VIT

Vellore Institute of Technology

Final Assessment Test – Jan / Feb 2023

Course: BMAT101L - Calculus | Slot: E1+TE1

Class NBR(s): 5565/ 5572 / 5585 / 5677 / 5695 / 5696 / 7323 | Max. Marks: 100

Time: Three Hours

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. a) Verify Lagrange's Mean Value theorem for the function $f(x)=\frac{x}{1+x}$ in [1, 3]. [5]

   b) Obtain the local extreme values of $f(x)=x^2e^{-x}$. [5]

2. Determine the volume of the solid obtained by rotating the region bounded by $y=x^2+1$ and $y=3-x^2$ about the $x-$axis.

3. Verify if the functions $u=x^2+y^2+xy+2z$, $v=x^2+y^2-5xy-3x+2z+3$ and $w=2xy+x-1$ are functionally dependent are not. If u, v and w are functionally dependent, then obtain the relation between them.

4. Obtain the second order Taylor series expansion for $f(x, y)=e^{-(x^2+y^2)}$ in powers of $x-1$ and $y-2$.

5. Let the profit function be $P(x, y)=(\sin x)(\sin y)\sin(x+y)$, where $0<x<\frac{\pi}{2}$ and $0<y<\frac{\pi}{2}$. Then obtain the point at which the maximum profit occurs.

6. Use change of order of integration, to evaluate $\int_0^{\infty}\int_0^{x} xe^{\frac{-x^2}{y}}\,dy\,dx$. $\int_0^{\infty} y e^{-y} dy$.

7. Evaluate $\iiint_E x^2 dV$, where E is the solid that lies with in the cylinder $x^2+y^2=1$, above the plane $z=0$ and below the cone $z^2=4x^2+4y^2$ using cylindrical polar coordinates.

   $\frac{2\pi}{5}$ $x = r\cos\theta$ $y = r\sin\theta$ $z = z$. (r,θ,z) $r\,dr\,d\theta\,dz$

8. a) Evaluate $\int_0^{\infty} 3^{-x^2}dx$ using gamma function. [5]

   b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^5\theta\,\cos^3\theta\,d\theta$ using beta and gamma functions. [5]

9. Find the directional derivative of the function $\phi=x^2-y^2+2z^2$ at the point $P(1,1,1)$ in the direction of the vector $\vec{i}+\vec{j}+\vec{k}$.

10. Obtain the values of a and b such that $\vec{f}=(axy+z^3)\vec{i}+(3x^2-z)\vec{j}+(bxz^2-y)\vec{k}$ is irrotational. Hence find its scalar potential.

$-4(x)(2x+y)$. $2x^2+xy)$ $-8x^2-4xy$

$-2.x.e^{-(x^2+y^2)}$