane containing $\mathbf{P}$ and $\mathbf{Q}$
2. The magnitude of $\mathbf{V}$ is the product of the magnitudes of $\mathbf{P}$ and $\mathbf{Q}$ and of the sine of the angle θ formed by $\mathbf{P}$ and $\mathbf{Q}$ (the measure of which is always 180°). We thus have

Magnitude of a vector product

$$V = PQ \sin \theta$$

3. The direction of $\mathbf{V}$ is obtained from the right-hand rule

Vector Products

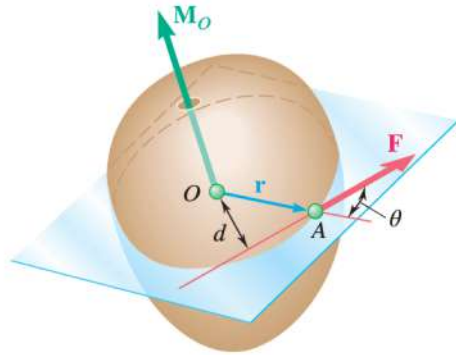


- Vector product : $\mathbf{V} = \mathbf{P} \times \mathbf{Q}$
- If the vectors $\mathbf{P}$ and $\mathbf{Q}$ have either the same direction or opposite directions, their vector product is zero.
- Magnitude $\mathbf{V}$ of the vector product of $\mathbf{P}$ and $\mathbf{Q}$ is equal to the area of the parallelogram that has $\mathbf{P}$ and $\mathbf{Q}$ for sides. The vector product $\mathbf{P} \times \mathbf{Q}$ is therefore unchanged if we replace $\mathbf{Q}$ by a vector $\mathbf{Q}'$ that is coplanar with $\mathbf{P}$ and $\mathbf{Q}$ such that the line joining the tips of $\mathbf{Q}$ and $\mathbf{Q}'$ is parallel to $\mathbf{P}$.

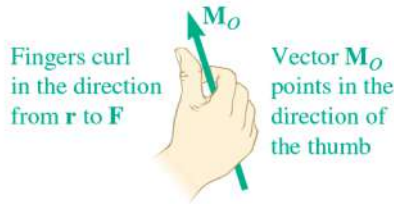
$$\mathbf{V} = \mathbf{P} \times \mathbf{Q} = \mathbf{P} \times \mathbf{Q}'$$

- Vector products are not commutative
 $\mathbf{P} \times \mathbf{Q} \neq \mathbf{Q} \times \mathbf{P}$
- Vectors can demonstrate distributive property
 $\mathbf{P} \times (\mathbf{Q}_1 + \mathbf{Q}_2) = \mathbf{P} \times \mathbf{Q}_1 + \mathbf{P} \times \mathbf{Q}_2$
- Associative law doesn't apply
 $(\mathbf{P} \times \mathbf{Q}) \times \mathbf{S} \neq \mathbf{P} \times (\mathbf{Q} \times \mathbf{S})$

Moment of Force about a Point



(a)



- Consider a force F acting on a rigid body (Fig. a). As we know, the force F is represented by a vector that defines its magnitude and direction. However, the effect of the force on the rigid body depends also upon its point of application A . The position of A can be conveniently defined by the vector $\mathbf{r}$ that joins the fixed reference point O with A ; this vector is known as the **position vector** of A .
- Moment of a force about a point O
$$M_O = \mathbf{r} \times \mathbf{F} = rF\sin\theta$$
- Experimentally, the tendency of a force F to make a rigid body rotate about a fixed axis perpendicular to the force depends upon the distance of F from that axis, as well as upon the magnitude of F .

Moment of Force about a Point

The perpendicular distance between the rotation point and the line of action of the force (often called the moment arm) is just as important.

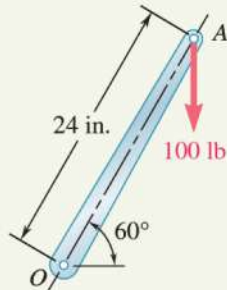
The magnitude of M_O measures the tendency of the force F to make the rigid body rotate about a fixed axis directed along M_O .

Restating principle of transmissibility: **Two forces F and F' are equivalent if, and only if, they are equal (i.e., have the same magnitude and same direction) and have equal moments about a given point O .**

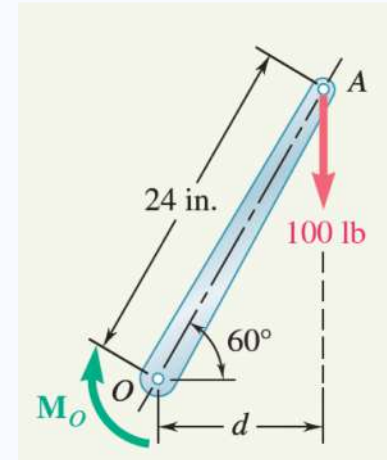
The necessary and sufficient conditions for two forces F and F' to be equivalent are thus $F = F'$ and $M_O = M'$

Problem 1

A 100-lb vertical force is applied to the end of a lever, which is attached to a shaft at O . Determine (a) the moment of the 100-lb force about O ; (b) the horizontal force applied at A that creates the same moment about O ; (c) the smallest force applied at A that creates the same moment about O ; (d) how far from the shaft a 240-lb vertical force must act to create the same moment about O ; (e) whether any one of the forces obtained in parts b , c , or d is equivalent to the original force.



For (a) part

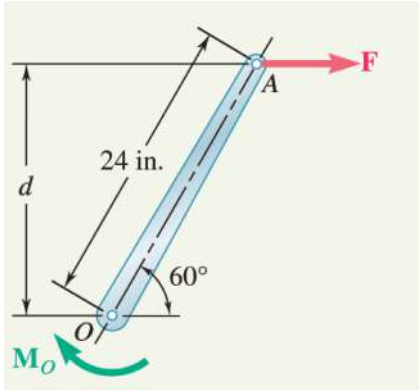


(a) **Moment about O.** The perpendicular distance from O to the line of action of the 100-lb force is

$$d = (24 \text{ in.}) \cos 60^\circ = 12 \text{ in.}$$

The magnitude of the moment about O of the 100-lb force is

$$M_O = Fd = (100 \text{ lb}) (12 \text{ in.}) = 1200 \text{ lb.in.}$$



b. Horizontal Force. In this case, we have

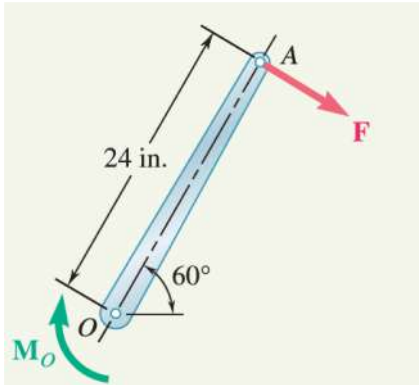
$$d = (24 \text{ in.}) \sin 60 = 20.8 \text{ in.}$$

Because the moment about O must be 1200 lb.in., you obtain

$$M_O = Fd$$

$$1200 \text{ lb.in.} = F (20.8 \text{ in.})$$

$$F = 57.7 \text{ lb}$$

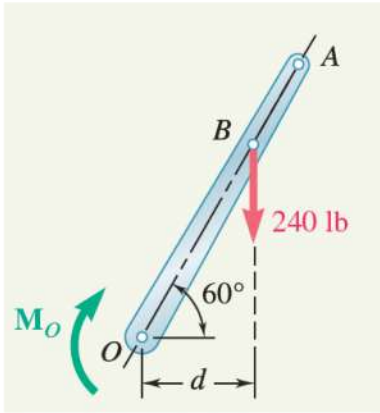


c. Smallest Force. Because $M_O = Fd$, the smallest value of F occurs when d is maximum. Choose the force perpendicular to OA and note that $d = 24$, thus,

$$M_O = Fd$$

$$1200 \text{ lb.in.} = F (24 \text{ in.})$$

$$F = 50 \text{ lb}$$



d. **240-lb Vertical Force.** In this case $M_O = Fd$ yields
 $1200 \text{ lb.in.} = (240 \text{ lb}) d$ $d = 5 \text{ in.}$

but

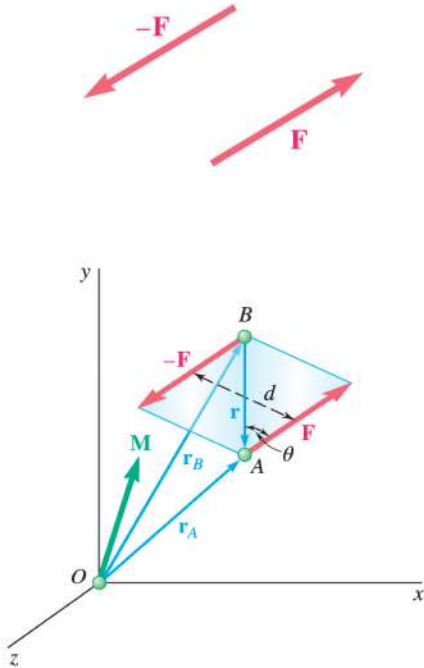
$$OB \cos 60^\circ = d$$

so

$$OB = 10 \text{ in.} \quad \blacktriangleleft$$

e. None of the forces considered in parts b, c, or d is equivalent to the original 100-lb force. Although they have the same moment about O, they have different x and y components. In other words, although each force tends to rotate the shaft in the same direction, each causes the lever to pull on the shaft in a different way.

Moment of couple



- Two forces F and $-F$, having the same magnitude, parallel lines of action, and opposite sense, are said to form a **couple**
- The sum of the components of the two forces in any direction is zero. The sum of the moments of the two forces about a given point, however, is not zero. The two forces do not cause the body on which they act to move along a line (translation), but they do tend to make it rotate.
- The sum of the moments of the two forces about O is

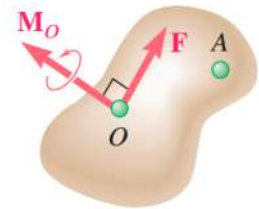
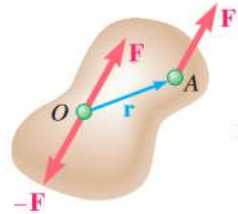
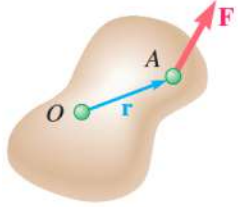
$$\mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

Setting $\mathbf{r}_A - \mathbf{r}_B = \mathbf{r}$, where $\mathbf{r}$ is the vector joining the points of application of the two forces, we conclude that the sum of the moments of F and $-F$ about O is represented by the vector

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

The vector $\mathbf{M}$ is called the **moment of the couple**
- $M = rF \sin \theta = Fd$

Force couple system

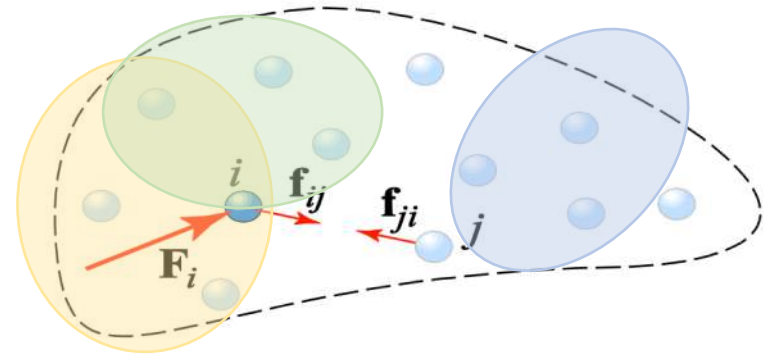


- Consider a force F acting on a rigid body at a point A defined by the position vector r (Fig. a).
- Suppose that for some reason it would simplify the analysis to have the force act at point O instead. Although we can move F along its line of action (principle of transmissibility), we cannot move it to a point O that does not lie on the original line of action without modifying the action of F on the rigid body.
- We can, however, attach two forces at point O, one equal to F and the other equal to $-F$, without modifying the action of the original force on the rigid body (Fig.b). As a result of this transformation, we now have a force F applied at O; the other two forces form a couple of moment $M_O = r \times F$.
- **Any force F acting on a rigid body can be moved to an arbitrary point O provided that we add a couple whose moment is equal to the moment of F about O.**

Equilibrium conditions

- What are the condition for a system a particles to be in equilibrium?
- Necessity condition: Resultant forces of particles and resultant moment of these forces about any point should vanish.

$$\sum F = 0 \qquad \sum M = 0$$



- Sufficient condition: The above condition becomes sufficient if every subsystem satisfies them.

Equilibrium of Rigid body

$$\sum \vec{F} = 0 \quad \sum \vec{M} = 0$$

- Writing in component form

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

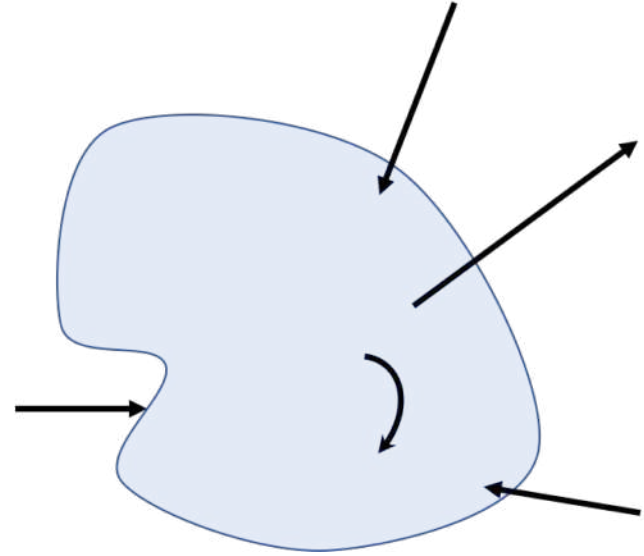
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- Unknown forces can be determined by applying above conditions

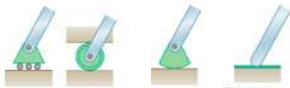
Equilibrium condition for coplanar force system

- In coplanar system, all forces are in the same planes
- No forces are perpendicular to the plane.
- Only moments perpendicular to the plane exist.

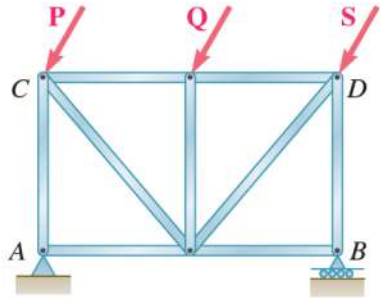
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$



Support Reactions in 2D

Support or Connection	Reaction	Number of Unknowns
 Rollers Rocker Frictionless surface	 Force with known line of action perpendicular to surface	1
 Short cable Short link	 Force with known line of action along cable or link	1
 Collar on frictionless rod Frictionless pin in slot	 Force with known line of action perpendicular to rod or slot	1
 Frictionless pin or hinge Rough surface	 Force of unknown direction	2
 Fixed support	 Force and couple	3

Equilibrium condition for 2D systems



Consider Fig., in which the truss shown is in equilibrium and is subjected to the given forces P, Q, and S.

- $\Sigma F_x = 0$

$$\Sigma F_y = 0$$

$$\Sigma M_A = 0$$

- $\Sigma F_x = 0$

$$\Sigma M_B = 0$$

$$\Sigma M_A = 0$$

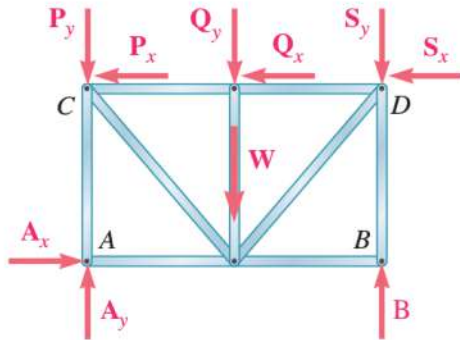
The moments are summed (in this case, point B) cannot lie on the line parallel to the y axis that passes through point A

- $\Sigma M_C = 0$

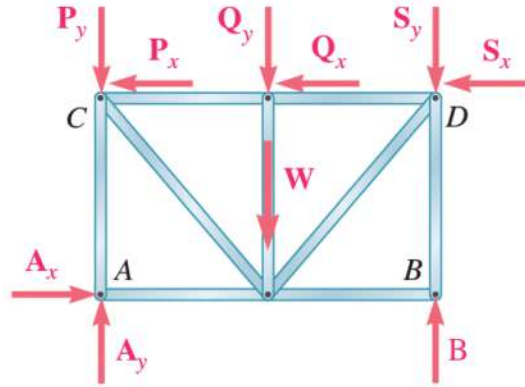
$$\Sigma M_B = 0$$

$$\Sigma M_A = 0$$

The points A, B, C do not lie in a straight line

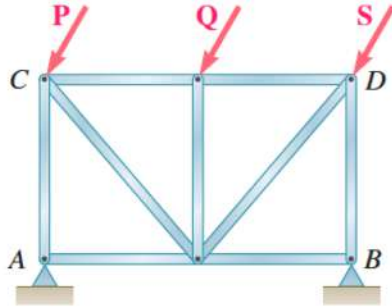


Statically determinate reactions and completely constrain



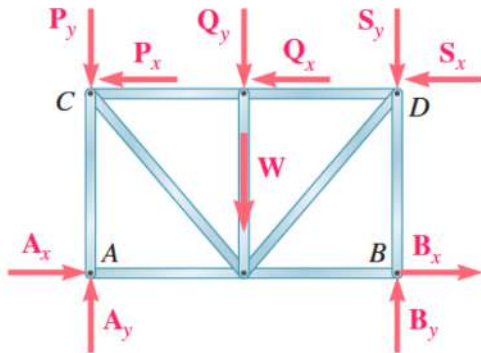
- The types of supports used were such that the rigid body could not possibly move under the given loads or under any other loading conditions – **Completely constrained**
- Involved three unknowns and could be determined by solving the three equations of equilibrium – **Statically determinate system**

Statically indeterminate reactions and completely constrain



(a)

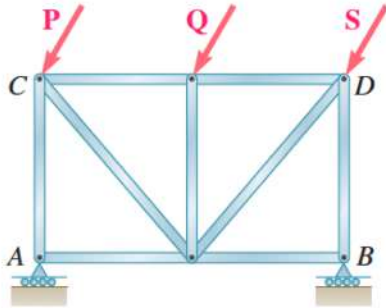
- The types of supports used were such that the rigid body could not possibly move under the given loads or under any other loading conditions – **Completely constrained**



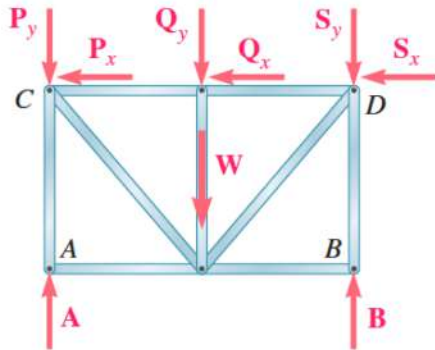
(b)

- Involved four unknowns and couldn't be determined by solving the three equations of equilibrium – **Statically indeterminate system**

Statically indeterminate reactions and partial constrained



(a)



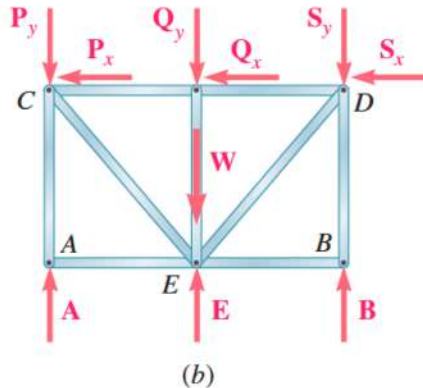
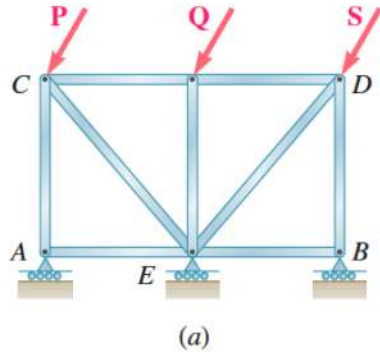
(b)

- The constraints provided by these supports are **not sufficient** to keep the truss from moving. Although they prevent any vertical motion, the **truss is free to move horizontally – partially constrained**.
- Involved two unknowns and could be determined by solving the two equations of equilibrium.

Statically indeterminate reactions and partial constrained

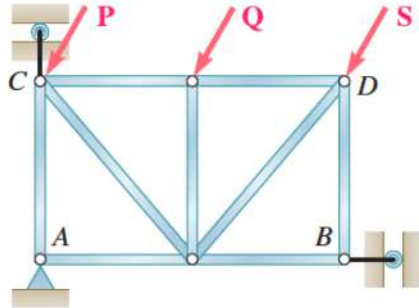
- It would appear that, if a rigid body is to be completely constrained and if the reactions at its supports are to be statically determinate, there must be as many unknowns as there are equations of equilibrium.
- When this condition is not satisfied, we can be certain that either the rigid body is not completely constrained or that the reactions at its supports are not statically determinate.
- It is also possible that the rigid body is not completely constrained and that the reactions are statically indeterminate.
- You should note, however, that, although this condition is necessary, it is not sufficient
- In other words, the fact that the number of unknowns is equal to the number of equations is no guarantee that a body is completely constrained or that the reactions at its supports are statically determinate.

Statically indeterminate reactions and partial constrained



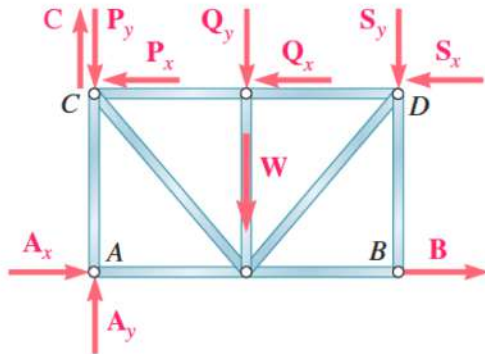
- We have three unknown reactions of A, B, and E, but the equation $\Sigma F_x = 0$ is not satisfied unless the sum of the horizontal components of the applied forces happens to be zero.
- Although there are a sufficient number of constraints, these constraints are not properly arranged, so the truss is free to move horizontally
- We say that the truss is **improperly constrained**.
- Because only two equilibrium equations are left for determining three unknowns, the reactions are statically indeterminate.
- Thus, improper constraints also produce static indeterminacy.

Statically indeterminate reactions and partial constrained

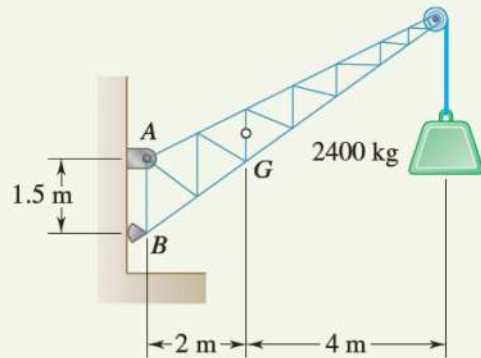


(a)

- Another example of improper constraints— and of static indeterminacy
- A rigid body is improperly constrained whenever the supports (even though they may provide a sufficient number of reactions) are arranged in such a way that the reactions must be either concurrent or parallel.
- To be sure that a two-dimensional rigid body is completely constrained and that the reactions at its supports are statically determinate, you should verify that the reactions involve three—and only three—unknowns and that the supports are arranged in such a way that they do not require the reactions to be either concurrent or parallel.



Problem 1



A fixed crane has a mass of 1000 kg and is used to lift a 2400-kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Determine the components of the reactions at A and B.

- **Determination of B.**

$$\Sigma M_A = 0:$$

$$+B(1.5 \text{ m}) - (9.81 \text{ kN})(2 \text{ m}) - (23.5 \text{ kN})(6 \text{ m}) = 0$$

$$B = +107.1 \text{ kN}$$

$$B = 107.1 \text{ kN} \rightarrow$$

- **Determination of A_x .** $\rightarrow + \Sigma F_x = 0:$

$$A_x + B = 0$$

$$A_x + 107.1 \text{ kN} = 0$$

$$A_x = -107.1 \text{ kN}$$

$$A_x = 107.1 \text{ kN} \leftarrow$$

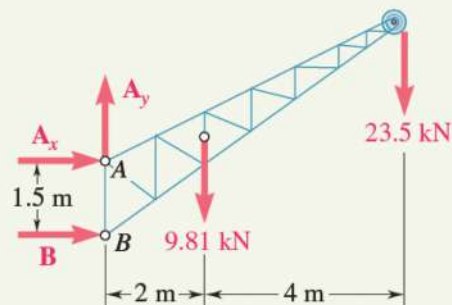
- **Determination of A_y .** $+ \uparrow \Sigma F_y = 0: A_y - 9.81 \text{ kN} - 23.5 \text{ kN} = 0$

$$A_y = +33.3 \text{ kN}$$

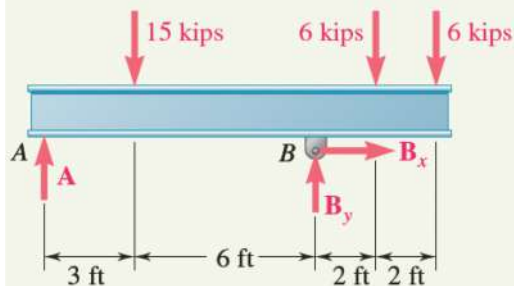
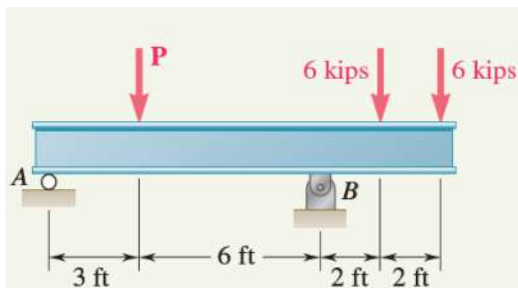
$$A_y = 33.3 \text{ kN} \uparrow$$

Adding the components A_x and A_y vectorially,

$$A \text{ is } 112.2 \text{ kN.}$$



Problem 2



Three loads are applied to a beam as shown. The beam is supported by a roller at A and by a pin at B. Neglecting the weight of the beam, determine the reactions at A and B when $P = 15$ kips.

STRATEGY: Draw a free-body diagram of the beam, then write the equilibrium equations, first summing forces in the x direction and then summing moments at A and at B.

MODELING:

Free-Body Diagram. The reaction at A is vertical and is denoted by **A** (Fig. 1). Represent the reaction at B by components **B_x** and **B_y**. Assume that each component acts in the direction shown.

ANALYSIS:

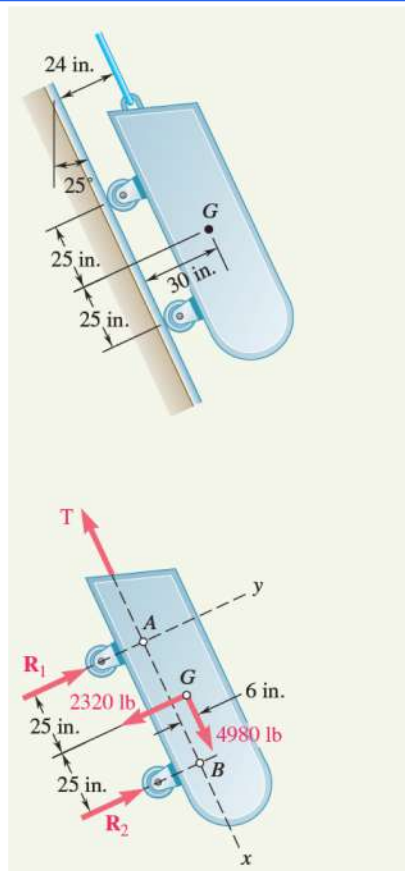
Equilibrium Equations. Write the three equilibrium equations and solve for the reactions indicated:

$$\rightarrow \Sigma F_x = 0: \quad B_x = 0 \quad \mathbf{B_x = 0} \quad \blacktriangleleft$$

$$\begin{aligned} +\circlearrowleft \Sigma M_A = 0: \\ -(15 \text{ kips})(3 \text{ ft}) + B_y(9 \text{ ft}) - (6 \text{ kips})(11 \text{ ft}) - (6 \text{ kips})(13 \text{ ft}) = 0 \\ B_y = +21.0 \text{ kips} \quad \mathbf{B_y = 21.0 \text{ kips} \uparrow} \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} +\circlearrowleft \Sigma M_B = 0: \\ -A(9 \text{ ft}) + (15 \text{ kips})(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (6 \text{ kips})(4 \text{ ft}) = 0 \\ A = +6.00 \text{ kips} \quad \mathbf{A = 6.00 \text{ kips} \uparrow} \quad \blacktriangleleft \end{aligned}$$

Problem 3



A loading car is at rest on a track forming an angle of 25° with the vertical. The gross weight of the car and its load is 5500 lb, and it acts at a point 30 in. from the track, halfway between the two axles. The car is held by a cable attached 24 in. from the track. Determine the tension in the cable and the reaction at each pair of wheels.

Free-Body Diagram. The reaction at each wheel is perpendicular to the track, and the tension force T is parallel to the track. Therefore, for convenience, choose the x axis parallel to the track and the y axis perpendicular to the track (Fig. 1). Then, resolve the 5500-lb weight into x and y components.

$$W_x = +(5500 \text{ lb}) \cos 25^\circ = +4980 \text{ lb}$$

$$W_y = -(5500 \text{ lb}) \sin 25^\circ = -2320 \text{ lb}$$

ANALYSIS:

Equilibrium Equations. Take moments about A to eliminate T and R_1 from the computation.

$$+\circlearrowleft \Sigma M_A = 0: \quad -(2320 \text{ lb})(25 \text{ in.}) - (4980 \text{ lb})(6 \text{ in.}) + R_2(50 \text{ in.}) = 0$$

$$R_2 = +1758 \text{ lb} \nearrow$$

Then, take moments about B to eliminate T and R_2 from the computation.

$$+\circlearrowleft \Sigma M_B = 0: \quad (2320 \text{ lb})(25 \text{ in.}) - (4980 \text{ lb})(6 \text{ in.}) - R_1(50 \text{ in.}) = 0$$

$$R_1 = +562 \text{ lb} \nearrow$$

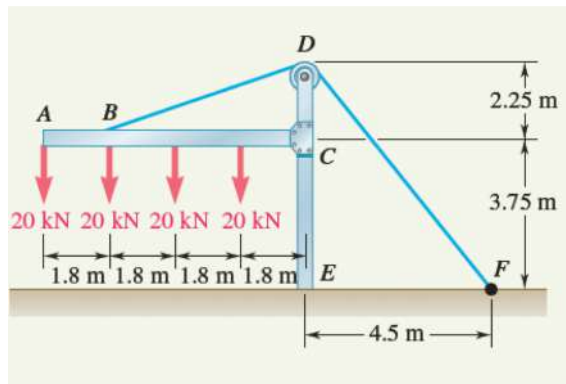
Determine the value of T by summing forces in the x direction.

$$\searrow + \Sigma F_x = 0: \quad +4980 \text{ lb} - T = 0$$

$$T = +4980 \text{ lb}$$

$$T = 4980 \text{ lb} \searrow$$

Problem 4



The frame shown supports part of the roof of a small building. Knowing that the tension in the cable is 150 kN, determine the reaction at the fixed end E.

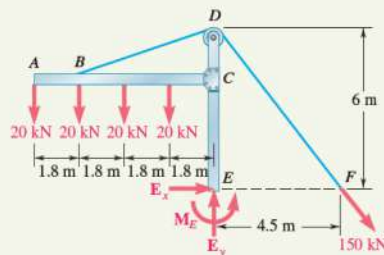


Fig. 1 Free-body diagram of frame.

ANALYSIS:

Equilibrium Equations. First note that

$$DF = \sqrt{(4.5 \text{ m})^2 + (6 \text{ m})^2} = 7.5 \text{ m}$$

Then, you can write the three equilibrium equations and solve for the reactions at E.

$$\begin{aligned} \rightarrow \Sigma F_x = 0: & \quad E_x + \frac{4.5}{7.5}(150 \text{ kN}) = 0 \\ & \quad E_x = -90.0 \text{ kN} \quad \mathbf{E_x = 90.0 \text{ kN} \leftarrow} \\ +\uparrow \Sigma F_y = 0: & \quad E_y - 4(20 \text{ kN}) - \frac{6}{7.5}(150 \text{ kN}) = 0 \\ & \quad E_y = +200 \text{ kN} \quad \mathbf{E_y = 200 \text{ kN} \uparrow} \\ +\circlearrowleft \Sigma M_E = 0: & \quad (20 \text{ kN})(7.2 \text{ m}) + (20 \text{ kN})(5.4 \text{ m}) + (20 \text{ kN})(3.6 \text{ m}) \\ & \quad + (20 \text{ kN})(1.8 \text{ m}) - \frac{6}{7.5}(150 \text{ kN})(4.5 \text{ m}) + M_E = 0 \\ & \quad M_E = +180.0 \text{ kN}\cdot\text{m} \quad \mathbf{M_E = 180.0 \text{ kN}\cdot\text{m} \circlearrowleft} \end{aligned}$$

Simplifies use of equilibrium equation



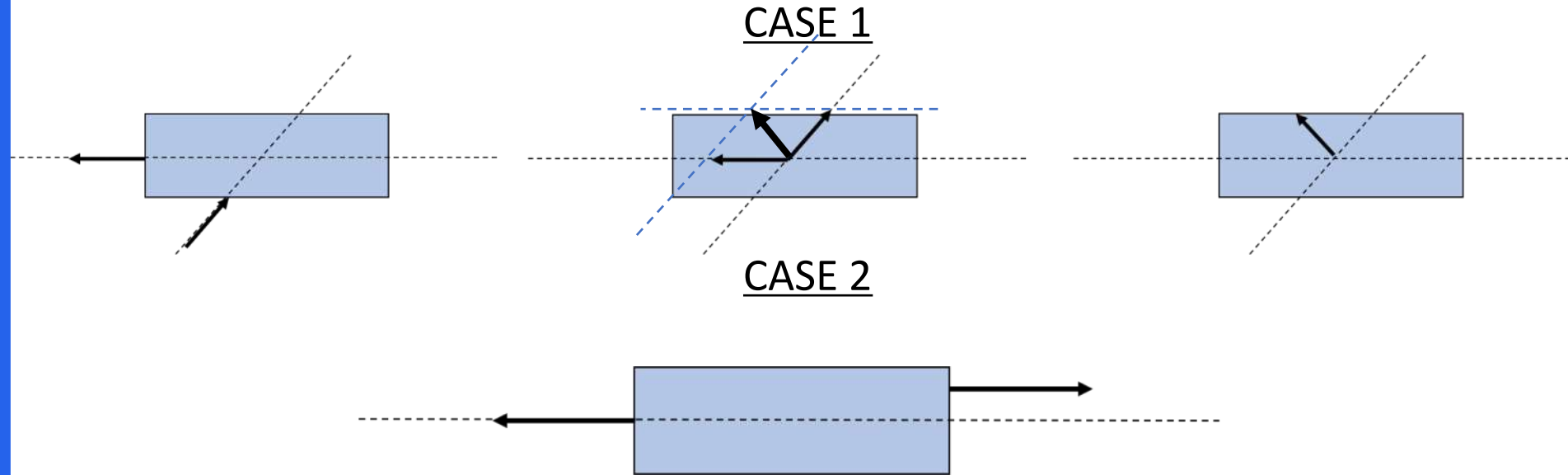
- At least two forces are required to keep the body in equilibrium



- A member acted by 2 forces: **two force member**
- How the forces need to be aligned for the member to be in equilibrium?

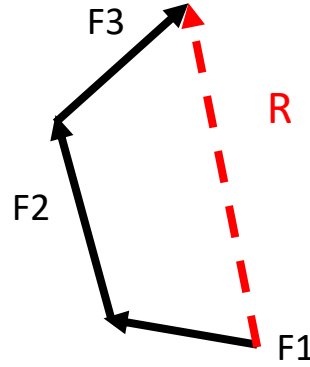
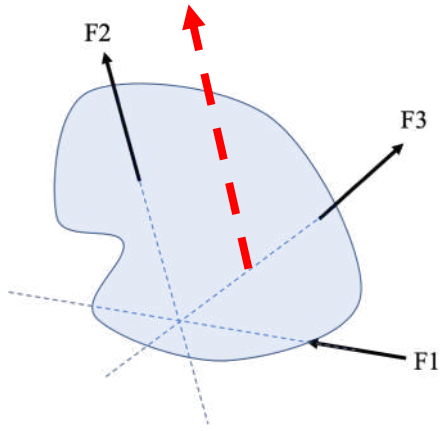
Collinear and equal and opposite

Simplifies use of equilibrium equation

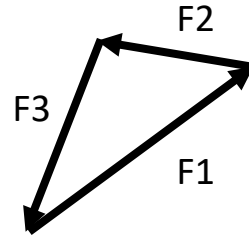
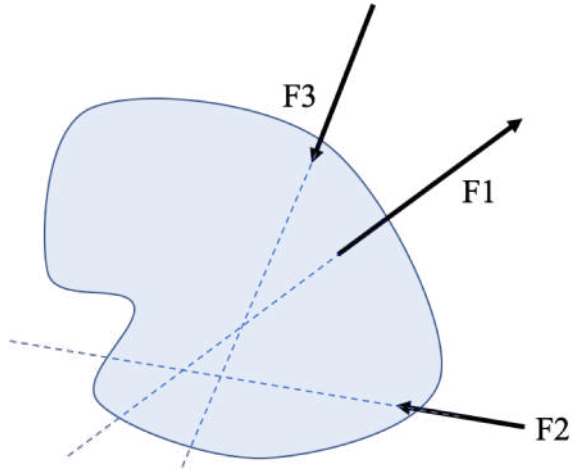


- Thus, **collinear, equal and opposite forces** are needed to keep the member in equilibrium

Equilibrium condition for three force member

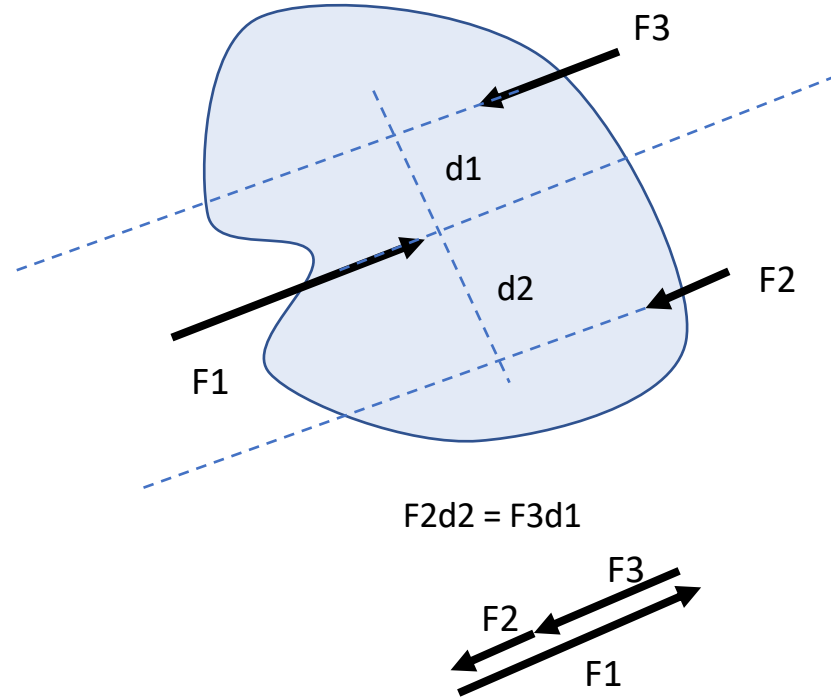
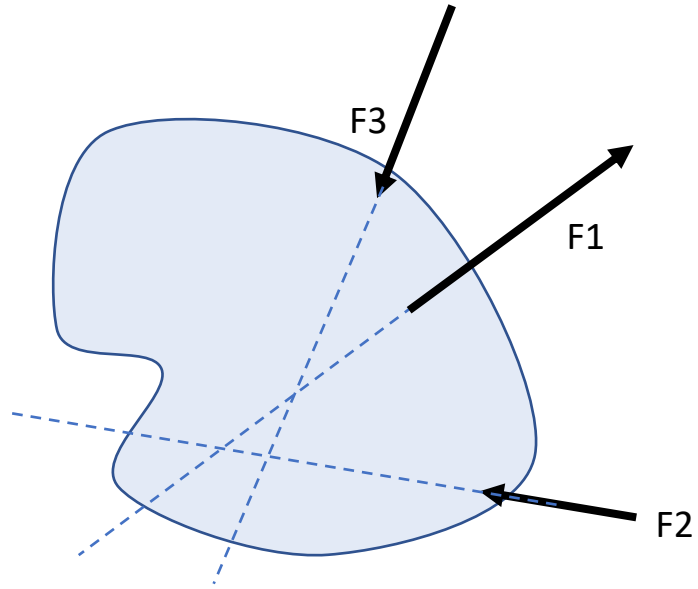


$$\Sigma \mathbf{F} \neq \mathbf{0}$$

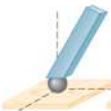
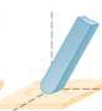
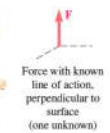
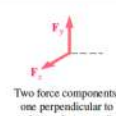
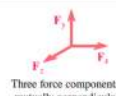
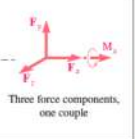
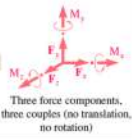


$$\Sigma \mathbf{M} \neq \mathbf{0}$$

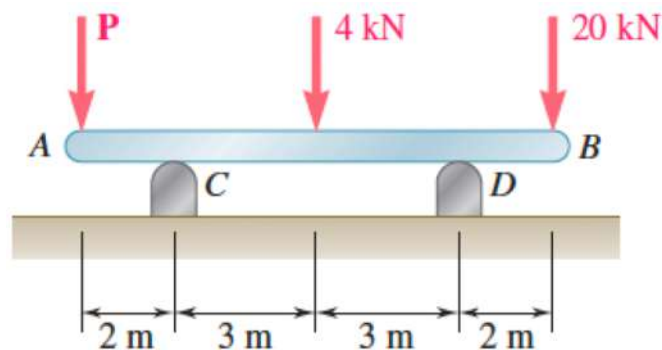
Equilibrium condition for three force member corrected



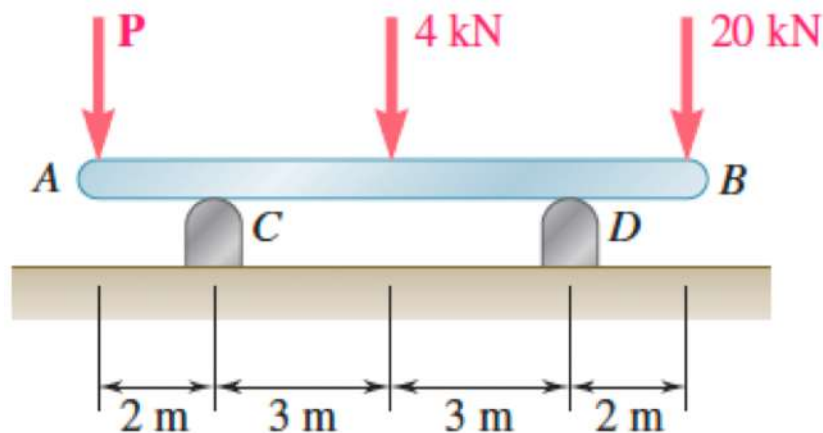
Support Reactions in 3D

 Ball	 Frictionless surface	 Force with known line of action, perpendicular to surface (one unknown)	 Cable  Force with known line of action, along cable (one unknown)		
 Roller on rough surface	 Wheel on rail	 Two force components, one perpendicular to surface and one parallel to axis of wheel			
 Rough surface	 Ball and socket	 Three force components, mutually perpendicular at point of contact			
 Universal joint	 Three force components, one couple				
 Fixed support	 Three force components, three couples (no translation, no rotation)				
 Hinge and bearing supporting radial load only		 Two force components and up to two couples			
 Pin and bracket	 Hinge and bearing supporting axial thrust and radial load		 Three force components and up to two couples		

The 10-m beam AB rests upon, but is not attached to, supports at C and D . Neglecting the weight of the beam, determine the range of values of P for which the beam will remain in equilibrium.



The maximum allowable value of each of the reactions is 50 kN, and each reaction must be directed upward. Neglecting the weight of the beam, determine the range of values of P for which the beam is safe.



A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two (a) rear wheels A , (b) front wheels B .

