10. Find an orthonormal basis for the subspace U of R^4 spanned by the vectors $v_1 = (1,1,1,1), v_2 = (1,2,4,5), v_3 = (1,-3,-4,-2)$ using Gram Schmidt orthogonalization process.

11. Find Characteristic equation of A and A^{-1} using Cayley Hamilton theorem, when $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$.

12. Using Gauss Jordan method, solve the following system of equations

$$x + y - z - w = 2,$$
$$x - y - z + w = 4,$$
$$2x + y - z + w = 1.$$

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Final Assessment Test – November/December 2023

VIT
Vellore Institute of Technology

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 2002 / 2003 / 2004 / 2005 / 2006 / 2016 / 2017 / 2019 / 2020 / 2021 / 2022 / 2023 / 2027 / 2028 / 5005

Slot: A1+TA1+TAA1

Time: Three Hours

Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions
(10 X 10 = 100 Marks)

1. Let $f = u + iv$ where $u = x^2 - y^2 + 2xy$ and $v = x^2 - y^2 - 2xy$. Show that u and v are harmonic and determine whether f is an analytic function in the complex plane.

2. Let ϕ be the potential function given by $\phi(x,y) = e^{2x}(x\cos(2y) - y\sin(2y))$. Find the stream function ψ such that $f = \phi + i\psi$ is analytic.

3. Let $f(z) = \sqrt{2}\, e^{i\pi/4} z + (1 - 2i)$.
   a) Find the coefficient of magnification and angle of rotation at the point $2 + 3i$.
   b) Find the image of the rectangular region $0 \le x \le 2,\ 0 \le y \le 2$ under the transformation of f.

4. Find the bilinear transformation which maps the points $z = 1, i, -1$ into $w = i, 0, -i$. Also, find the image of $|z| < 1$.

5. Expand $f(z) = \frac{z+3}{z(z^2-z-2)}$ the Laurent's series for the region
   a) $|z| < 1$
   b) $1<|z|<2$
   c) $|z|>2$.

6. Let $f(z) = \frac{z+2}{(z+1)^2(z-2)}$.
   a) Find the residue of f about each singularity.
   b) Evaluate $\oint f(z)dz$ over the curve $C: |z - i| = 2$ using Cauchy's residue theorem.

7. Let $A = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 1 \\ -1 & 2 & 7 \end{bmatrix}$. Find the basis of $R(A), C(A)$ and $N(A)$. Also determine, whether the vector $(1,1,1)$ is in $R(A)$.

8. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a transformation defined by $T(x,y,z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is invertible and find $T^{-1}(u,v,w)$.

9. Find the matrix of change of basis A from the basis $\alpha = \{x^2 + x + 1, x^2 + 1, x - 1\}$ and $\beta = \{2x^2 + 3x + 1, 2x^2 + 2x + 1, -x^2 - 2\}$ for $P_2(x)$. Use this result and find the change of basis from β to α.