VIT
Vellore Institute of Technology

# Final Assessment Test – November/December 2023

Course: BMAT101L - Calculus

Time: Three Hours Max. Marks: 100

**KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE**

**Answer any TEN Questions**

**(10 X 10 = 100 Marks)**

1. Consider the function $f(x) = (2 - x^2)^{2/3}$
   (i) Identify where the extrema of $f$ occur.
   (ii) Find the intervals on which $f$ is increasing and decreasing.
   (iii) Find the intervals on which $f$ is concave up and concave down.

2. (i) Find the area of the region enclosed by the line $y = 2$ and the curve $y = x^2 - 2$. **[5]**
   (ii) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$. **[5]**

3. Examine whether the functions
   $u = x + y + z,\ v = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ and
   $w = x^3 + y^3 + z^3 - 3xyz$ are functionally dependent or not.

4. Using Taylor's series, expand $f(x, y) = e^x \log(1 + y)$ in powers of $x$ and $y$ up to third degree terms.

5. Find the dimensions of rectangular box, without top, of maximum capacity and surface area 432 square meters.

6. By changing the order of integration, evaluate $\int_0^1 \int_y^{2-y} xy\,dx\,dy$.

7. Transforming into spherical polar co-ordinates, evaluate $\iiint \frac{dx\,dy\,dz}{\sqrt{x^2+y^2+z^2}}$ taken throughout the volume contained in the positive octant of the sphere $x^2 + y^2 + z^2 = 1$.

8. Using Gamma function, evaluate $\left(\int_0^{\pi/2} \frac{dx}{\sqrt{\cos x}}\right)\left(\int_0^{\pi/2} \sqrt{\cos x}\,dx\right)$.

9. (i) Obtain the directional derivative of $\varphi = 2xy + z^2$ at the point $(1, -1, 3)$ in the direction of $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. **[5]**
   (ii) Find the value of $\lambda$, if $\vec{F} = (x + 2y)\hat{\imath} + (\lambda y + 4z)\hat{\jmath} + (5z + 6x)\hat{k}$ is solenoidal. **[5]**

10. Find the constants $a, b, c$ so that
    $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ may be irrotational.
    For these values of $a, b, c$ find its scalar potential $\varphi$.

11. Verify Green's theorem in the plane for $\int_C [(2x - y)dx + (x + y)dy]$ where $C$ is the boundary of the circle $x^2 + y^2 = a^2$.

12. Verify Stokes theorem for a vector field defined by $\vec{F} = (x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath}$ in the rectangular region in the $xy$-plane bounded by the lines $x = 0, x = a, y = 0, y = b$.

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