19 Three phase systems

At the end of this chapter you should be able to:

- describe a single-phase supply
- describe a three-phase supply
- understand a star connection, and recognize that $I_L = I_p$ and $V_L = \sqrt{3}V_p$
- draw a complete phasor diagram for a balanced, star connected load
- understand a delta connection, and recognize that $V_L = V_p$ and $I_L = \sqrt{3}I_p$
- draw a phasor diagram for a balanced, delta connected load
- calculate power in three-phase systems using $P = \sqrt{3}V_L I_L \cos\phi$
- appreciate how power is measured in a three-phase system, by the one, two and three-wattmeter methods
- compare star and delta connections
- appreciate the advantages of three-phase systems

19.1 Introduction

Generation, transmission and distribution of electricity via the National Grid system is accomplished by three-phase alternating currents.

The voltage induced by a single coil when rotated in a uniform magnetic field is shown in Figure 19.1 and is known as a **single-phase voltage**. Most consumers are fed by means of a single-phase a.c. supply. Two wires are used, one called the live conductor (usually coloured red) and the other is called the neutral conductor (usually coloured black). The neutral is usually connected via protective gear to earth, the earth wire being coloured green. The standard voltage for a single-phase a.c. supply is 240 V. The majority of single-phase supplies are obtained by connection to a three-phase supply (see Figure 19.5, page 299).

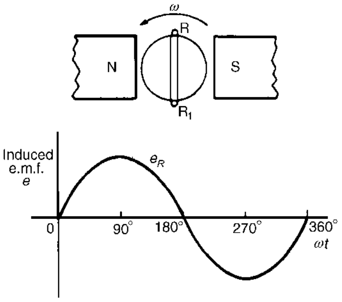


Figure 19.1

19.2 Three-phase supply

A three-phase supply is generated when three coils are placed 120° apart and the whole rotated in a uniform magnetic field as shown in Figure 19.2(a). The result is three independent supplies of equal voltages which are each displaced by 120° from each other as shown in Figure 19.2(b).

(i) The convention adopted to identify each of the phase voltages is: R-red, Y-yellow, and B-blue, as shown in Figure 19.2.

(ii) The **phase-sequence** is given by the sequence in which the conductors pass the point initially taken by the red conductor. The national standard phase sequence is R, Y, B.

A three-phase a.c. supply is carried by three conductors, called **‘lines’** which are coloured red, yellow and blue. The currents in these conductors are known as line currents (I_L) and the p.d.’s between them are known as line voltages (V_L). A fourth conductor, called the **neutral** (coloured black, and connected through protective devices to earth) is often used with a three-phase supply.

If the three-phase windings shown in Figure 19.2 are kept independent then six wires are needed to connect a supply source (such as a generator) to a load (such as motor). To reduce the number of wires it is usual to interconnect the three phases. There are two ways in which this can be done, these being:

(a) a **star connection**, and (b) a **delta**, or **mesh, connection**. Sources of three-phase supplies, i.e. alternators, are usually connected in star, whereas three-phase transformer windings, motors and other loads may be connected either in star or delta.

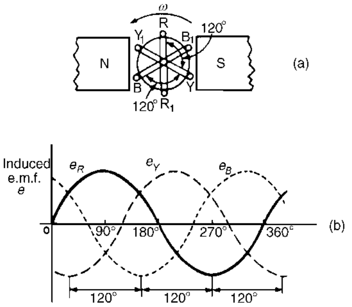


Figure 19.2

19.3 Star connection

(i) A **star-connected load** is shown in Figure 19.3 where the three line conductors are each connected to a load and the outlets from the loads are joined together at N to form what is termed the **neutral point** or the **star point**.

(ii) The voltages, V_R, V_Y and V_B are called **phase voltages** or line to neutral voltages. Phase voltages are generally denoted by V_p

(iii) The voltages, V_{RY}, V_{YB} and V_{BR} are called **line voltages**

(iv) From Figure 19.3 it can be seen that the phase currents (generally denoted by I_p) are equal to their respective line currents I_R, I_Y and I_B, i.e. for a star connection:

$$\boxed{I_L = I_p}$$

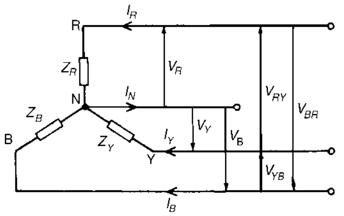


Figure 19.3

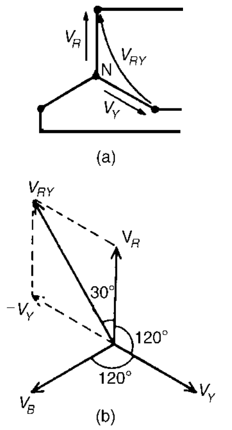
(v) For a balanced system: $I_R = I_Y = I_B, \quad V_R = V_Y = V_B$

$$V_{RY} = V_{YB} = V_{BR}, \quad Z_R = Z_Y = Z_B$$

and the current in the neutral conductor, $I_N = 0$
When a star connected system is balanced, then the neutral conductor is unnecessary and is often omitted.

(vi) The line voltage, V_{RY}, shown in Figure 19.4(a) is given by $\boldsymbol{V_{RY} = V_R - V_Y}$ (V_Y is negative since it is in the opposite direction to V_{RY}). In the phasor diagram of Figure 19.4(b), phasor V_Y is reversed (shown by the broken line) and then added phasorially to V_R (i.e. $\boldsymbol{V_{RY} = V_R + (-V_Y)}$). By trigonometry, or by measurement, $V_{RY} = \sqrt{3}V_R$, i.e. for a balanced star connection:

$$\boxed{V_L = \sqrt{3}\ V_p}$$



(a)

(b)

Figure 19.4

(See problem 3 following for a complete phasor diagram of a star-connected system.)

(vii) The star connection of the three phases of a supply, together with a neutral conductor, allows the use of two voltages — the phase voltage and the line voltage. A 4-wire system is also used when the load is not balanced. The standard electricity supply to consumers in Great Britain is 415/240 V, 50 Hz, 3-phase, 4-wire alternating current, and a diagram of connections is shown in Figure 19.5.

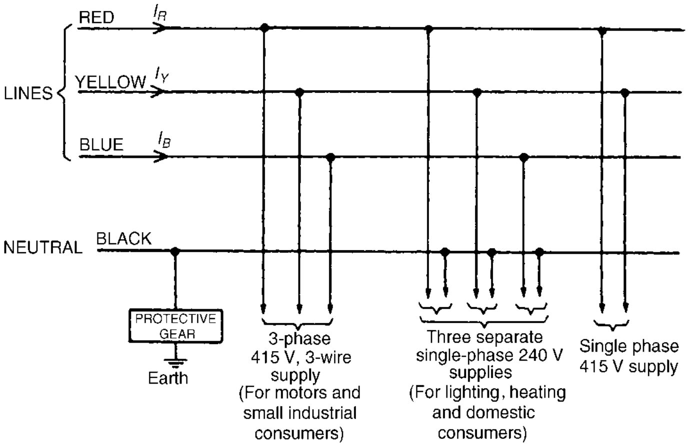


Figure 19.5

Problem 1. Three loads, each of resistance 30 Ω, are connected in star to a 415 V, 3-phase supply. Determine (a) the system phase voltage, (b) the phase current and (c) the line current.

A '415 V, 3-phase supply' means that 415 V is the line voltage, V_L

(a) For a star connection, $V_L = \sqrt{3}V_p$

Hence phase voltage, $V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = \mathbf{239.6\ V\ or\ 240\ V}$ correct to 3 significant figures

(b) Phase current, $I_p = \frac{V_p}{R_p} = \frac{240}{30} = \mathbf{8\ A}$

(c) For a star connection, $I_p = I_L$

Hence the line current, $I_L = \mathbf{8\ A}$

Problem 2. A star-connected load consists of three identical coils each of resistance 30 Ω and inductance 127.3 mH. If the line current is 5.08 A, calculate the line voltage if the supply frequency is 50 Hz

Inductive reactance $X_L = 2\pi f L = 2\pi(50)(127.3 \times 10^{-3}) = 40\ \Omega$

Impedance of each phase $Z_p = \sqrt{(R^2 + X_L{}^2)} = \sqrt{(30^2 + 40^2)} = 50\ \Omega$

For a star connection $I_L = I_p = \dfrac{V_p}{Z_p}$

Hence phase voltage $V_p = I_p Z_p = (5.08)(50) = 254$ V

Line voltage $V_L = \sqrt{3}V_p = \sqrt{3}(254) = \mathbf{440\ V}$

Problem 3. A balanced, three-wire, star-connected, 3-phase load has a phase voltage of 240 V, a line current of 5 A and a lagging power factor of 0.966. Draw the complete phasor diagram.

The phasor diagram is shown in Figure 19.6.

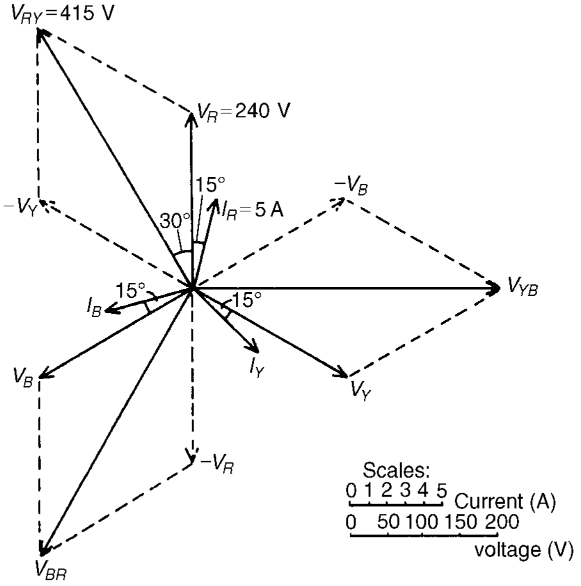


Figure 19.6

Procedure to construct the phasor diagram:

(i) Draw $V_R = V_Y = V_B = 240$ V and spaced 120° apart. (Note that V_R is shown vertically upwards — this however is immaterial for it may be drawn in any direction.)

(ii) Power factor $= \cos\phi = 0.966$ lagging. Hence the load phase angle is given by arccos 0.966, i.e. 15° lagging. Hence $I_R = I_Y = I_B = 5$ A, lagging V_R, V_Y and V_B respectively by 15°

(iii) $V_{RY} = V_R - V_Y$ (phasorially). Hence V_Y is reversed and added phasorially to V_R. By measurement, $V_{RY} = 415$ V (i.e. $\sqrt{3}(240)$) and leads V_R by 30°. Similarly, $V_{YB} = V_Y - V_B$ and $V_{BR} = V_B - V_R$

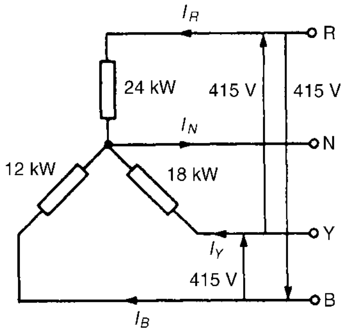


Figure 19.7

Problem 4. A 415 V, 3-phase, 4 wire, star-connected system supplies three resistive loads as shown in Figure 19.7. Determine (a) the current in each line and (b) the current in the neutral conductor.

(a) For a star-connected system $V_L = \sqrt{3}V_p$

Hence $$V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240 \text{ V}$$

Since current $I = \dfrac{\text{Power } P}{\text{Voltage } V}$ for a resistive load

then $$I_R = \frac{P_R}{V_R} = \frac{24\,000}{240} = \mathbf{100\ A}$$

$$I_Y = \frac{P_Y}{V_Y} = \frac{18\,000}{240} = \mathbf{75\ A}$$

and $$I_B = \frac{P_B}{V_B} = \frac{12\,000}{240} = \mathbf{50\ A}$$

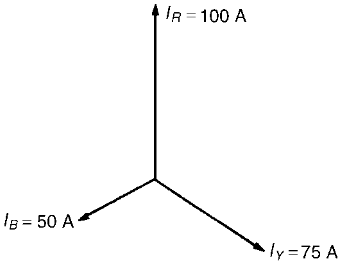


Figure 19.8

(b) The three line currents are shown in the phasor diagram of Figure 19.8. Since each load is resistive the currents are in phase with the phase voltages and are hence mutually displaced by 120°. The current in the neutral conductor is given by:

$$I_N = I_R + I_Y + I_B \text{ phasorially.}$$

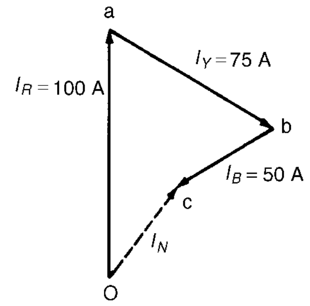


Figure 19.9

Figure 19.9 shows the three line currents added phasorially. oa represents I_R in magnitude and direction. From the nose of oa, ab is drawn representing I_Y in magnitude and direction. From the nose of ab, bc is drawn representing I_B in magnitude and direction. oc represents the resultant, I_N.

By measurement, $\boldsymbol{I_N} = \mathbf{43\ A}$

Alternatively, by calculation, considering I_R at 90°, I_B at 210° and I_Y at 330°:

Total horizontal component $= 100\cos 90° + 75\cos 330° + 50\cos 210°$

$= 21.65$

Total vertical component $= 100\sin 90° + 75\sin 330° + 50\sin 210°$

$= 37.50$

Hence magnitude of $I_N = \sqrt{(21.65^2 + 37.50^2)} = \mathbf{43.3\ A}$

19.4 Delta connection

(i) A **delta (or mesh) connected load** is shown in Figure 19.10 where the end of one load is connected to the start of the next load.

(ii) From Figure 19.10, it can be seen that the line voltages V_{RY}, V_{YB} and V_{BR} are the respective phase voltages, i.e. for a delta connection:

$$\boxed{V_L = V_p}$$

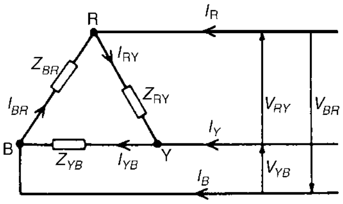


Figure 19.10

(iii) Using Kirchhoff's current law in Figure 19.10, $I_R = I_{RY} - I_{BR} = I_{RY} + (-I_{BR})$. From the phasor diagram shown in Figure 19.11, by trigonometry or by measurement, $I_R = \sqrt{3} I_{RY}$, i.e. for a delta connection:

$$\boxed{I_L = \sqrt{3} I_p}$$

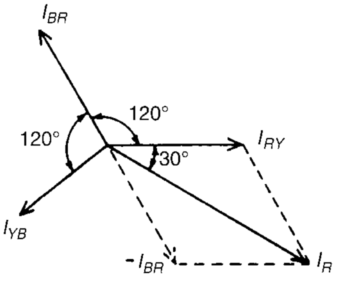


Figure 19.11

Problem 5. Three identical coils each of resistance 30 Ω and inductance 127.3 mH are connected in delta to a 440 V, 50 Hz, 3-phase supply. Determine (a) the phase current, and (b) the line current.

Phase impedance, $Z_p = 50\ \Omega$ (from problem 2) and for a delta connection, $V_p = V_L$

(a) Phase current, $I_p = \dfrac{V_p}{Z_p} = \dfrac{V_L}{Z_p} = \dfrac{440}{50} = \mathbf{8.8\ A}$

(b) For a delta connection, $I_L = \sqrt{3} I_p = \sqrt{3}(8.8) = \mathbf{15.24\ A}$

Thus when the load is connected in delta, three times the line current is taken from the supply than is taken if connected in star.

Problem 6. Three identical capacitors are connected in delta to a 415 V, 50 Hz, 3-phase supply. If the line current is 15 A, determine the capacitance of each of the capacitors.

For a delta connection $I_L = \sqrt{3} I_p$

Hence phase current $I_p = \dfrac{I_L}{\sqrt{3}} = \dfrac{15}{\sqrt{3}} = 8.66\ A$

Capacitive reactance per phase, $X_C = \dfrac{V_p}{I_p} = \dfrac{V_L}{I_p}$ (since for a delta connection $V_L = V_p$)

Hence $X_C = \dfrac{415}{8.66} = 47.92\ \Omega$

$X_C = \dfrac{1}{2\pi f C}$, from which capacitance, $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi(50)(47.92)}$ F

$= \mathbf{66.43\ \mu F}$

Problem 7. Three coils each having resistance 3 Ω and inductive reactance 4 Ω are connected (i) in star and (ii) in delta to a 415 V, 3-phase supply. Calculate for each connection (a) the line and phase voltages and (b) the phase and line currents.

(i) **For a star connection:** $I_L = I_p$ and $V_L = \sqrt{3}V_p$

(a) A 415 V, 3-phase supply means that the

line voltage, $V_L = \mathbf{415\ V}$

Phase voltage, $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{415}{\sqrt{3}} = \mathbf{240\ V}$

(b) Impedance per phase, $Z_p = \sqrt{(R^2 + X_L{}^2)} = \sqrt{(3^2 + 4^2)}$

$= 5\ \Omega$

Phase current, $I_p = \dfrac{V_p}{Z_p} = \dfrac{240}{5} = \mathbf{48\ A}$

Line current, $I_L = I_p = \mathbf{48\ A}$

(ii) **For a delta connection:** $V_L = V_p$ and $I_L = \sqrt{3}I_p$

(a) Line voltage, $V_L = 415$ V

Phase voltage, $V_p = V_L = \mathbf{415\ V}$

(b) Phase current, $I_p = \dfrac{V_p}{Z_p} = \dfrac{415}{5} = \mathbf{83\ A}$

Line current, $I_L = \sqrt{3}I_p = \sqrt{3}(83) = \mathbf{144\ A}$

Further problems on star and delta connections may be found in Section 19.9, problems 1 to 7, page 312.

19.5 Power in three-phase systems

The power dissipated in a three-phase load is given by the sum of the power dissipated in each phase. If a load is balanced then the total power P is given by: $P = 3 \times$ power consumed by one phase.

The power consumed in one phase $= I_p{}^2 R_p$ or $V_p I_p \cos\phi$ (where ϕ is the phase angle between V_p and I_p)

For a star connection, $V_p = \dfrac{V_L}{\sqrt{3}}$ and $I_p = I_L$ hence

$$P = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\phi = \sqrt{3} V_L I_L \cos\phi$$

For a delta connection, $V_p = V_L$ and $I_p = \dfrac{I_L}{\sqrt{3}}$ hence

$$P = 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\phi = \sqrt{3}V_LI_L\cos\phi$$

Hence for either a star or a delta balanced connection the total power P is given by:

$$\boxed{\boldsymbol{P = \sqrt{3}V_LI_L\cos\phi}\ \textbf{watts}}\ \textbf{or}\ \boxed{\boldsymbol{P = 3I_p{}^2R_p}\ \textbf{watts.}}$$

Total volt-amperes, $\boxed{\boldsymbol{S = \sqrt{3}V_LI_L}\ \textbf{volt-amperes}}$

Problem 8. Three 12 Ω resistors are connected in star to a 415 V, 3-phase supply. Determine the total power dissipated by the resistors.

Power dissipated, $P = \sqrt{3}V_LI_L\cos\phi$ or $P = 3I_p{}^2R_p$

Line voltage, $V_L = 415$ V and phase voltage $V_p = \dfrac{415}{\sqrt{3}} = 240$ V
(since the resistors are star-connected)

Phase current, $I_p = \dfrac{V_p}{Z_p} = \dfrac{V_p}{R_p} = \dfrac{240}{12} = 20$ A

For a star connection $I_L = I_p = 20$ A

For a purely resistive load, the power factor $= \cos\phi = 1$

Hence power $P = \sqrt{3}V_LI_L\cos\phi = \sqrt{3}(415)(20)(1) = \mathbf{14.4\ kW}$

or power $P = 3I_p{}^2R_p = 3(20)^2(12) = \mathbf{14.4\ kW}$

Problem 9. The input power to a 3-phase a.c. motor is measured as 5 kW. If the voltage and current to the motor are 400 V and 8.6 A respectively, determine the power factor of the system.

Power, $P = 5000$ W; Line voltage $V_L = 400$ V; Line current, $I_L = 8.6$ A

Power, $P = \sqrt{3}V_LI_L\cos\phi$

Hence power factor $= \cos\phi = \dfrac{P}{\sqrt{3}V_LI_L} = \dfrac{5000}{\sqrt{3}(400)(8.6)} = \mathbf{0.839}$

Problem 10. Three identical coils, each of resistance 10 Ω and inductance 42 mH are connected (a) in star and (b) in delta to a 415 V, 50 Hz, 3-phase supply. Determine the total power dissipated in each case.

(a) **Star connection**

Inductive reactance $X_L = 2\pi f L = 2\pi(50)(42 \times 10^{-3})$

$= 13.19\ \Omega$

Phase impedance $Z_p = \sqrt{(R^2 + X_L^2)} = \sqrt{(10^2 + 13.19^2)}$

$= 16.55\ \Omega$

Line voltage $V_L = 415$ V and

phase voltage, $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{415}{\sqrt{3}} = 240$ V

Phase current, $I_p = \dfrac{V_p}{Z_p} = \dfrac{240}{16.55} = 14.50$ A

Line current, $I_L = I_p = 14.50$ A

Power factor $= \cos\phi = \dfrac{R_p}{Z_p} = \dfrac{10}{16.55} = 0.6042$ lagging

Power dissipated, $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3}(415)(14.50)(0.6042)$

$= \mathbf{6.3\ kW}$

(Alternatively, $P = 3 I_p{}^2 R_p = 3(14.50)^2(10) = \mathbf{6.3\ kW}$)

(b) **Delta connection**

$V_L = V_p = 415$ V, $Z_p = 16.55\ \Omega$,

$\cos\phi = 0.6042$ lagging (from above).

Phase current, $I_p = \dfrac{V_p}{Z_p} = \dfrac{415}{16.55} = 25.08$ A

Line current, $I_L = \sqrt{3} I_p = \sqrt{3}(25.08) = 43.44$ A

Power dissipated, $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3}(415)(43.44)(0.6042)$

$= \mathbf{18.87\ kW}$

(Alternatively, $P = 3 I_p{}^2 R_p = 3(25.08)^2(10) = \mathbf{18.87\ kW}$)

Hence loads connected in delta dissipate three times the power than when connected in star, and also take a line current three times greater.

Problem 11. A 415 V, 3-phase a.c. motor has a power output of 12.75 kW and operates at a power factor of 0.77 lagging and with an efficiency of 85%. If the motor is delta-connected, determine (a) the power input, (b) the line current and (c) the phase current.

(a) Efficiency $= \dfrac{\text{power output}}{\text{power input}}$, hence $\dfrac{85}{100} = \dfrac{12\,750}{\text{power input}}$

from which, **power input** $= \dfrac{12\,750 \times 100}{85} = \mathbf{15\,000\ W\ or\ 15\ kW}$

(b) Power, $P = \sqrt{3} V_L I_L \cos\phi$, hence

line current, $I_L = \dfrac{P}{\sqrt{3} V_L \cos\phi} = \dfrac{15\,000}{\sqrt{3}(415)(0.77)} = \mathbf{27.10\ A}$

(c) For a delta connection, $I_L = \sqrt{3} I_p$,

hence **phase current**, $I_p = \dfrac{I_L}{\sqrt{3}} = \dfrac{27.10}{\sqrt{3}} = \mathbf{15.65\ A}$

19.6 Measurement of power in three-phase systems

Power in three-phase loads may be measured by the following methods:

(i) **One-wattmeter method for a balanced load**
Wattmeter connections for both star and delta are shown in Figure 19.12.

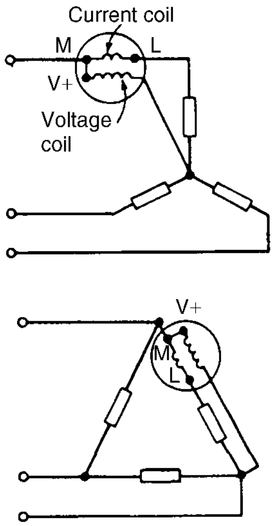


Figure 19.12

$$\boxed{\textbf{Total power} = 3 \times \textbf{wattmeter reading}}$$

(ii) **Two-wattmeter method for balanced or unbalanced loads**
A connection diagram for this method is shown in Figure 19.13 for a star-connected load. Similar connections are made for a delta-connected load.

$$\boxed{\textbf{Total power} = \textbf{sum of wattmeter readings} = P_1 + P_2}$$

The power factor may be determined from:

$$\boxed{\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)}$$ (see Problems 12 and 15 to 18)

It is possible, depending on the load power factor, for one wattmeter to have to be 'reversed' to obtain a reading. In this case it is taken as a negative reading (see Problem 17).

(iii) **Three-wattmeter method for a three-phase, 4-wire system for balanced and unbalanced loads.** (see Figure 19.14).

$$\boxed{\textbf{Total power} = P_1 + P_2 + P_3}$$

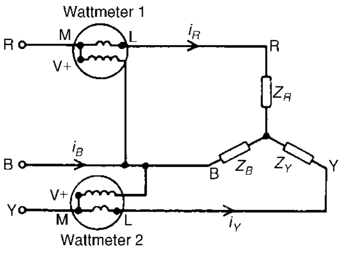


Figure 19.13

Problem 12. (a) Show that the total power in a 3-phase, 3-wire system using the two-wattmeter method of measurement is given by the sum of the wattmeter readings. Draw a connection diagram. (b) Draw a phasor diagram for the two-wattmeter method for a balanced load. (c) Use the phasor diagram of part (b) to derive a formula from which the power factor of a 3-phase system may be determined using only the wattmeter readings.