S/K/TY

Reg. No: 25BLE2811

Final Assessment Test – November 2025

VIT® Vellore Institute of Technology

Course: BAMAT101 - Multivariable Calculus and Differential Equations

Class NBR(s): 5330 / 5333 / 5336 / 5347 / 5349 / 5354 / 5366 / 5370 / 5373 / 5387 / 5390 / 5397 / 5400 / 5406 / 5409 / 5413 / 5417 / 5419 / 5421 / 8710

Slot: B1+TB1

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

COs	CO Statements
CO1	Evaluate partial derivatives and solve optimization problems involving several variables with or without constraints
CO2	Solve multiple integrals in cartesian, polar, cylindrical and spherical coordinates.
CO3	Understand gradient, directional derivatives, divergence, curl, Green's, Stokes and Gauss Divergence theorems.
CO4	Utilize different solution methods to obtain the solution for second-order ordinary differential equations.
CO5	Demonstrate proficiency in solving linear and nonlinear first-order partial differential equations.

BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)

Answer ALL Questions

(10 X 10 = 100 Marks)

1. Show that $u = x^3 - y^3z^3$, $v = x^2 + y^2z^2 + xyz$ and $w = x - yz$ are functionally dependent and also find the relation. **CO1 BL2**

2. A simple machine-learning model for a task uses two parameters w and v. The loss function (to be minimized) for a single example is modeled as **CO1 BL2**

$$L(w, v) = w^2 + wv + v^2 - v.$$

Find the parameter values (w, v) that minimize the loss, and state the minimum loss. Also say whether any maximum exists.

3. Change the order of integration, and evaluate $\int_0^1 \int_x^1 e^{y^2}\, dydx$. **CO2 BL3**

4.a) A data centre has a cuboid server room with dimensions x from 0 to 2 meters, y from 0 to 3 meters, and z from 0 to 4 meters. The temperature distribution in the room is modelled by: $T(x, y, z) = x + 2y + 3z$ (in degrees Celsius) Compute the total "heat content" of the room assuming a uniform density and unit volume coefficient (i.e., integrate $T(x, y, z)$over the room). **CO2 BL3**

OR

4.b) A spherical sensor network node has a signal strength distribution in space given by: $S(x, y, z) = x^2 + y^2 + z^2$. The node has coverage radius $R = 2$ meters. Compute the total signal strength within the spherical coverage region. **CO2 BL**

5. Let the vector field be: $\bar{F}(x,y,z) = (xy)\,\hat{i} + (y^2z)\,\hat{j} + (xz^2)\,\hat{k}$. CO3 BL3

(i) Find the gradient of the scalar function $f(x,y,z) = x^2y + yz^2$.

(ii) Find the divergence $\nabla \cdot \bar{F}$.

(iii) Find the curl $\nabla \times \bar{F}$.

(iv) Evaluate the divergence and curl at the point $(1,1,1)$.

6.a) A particle moves along the path C from point $A(0,0)$ to point $B(1,2)$ under the force field: $\bar{F}(x,y) = (2x+y)\,\hat{i} + (x+3y)\,\hat{j}$ CO3 BL3

(i) Find the work done by the force along the straight-line path from A to B.

(ii) Determine if the force field is conservative.

(iii) If conservative, find the potential function $\phi(x,y)$.

OR

6.b) Let the vector field be: $\bar{F}(x,y,z) = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$. Compute the surface integral of F over the surface of the unit sphere $x^2 + y^2 + z^2 = 1$ using the divergence theorem. Also, interpret the result in a computer science context, e.g., total "flow" of data in a spherical network node. CO3 BL3

7. Solve the differential equation: $y'' - 3y' + 2y = e^x + x$ using the method of undetermined coefficients. CO4 BL3

8. A network buffer receives packets at a varying rate modelled by $u(t) = 10e^{-t}$ packets/sec. The buffer has damping effects (due to congestion control) and natural flow characteristics. The number of packets in the buffer, $y(t)$, satisfies: $y'' + 3y' + 2y = 10e^{-t}, y(0) = 0, y'(0) = 0$. Find the number of packets in the buffer over time. CO4 BL3

9. (a) Form partial differential equation (PDE) by eliminating the arbitrary function: $xyz = f(x+y+z)$. [5] CO5 BL3

(b) Solve the PDE : $z = px + qy + e^p + \ln q$.

10. Solve $(y - xz)p + (yz - x)q = (x - y)(x + y)$. [5] CO5 BL3

⇔⇔⇔ S/K/TY ⇔⇔⇔