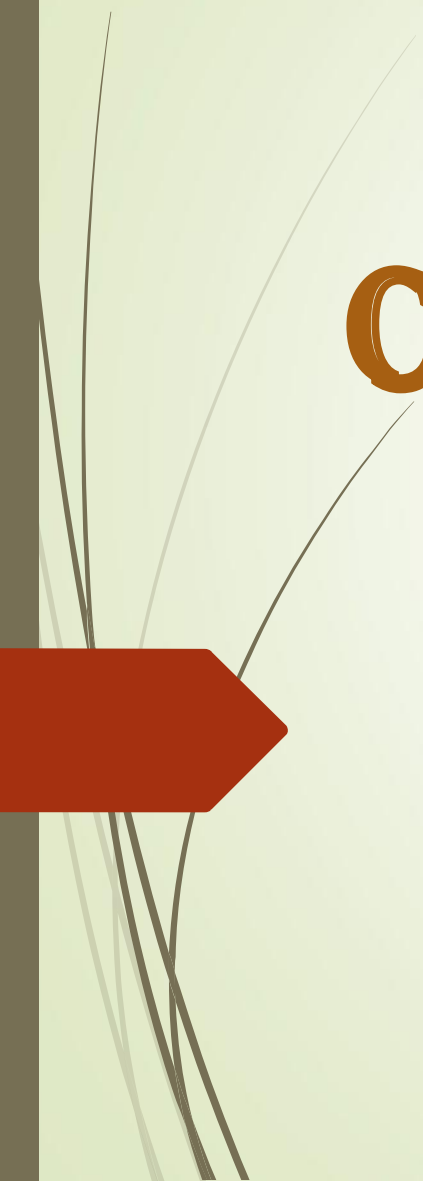




BEEE203L

CIRCUIT THEORY

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MODULE – IV

Analysis of Transient Response of Circuits

Contents

- Review of Laplace transformation;
- Laplace transform of network
- Time domain solution for RL,RC, RLC networks for DC excitation
- Time domain solution for RL,RC, RLC networks for AC excitation
- Transient Behaviour of circuit elements under switching conditions and their representations
- Evaluation of initial and final conditions in RL,RC,RLC circuits with DC excitation
- Evaluation of initial and final conditions in RL,RC,RLC circuits with AC excitation

Transient Response Analysis

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Introduction

- So far steady state analysis of electric circuits was discussed.
- Electric circuits will be subjected to sudden changes which may be in the form of opening and closing of switches or sudden changes in sources etc.
- Whenever such a change occurs, the circuit which was in a particular steady state condition will go to another steady state condition.
- Transient analysis is the analysis of the circuits during the time it changes from one steady state condition to another steady state condition.
- Transient analysis will reveal how the currents and voltages are changing during the transient period.

- ▶ To get such time responses, the mathematical models should necessarily be a set of differential equations.
- ▶ Transient response of simple circuits use classical method of solving differential equations
- ▶ Laplace Transform is a very useful tool for solving differential equations.

Transient Response Analysis

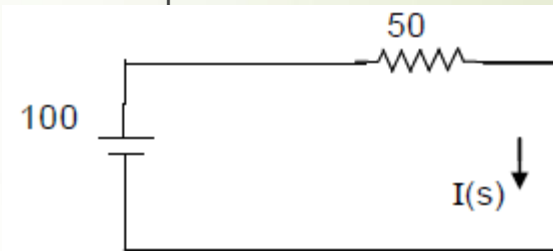
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Steady state

A circuit having voltage source or current source is said to be in steady state if voltage or currents does not change with respect to time

Ex in figure at $t = 2$ sec, $V = 100\text{V}$, $I = 2$ A

at $t = 5$ sec, $V = 100\text{V}$, $I = 2$ A

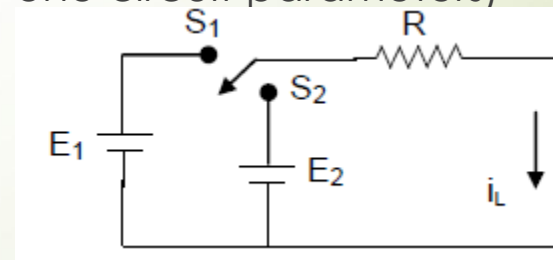


Transient state

A circuit having voltage source or current source is said to be in transient state if voltage or currents changes with respect to time (Changes may be due to change in voltage sources or any one circuit parameters)

Ex in figure at $t = 2$ sec, $V = E_1 = 100\text{V}$, $I = 2$ A

at $t = 5$ sec, $V = E_2 = 50\text{V}$, $I = 1$ A



Transient time

The time required or changing the voltage or current from one state to another state is called the Transient time.

Natural response

The natural response tells us what the circuit does as its internal stored energy is allowed to dissipate when there are no external influences

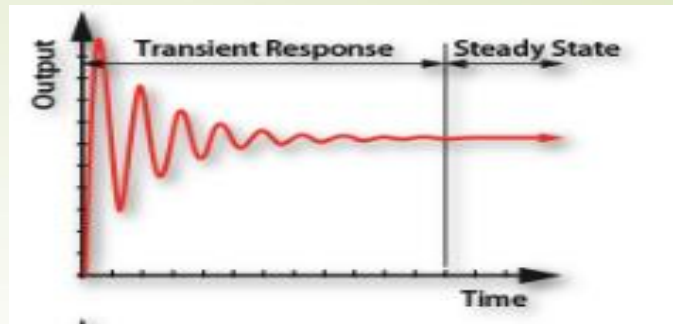
The "destination" of the natural response is always zero voltage and zero current..

Forced Response

The forced response is what the circuit does with the sources turned on, but with the initial conditions set to zero.

The total response is the sum of the forced response plus the natural response.

$$\text{Complete response} = \underset{\text{stored energy}}{\text{natural response}} + \underset{\text{independent source}}{\text{forced response}}$$



Steady state response

Steady state response refers to the behavior of a system under **stable input conditions** over time once **all initial transient behavior** has subsided.

Transient Response

The transient response of a circuit is a **temporary change in the way that it behaves due to an external excitation**, that will disappear with time.

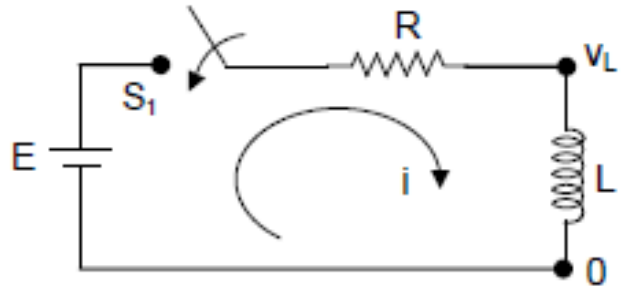
Total Response

In electrical engineering specifically, The total response is **sum of transient and steady state response**

$$\text{Complete response} = \underset{\text{temporary part}}{\text{transient response}} + \underset{\text{permanent part}}{\text{steady-state response}}$$

Transient response of RL circuit – Time domain approach

Consider the RL circuit shown in Fig. Assume that the switch is closed at time $t = 0$ and assume that the current i at the time of switching is zero.



Applying Kirchhoff's voltage law to the above circuit we get

$$V = V_R(t) + V_L(t)$$

$$i(t) = 0 \text{ for } t < 0 \quad \text{--(1)}$$

and Using the standard relationships of Voltage and Current for the Resistors and Inductors we can rewrite the above equations as

$$V = Ri + L \frac{di}{dt} \text{ for } t > 0$$

divide both sides with 'L'

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \longrightarrow (2)$$

It is a non-homogeneous equation

$$\frac{di}{dt} + P i = Q, \text{ where } P = \frac{R}{L}, Q = \frac{V}{L}$$

The solution for eqn (2) is

$$i(t) = K e^{-Pt} + \frac{Q}{P}$$

$$i(t) = K e^{-\frac{R}{L}t} + \frac{V}{R} \longrightarrow (3)$$

Subst. eqn (1) in eqn (3)

$$0 = K e^0 + \frac{V}{R}$$

$$\boxed{K = -\frac{V}{R}} \longrightarrow (4)$$

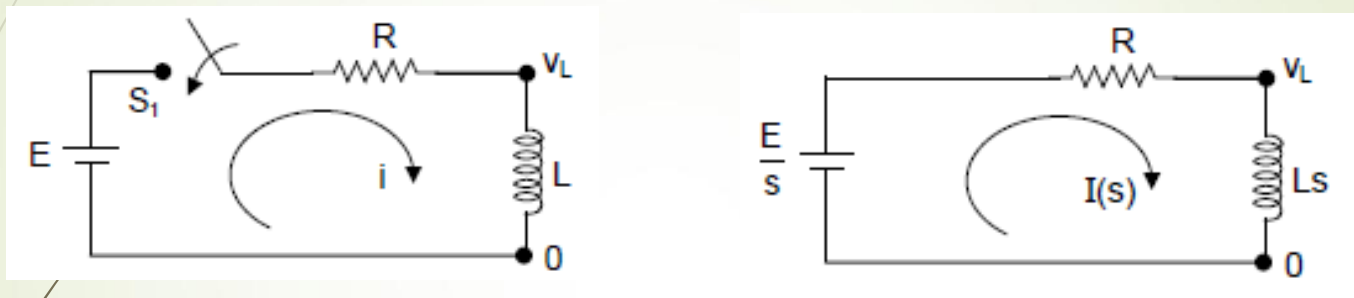
Subst. eqn (4) in eqn (3)

$$i(t) = -\frac{V}{R} e^{-\frac{R}{L}t} + \frac{V}{R}$$

$$\boxed{i(t) = \frac{V}{R} (1 - e^{-\frac{R}{L}t})} \longrightarrow (5)$$

Transient response of RL circuit – Laplace Transform approach

The transform circuit in s domain is shown in Fig.



$$I(s) = \frac{E/s}{R + Ls} = \frac{E/L}{s(s + \frac{R}{L})} = \frac{E/L}{R/L} \left(\frac{1}{s} - \frac{1}{s + \frac{R}{L}} \right) = \frac{E}{R} \left(\frac{1}{s} - \frac{1}{s + \frac{R}{L}} \right)$$

Taking inverse LT

$$i(t) = \frac{E}{R} (1 - e^{-\frac{R}{L}t})$$

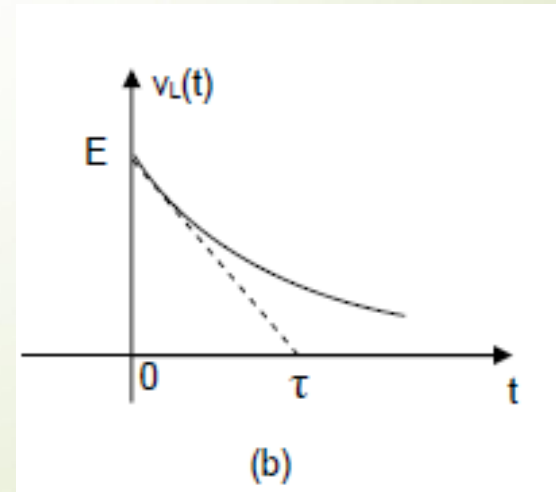
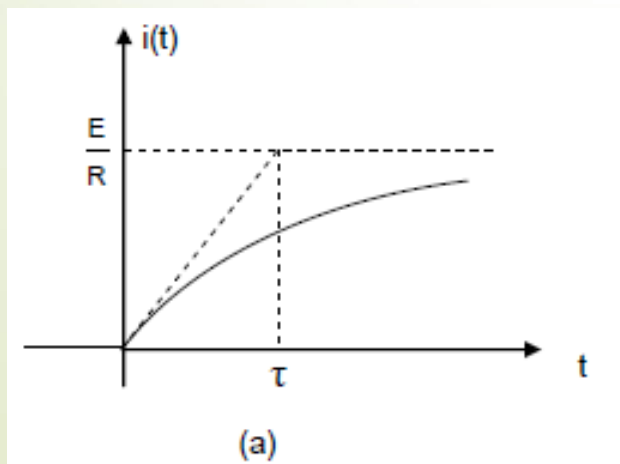
Thus, inductor current rises exponentially with time constant L / R .

Voltage across the inductor is given by

$$V(s) = L s I(s) = \frac{E}{s + \frac{R}{L}}$$

Taking inverse LT $v_L(t) = E e^{-\frac{R}{L}t}$

Inductor voltage increases exponentially with time constant L / R . The current and voltage transients are shown in Fig. (a) and (b)

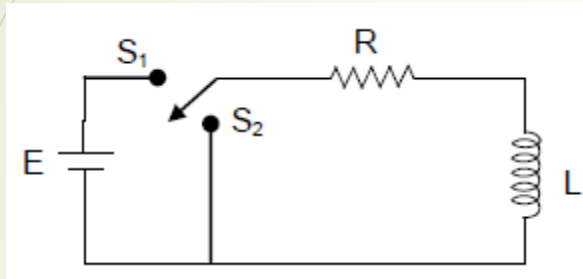


Transient response of RL circuit –

With initial conditions applied

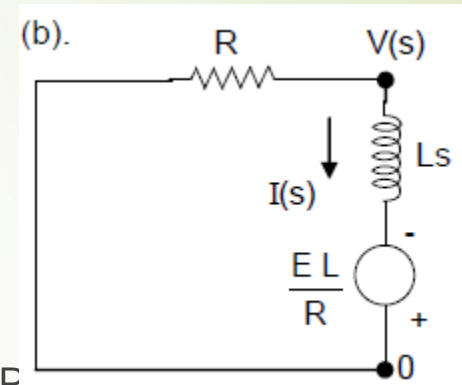
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Consider the circuit shown in Fig.(a). Let us say that with the switch in position S1, steady state condition is reached.



The current flowing through the inductor is E / R .

At time $t = 0$, the switch is turned to position S2. Then $i(0+) = i(0-) = E / R$. The transform circuit for time $t > 0$ is shown in Fig. (b).



Considering the transformed circuit

$$I(s) = \frac{\frac{EL}{R}}{R + Ls} = \frac{\frac{E}{R}}{s + \frac{R}{L}}$$

Taking inverse LT $i(t) = \frac{E}{R} e^{-\frac{R}{L}t}$

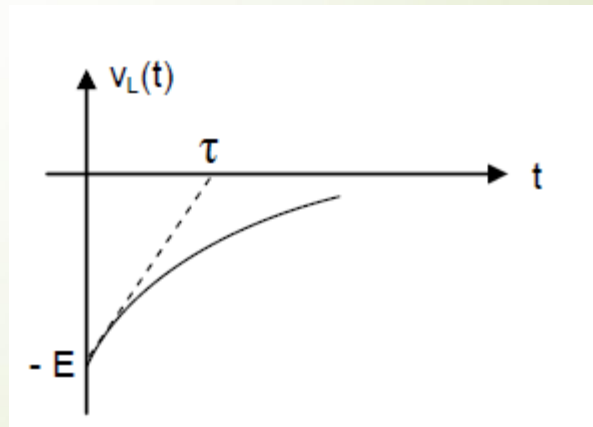
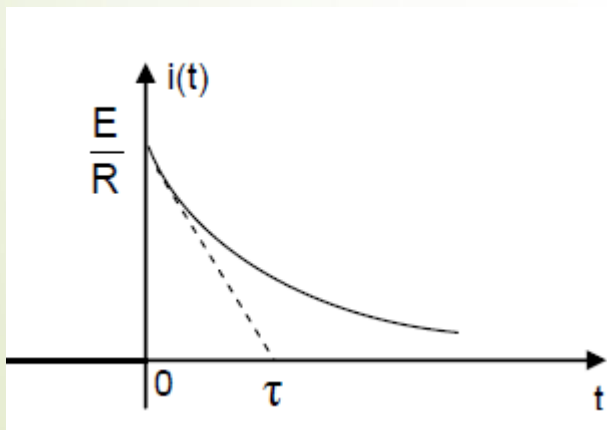
The current decays exponentially with time constant L / R .

- Since $R I(s) + V(s) = 0$ the voltage across the inductor is

$$V(s) = -R I(s) = -\frac{E}{s + \frac{R}{L}}$$

- Taking inverse LT $v_L(t) = -E e^{-\frac{R}{L}t}$

- The inductor voltage exponentially changes from $-E$ to zero with time constant L / R . The current and voltage transients are given by the above two equations are shown



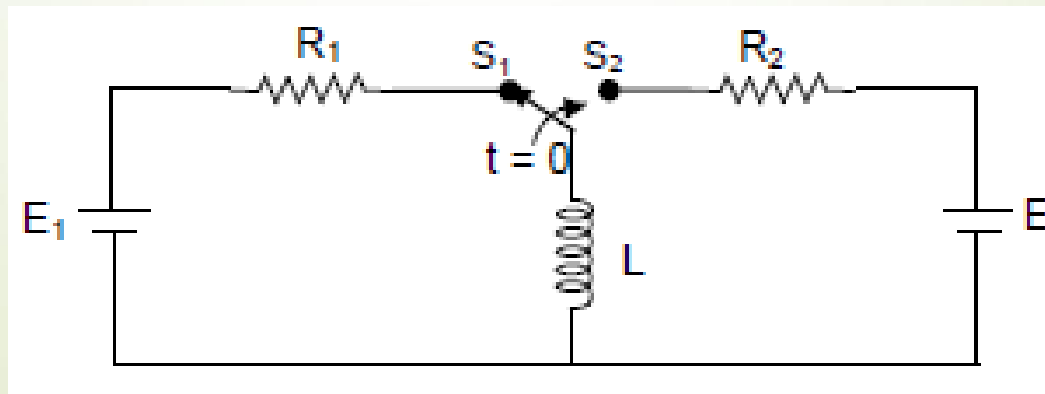
General case of finding the inductor current

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In the previous discussion, it was assumed that the initial inductor current $i_L(0) = 0$.

There may be very many situations wherein initial inductor current is not zero. The circuit arrangements can cause non-zero initial inductor current.

For this purpose consider the circuit shown below. The switch was in position S_1 for a long time. It is moved from position S_1 to S_2 at time $t = 0$.



We shall assume the following:

1. At time $t = 0^-$ the circuit was at steady state condition with the switch in position S_1
2. After switching to position S_2 , the circuit is allowed to reach the steady state condition. Thus, we are interested about the transient analysis for one switching period only.

Initial inductor current $i_L(0)$ is E_1/R_1 and the final inductor current $i_L(\infty)$, will be E_2/R_2 .

The more general expression for the inductor current can be obtained as

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-(R/L)t}$$

Circuit with multiple resistances

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In order to calculate the time constant L/R for circuits with multiple resistances, following steps to be followed

1. Calculate the Thevenin's Equivalent resistance across the Inductor (i.e. inductor as the load)
2. Time constant is given by L/R_{th}

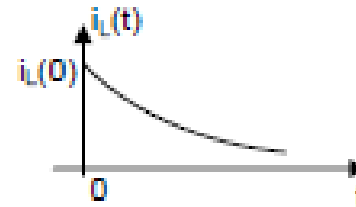
Summary of formulae useful for transient analysis on RL circuits

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1. Time constant $\tau = L / R$ Hence $\alpha = R / L$

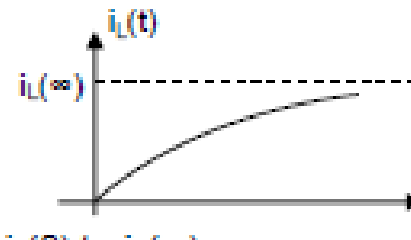
2. When the inductor current is decaying from the initial value of $i_L(0)$ to zero

$$i_L(t) = i_L(0) e^{-\frac{R}{L}t}$$



3. When the inductor current is exponentially increasing from zero to $i_L(\infty)$

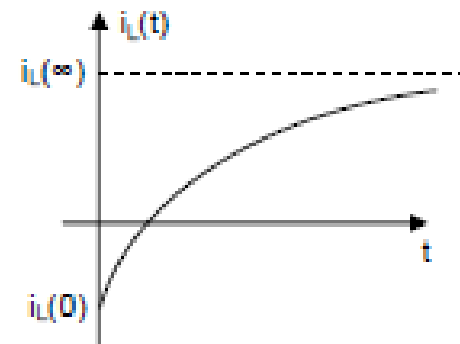
$$i_L(t) = i_L(\infty) (1 - e^{-\frac{R}{L}t})$$



4. When the inductor current changes from $i_L(0)$ to $i_L(\infty)$

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)] e^{-\frac{R}{L}t}$$

Plot of $i_L(t)$ depends on values of $i_L(0)$ and $i_L(\infty)$



5. Inductor voltage $v_L(t) = L \frac{di_L(t)}{dt}$

$$v_L(t) = -E e^{-\frac{R}{L}t}$$

Problems

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P.4.8 A series RL circuit has $R = 25 \Omega$ and $L = 5$ Henry. A dc voltage V of 100 V is applied to this circuit at $t = 0$ secs. Find :

- (a) The equations for the charging current , and voltage across R & L
- (b) The current in the circuit 0.5 secs after the voltage is applied.
- (c) The time at which the drops across R and L are equal.

Solution: The solutions for (a) and (b) are straightforward

(a) Time constant τ of the series RL circuit which is given by $\tau = L/R$ secs

$$\therefore \tau = 5/25 = 1/5 \text{ secs}$$

➡ The charging current is given by $i(t) = V/R (1 - e^{-t/\tau})$

$$= 100/25 (1 - e^{-t/(1/5)})$$
$$= 4 (1 - e^{-5t}) \text{ Amps}$$

$$\text{so } i(t) = 4 (1 - e^{-5t}) \text{ Amps}$$

- The voltage across R is given by $v_R = i(t) \cdot R = V/R (1 - e^{-t/\tau}) \cdot R$

$$= V (1 - e^{-t/\tau})$$

$$\text{So } v_R = 100 (1 - e^{-5t})$$

- The voltage drop across L can be found in two ways.

1. Voltage across Inductor $v_L = L di/dt$

2. Voltage across Inductor $v_L = V - V_R$

But it is easier to find using the second method.

$$\therefore v_L = 100 - 100 (1 - e^{-5t})$$

$$v_L = 100e^{-5t}$$

(b) At time $t = 0.5$ secs, $i(t) = 4 (1 - e^{-5t}) = 4 (1 - e^{-2.5}) = 3.67$ Amps

(c) To find out the time at which the voltages across the Inductor and resistor are equal

we know that the applied voltage is 100 V

the condition $V_R = V_L$ will be satisfied when $V_R = V_L = 50$ V. i.e

$$V_R = 100 (1 - e^{-5t}) = 50 \text{ volts and } V_L = 100 \cdot e^{-5t} = 50 \text{ V.}$$

$$V_L = 100.e^{-5t} = 50 \text{ V.}$$

$$e^{-5t} = 50/100 = 0.5 .$$

Taking natural logarithm on both sides we get:

$$- 5t .\ln(e) = \ln 0.5$$

$$- 5t .1 = - 0.693$$

$$\text{or } t = 0.693/5 = 0.139 \text{ secs}$$

$\therefore$ The voltages across the resistance and the Inductance are equal at time $t = 0.139$ secs

P.4.9

Initially relaxed series RL circuit with $R = 100 \, \Omega$ and $L = 20 \text{ H}$ has dc voltage of 200 V applied at time $t = 0$. Find (a) the equation for current and voltages across different elements (b) the current at time $t = 0.5 \text{ s}$ and 1.0 s (c) the time at which the voltages across the resistor and inductor are equal.

Solution : Transform circuit for time $t > 0$ is shown.

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$$(a) I(s) = \frac{\frac{200}{s}}{100 + 20s} = \frac{10}{s(s+5)} = 2 \left(\frac{1}{s} - \frac{1}{s+5} \right)$$

Therefore, current $i(t) = 2 (1 - e^{-5t})$ A

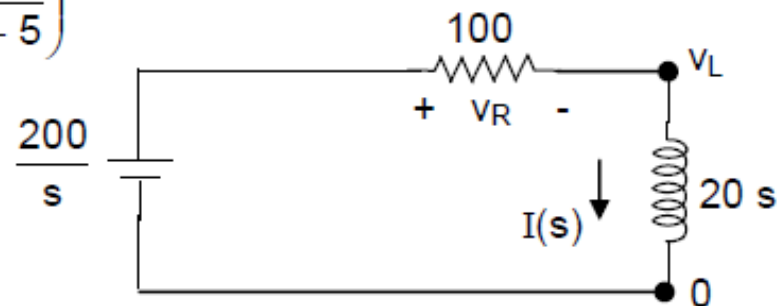
Voltage $v_R(t) = R i(t) = 200 (1 - e^{-5t})$ V

Voltage $v_L(t) = L \frac{di}{dt} = 20 \times 2 \times 5 e^{-5t} = 200 e^{-5t}$ V

(b) $i(0.5) = 2 (1 - e^{-2.5}) = 1.8358$ A $i(1.0) = 2 (1 - e^{-5}) = 1.9865$ A

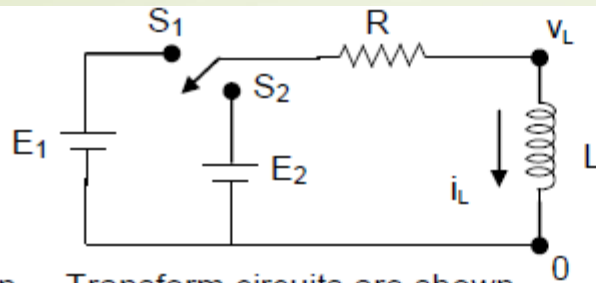
(c) Let t_1 be the time at which $v_R(t) = v_L(t)$. Then

$200 (1 - e^{-5t_1}) = 200 e^{-5t_1}$ i.e. $e^{-5t_1} = 0.5$ This gives $t_1 = 0.1386$ s



P.4.10 For the circuit shown, with zero inductor current the switch is closed on to position S1 at time $t = 0$. At one mille second it is moved to position S2 Obtain the equation for the currents in both the intervals.

Method 1

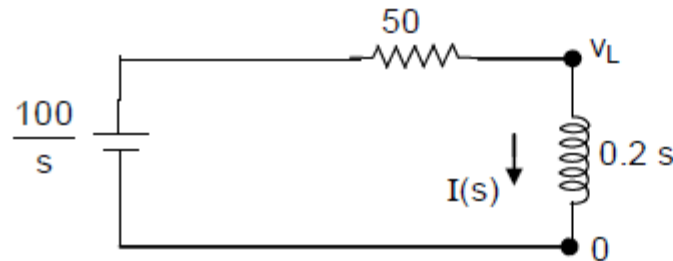


$$E_1 = 100 \text{ V}; E_2 = 50 \text{ V}$$

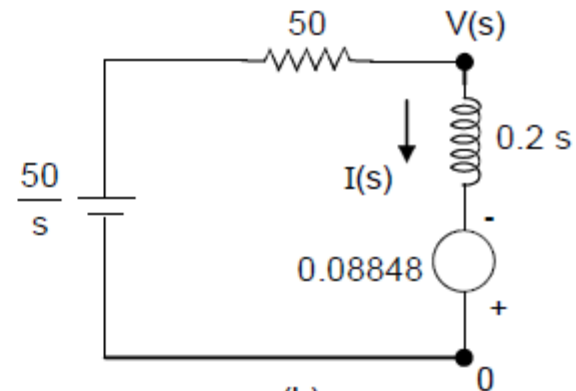
$$R = 50 \Omega$$

$$L = 0.2 \text{ H}$$

Solution Transform circuits are shown.



(a)



(b)

$$I(s) = \frac{\frac{100}{s}}{50 + 0.2s} = \frac{500}{s(s + 250)} = 2 \left(\frac{1}{s} - \frac{1}{s + 250} \right)$$

$$\text{Thus, } i(t) = 2 (1 - e^{-250t}) \text{ A} \quad i(0.001) = 2 (1 - e^{-0.25}) = 0.4424 \text{ A}$$

At time $t = 0.001 \text{ s}$, the switch is moved to position S_2 . We shall say that this is done at time $t' = 0$. The transform circuit for time $t' > 0$ is shown in Fig. (b) in which $L i(0+) = 0.2 \times 0.4424 = 0.08848$

$$\text{Now, } I(s) = \frac{\frac{50}{s} + 0.08848}{50 + 0.2s} = \frac{50 + 0.08848s}{s(50 + 0.2s)} = \frac{250 + 0.4424s}{s(s + 250)} = \frac{K_1}{s} + \frac{K_2}{s + 250}$$

$$K_1 = \left. \frac{250 + 0.4424s}{s + 250} \right|_{s=0} = 1 \quad K_2 = \left. \frac{250 + 0.4424s}{s} \right|_{s=-250} = -0.5576$$

$$\text{Thus, } I(s) = \frac{1}{s} - \frac{0.5576}{s + 250}$$

Taking inverse LT we get, current

$$i(t') = 1 - 0.5576 e^{-250t'}$$

Where $t' = t - 0.001$

Thus for the two intervals currents are given by

$$i(t) = 2 (1 - e^{-250t}) \text{ A} \quad 0.001 \geq t > 0$$

$$i(t') = 1 - 0.5576 e^{-250t'} \text{ A} \quad t > 0.001$$

Method 2

1. $t < 0$: $i_L(0) = 0$;

2. $0.001 \geq t > 0$: $i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-(R/L)t}$

$$i_L(\infty) = V/R = 100/50 = 2A$$

$$R = 50, L = 0.2, R/L = 250$$

So $i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-(R/L)t}$

$$= 2 + [0 - 2]e^{-250t}$$

$$= 2(1 - e^{-250t})$$

3. $t > 0.001$: $i_L(t) = i_L(\infty) + [i_L(0.001) - i_L(\infty)]e^{-(R/L)t-0.001}$

$$i_L(0.001) = 2(1 - e^{-250 \times 0.001}) = 0.4424A$$

$$i_L(\infty) = V/R = 50/50 = 1A$$

$$R = 50, L = 0.2, R/L = 250$$

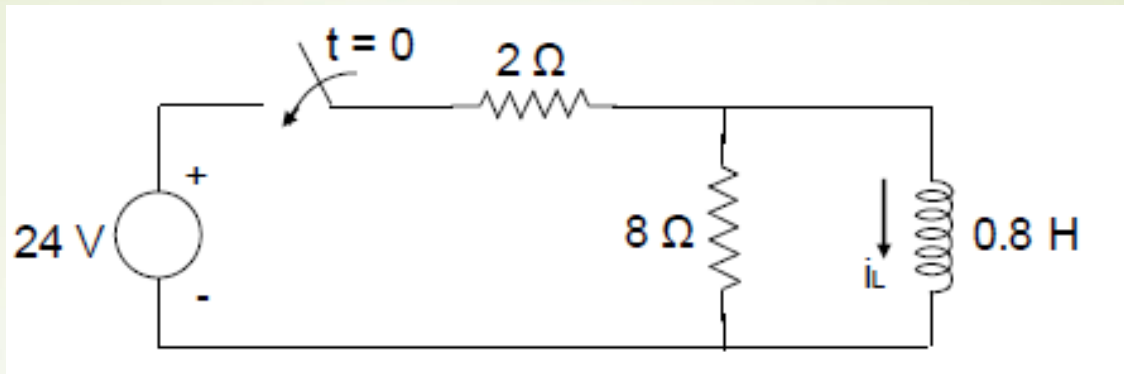
$$i_L(t) = i_L(\infty) + [i_L(0.001) - i_L(\infty)]e^{-(R/L)t-0.001}$$

$$= 1 + [0.4424 - 1] e^{-250(t-0.001)}$$

$$= 1 - 0.5576 e^{-250(t-0.001)}$$

P.4.11**26**

The switch in the circuit shown was in open position for a long time. It is closed at time $t = 0$. Find $i_L(t)$ for time $t > 0$.

**Solution :**

Switch is open - Current $i_L(0) = 0$;

Switch is closed - Current $i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-(R/L)t}$

Current $i_L(\infty) = 24 / 2 = 12\text{ A}$

Thevenin's resistance = (8 in parallel with 2) = 1.6Ω

$L = 0.8\text{ H}$

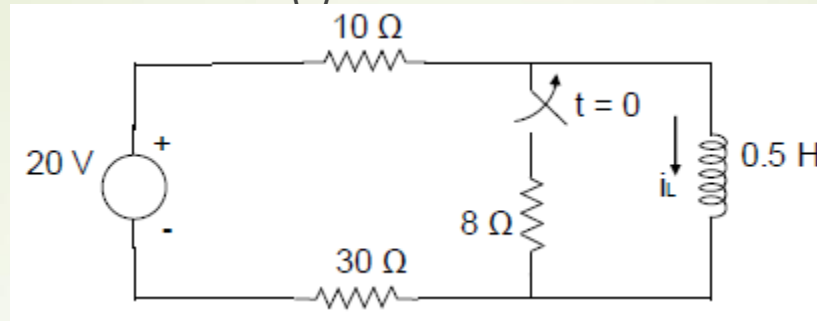
So $i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-(R/L)t}$

$$= 12 + [0 - 12]e^{-(1.6/0.8)t}$$

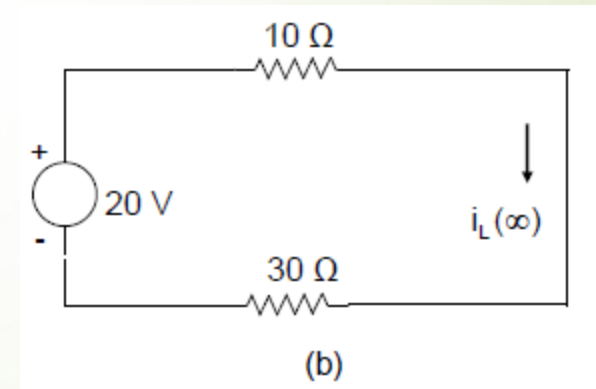
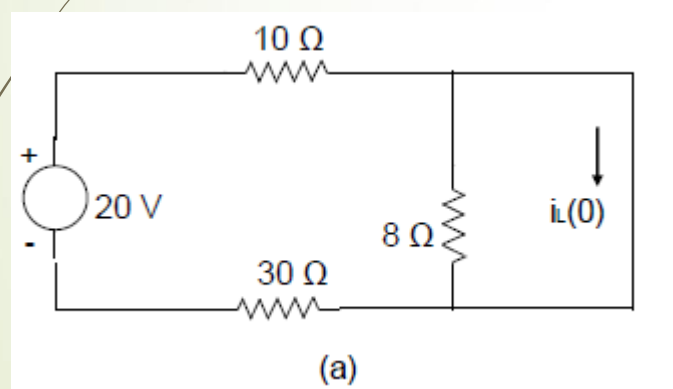
$$= 12(1 - e^{-2t})$$

P.4.12**27**

The switch in the circuit shown was in closed position for a long time. Find current $i_L(t)$ for time $t > 0$.



Solution: Circuit for $t = 0^-$ and $t = \infty$ are shown in Fig. (a) and (b) below.



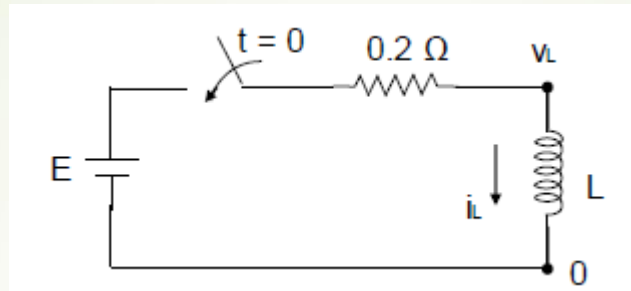
$$\text{Current } i_L(0) = 20 / 40 = 0.5 \text{ A}$$

$$\text{Current } i_L(\infty) = 20 / 40 = 0.5 \text{ A}$$

$$\text{Therefore, current } i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-(R/L)t} = 0.5 \text{ A}$$

P.4.13**28**

In the RL circuit shown in Fig. below, the voltage across the inductor for $t > 0$ is given by $V_L(t) = 0.16e^{-200t}V$. Determine the value of the inductor L and obtain the equation for current $i_L(t)$. Also compute the value of voltage E .



Solution: $R = 0.2 \Omega$; $R/L = 200$, So $L = R/200 = 0.2/200 = 1\text{mH}$

$V_L(t) = 0.16e^{-200t}V = -Ee^{-R/Lt}$, So, $E = 0.16V$

When the switch is closed inductor current exponentially increases from 0 to $i_L(\infty)$.

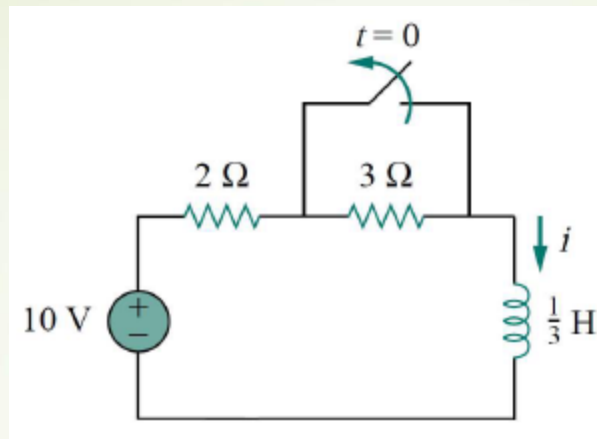
$$i_L(0) = 0$$

$$i_L(\infty) = E/R = 0.16/0.2 = 0.8A$$

$$\begin{aligned} \text{current } i_L(t) &= i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-(R/L)t} \\ &= 0.8 + [0 - 0.8] e^{-200t} \\ &= 0.8(1 - e^{-200t})A \end{aligned}$$

P.4.14**29**

Find $i(t)$ in the circuit for $t > 0$. Assume that the switch has been closed for a long time



Solution:

$$\text{current } i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-(R/L)t}$$

$$i_L(0) = 10/2 = 5\text{A}$$

$$i_L(\infty) = 10/5 = 2\text{A}$$

$$R/L = 5/(1/3) = 15$$

$$\text{So } i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-(R/L)t}$$

$$= 2 + [5 - 2]e^{-15t}$$

$$i_L(t) = 2 + 3e^{-15t} \text{ A}$$