



VIT

Vellore Institute of Technology

FALL SEMESTER 2022-2023

SLOT: G1+TG1

CONTINUOUS ASSESSMENT TEST - I

Programme Name & Branch : B.Tech. (SCE, SCHEME, SCOPE, SMEC)  
 Course Code / Course Name : BMAT101L - CALCULUS  
 Class Number(s) : VL202223010 5494, 5441, 7338, 5429, 5434, 544  
 Duration : 90 minutes Max. Marks : 50

Answer ALL the questions

Q.No	Question	Marks
1.	Find the area of the region enclosed by the line $x=2y$ and the curve $x=y^3-y^2$	10
2.	Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1} \frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem	10
3.	a) Find the volume of the solid generated by the revolution of the area bounded by $y=x^2$ and $y=x$ about $y$ -axis. (5 Marks) b) Discuss the concavity and find the extremities of a function $y=f(x)$ , if $y'=(x^2-2x)(x-5)^2$ (5 Marks)	10
4.	a) Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for the function $u = \tan^{-1} \frac{x}{y}$ (5 Marks) b) If $u = x \log xy$ where $x^3 + y^3 + 3xy = 1$ find $\frac{dy}{dx}$ (5 Marks)	10
5.	If $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$ , Show that the Jacobian of $y_1, y_2, y_3$ with respect to $x_1, x_2, x_3$ is 4.	10

*Handwritten notes and calculations:*

- Top right:  $\frac{y}{y^2+x^2}$
- Right side:  $\frac{3}{2}, \frac{1}{2}, -2x+1, -18, 90$
- Left side:  $-12$
- Bottom left:  $\frac{\partial y}{\partial x} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$ ,  $\frac{x^2+y^2}{x^2+y^2}$ ,  $x \log x - x \log y$
- Bottom middle:  $(x^2-2x)(x^2+25-10x)$ ,  $x^4 + 25x^2 - 10x^3 - 2x^3 - 50x + 250x$ ,  $(x-x^2)^2$ ,  $y^{-1}-1$ ,  $\frac{1}{y}$ ,  $-1y^{-2}$ ,  $\frac{1}{y^2}$
- Bottom right:  $Bx^2 + By^2 \frac{dy}{dx} + 3x \frac{dy}{dx}$ ,  $(y^2+x) \frac{dy}{dx} = -x^2$ ,  $\frac{dy}{dx} = \frac{-x^2}{y^2+x}$ ,  $\frac{1}{12} - \frac{3}{12} - \frac{4}{12}$