



Digital Image Processing

Chapter 5: Image Restoration

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A Model of the Image Degradation/Restoration Process

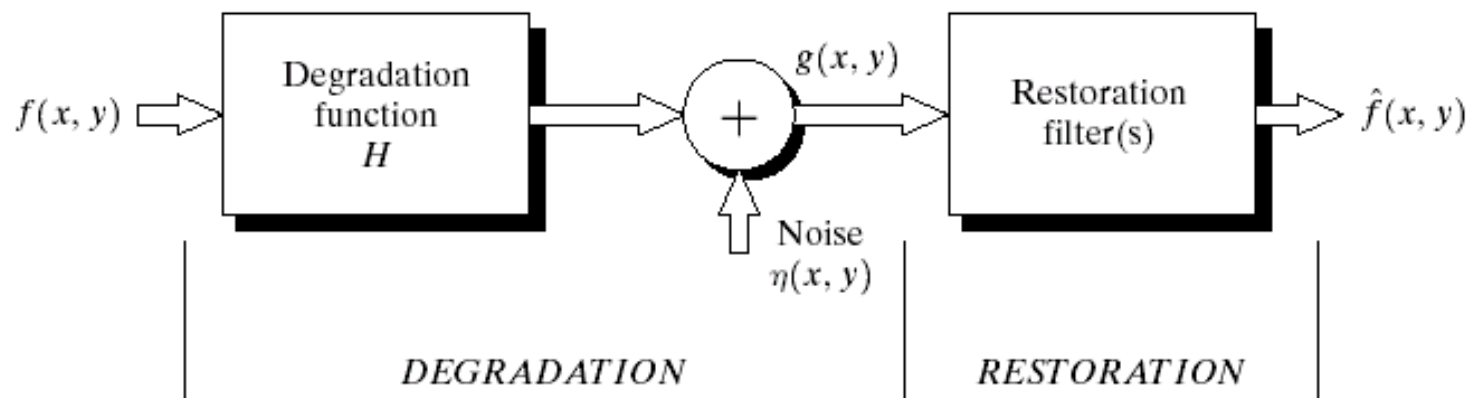


FIGURE 5.1 A model of the image degradation/restoration process.



○ Degradation

- Degradation function H
- Additive noise $\eta(x, y)$
- Spatial domain

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

- Frequency domain

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

- Restoration

$$g(x, y) \Rightarrow \text{Restoration Filter} \Rightarrow \hat{f}(x, y)$$



Noise Models

- Sources of noise
 - Image acquisition, digitization, transmission
- White noise
 - The Fourier spectrum of noise is constant
- Assuming
 - Noise is independent of spatial coordinates
 - Noise is uncorrelated with respect to the image itself



○ Gaussian noise

- The PDF of a Gaussian random variable, z ,

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2 / 2\sigma^2}$$

- Mean: μ
- Standard deviation: σ
- Variance: σ^2

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- 70% of its values will be in the range
$$[(\mu - \sigma), (\mu + \sigma)]$$
 - 95% of its values will be in the range
$$[(\mu - 2\sigma), (\mu + 2\sigma)]$$



○ Rayleigh noise

- The PDF of Rayleigh noise,

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

- Mean: $\mu = a + \sqrt{\pi b/4}$
- Variance: $\sigma^2 = \frac{b(4-\pi)}{4}$

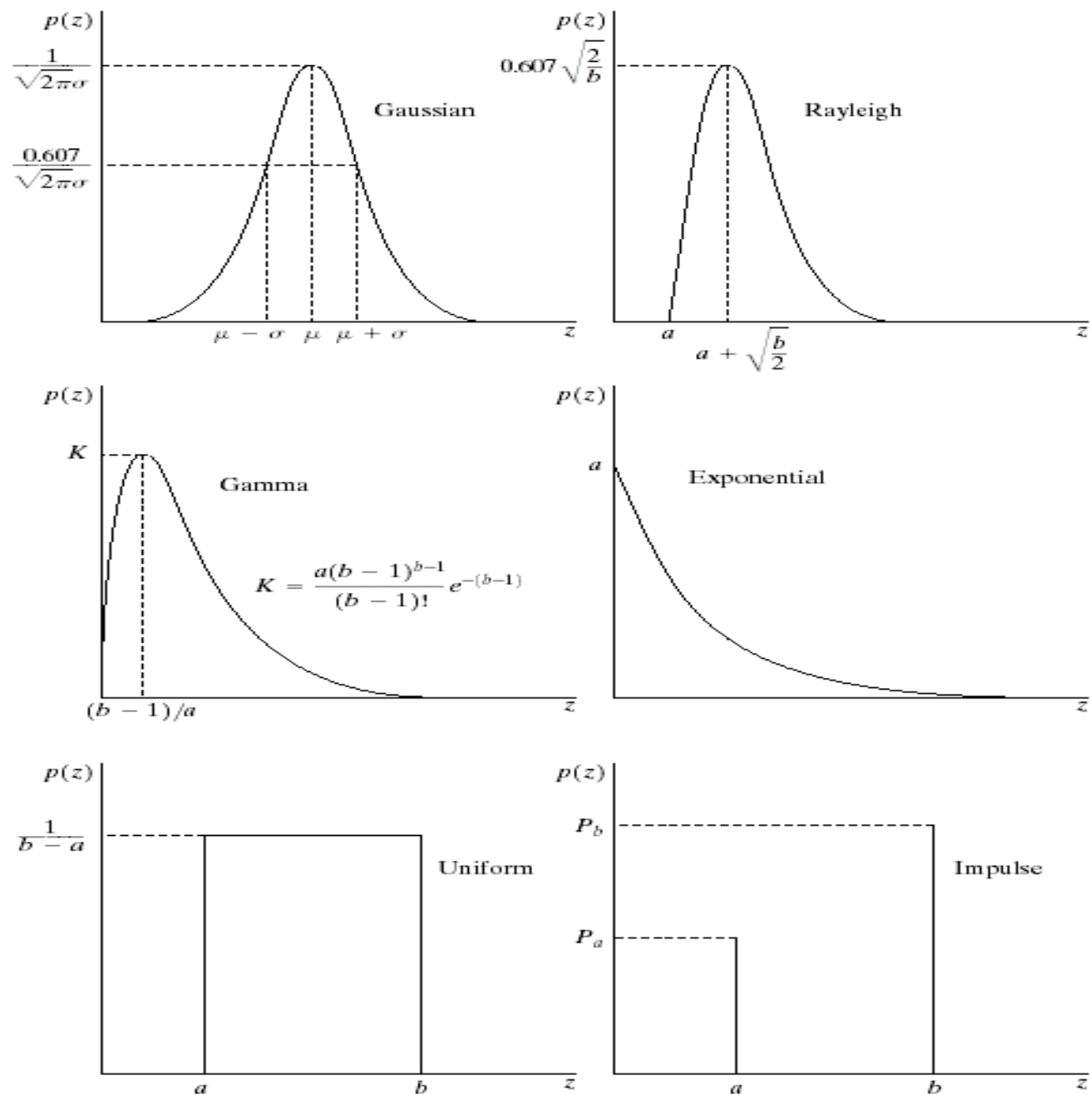


FIGURE 5.2 Some important probability density functions.

- Erlang (Gamma) noise

- The PDF of Erlang noise, $a > 0$, b is a positive integer,

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-a z} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

- Mean: $\mu = \frac{b}{a}$
- Variance: $\sigma^2 = \frac{b}{a^2}$



○ Exponential noise

- The PDF of exponential noise, $a > 0$,

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

- Mean: $\mu = \frac{1}{a}$
- Variance: $\sigma^2 = \frac{1}{a^2}$



○ Uniform noise

- The PDF of uniform noise,

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

- Mean: $\mu = \frac{a+b}{2}$
- Variance: $\sigma^2 = \frac{(b-a)^2}{12}$

- 
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- Impulse (salt-and-pepper) noise
 - The PDF of (bipolar) impulse noise,

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

- $b > a$: gray-level b will appear as a light dot, while level a will appear like a dark dot
- Unipolar: either P_a or P_b is zero

- 
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- Usually, for an 8-bit image, $a = 0$ (black) and $b = 0$ (white)



○ Modeling

- Gaussian

- Electronic circuit noise, sensor noise due to poor illumination and/or high temperature

- Rayleigh

- Range imaging

- Exponential and gamma

- Laser imaging

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- Impulse
 - Quick transients, such as faulty switching
 - Uniform
 - Least descriptive
 - Basis for numerous random number generators

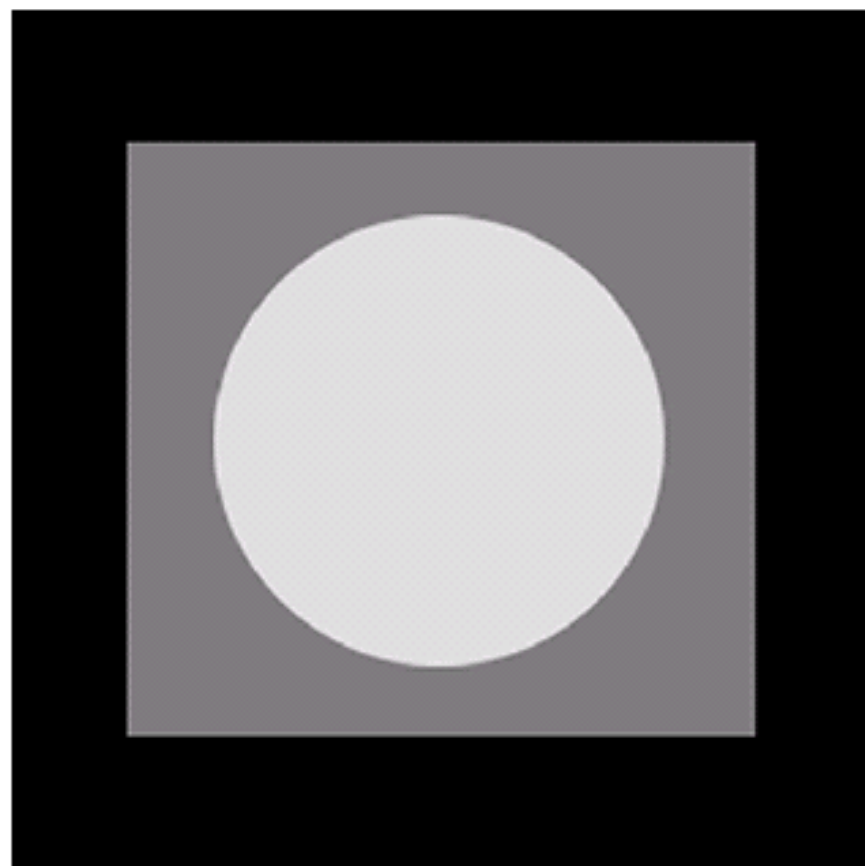
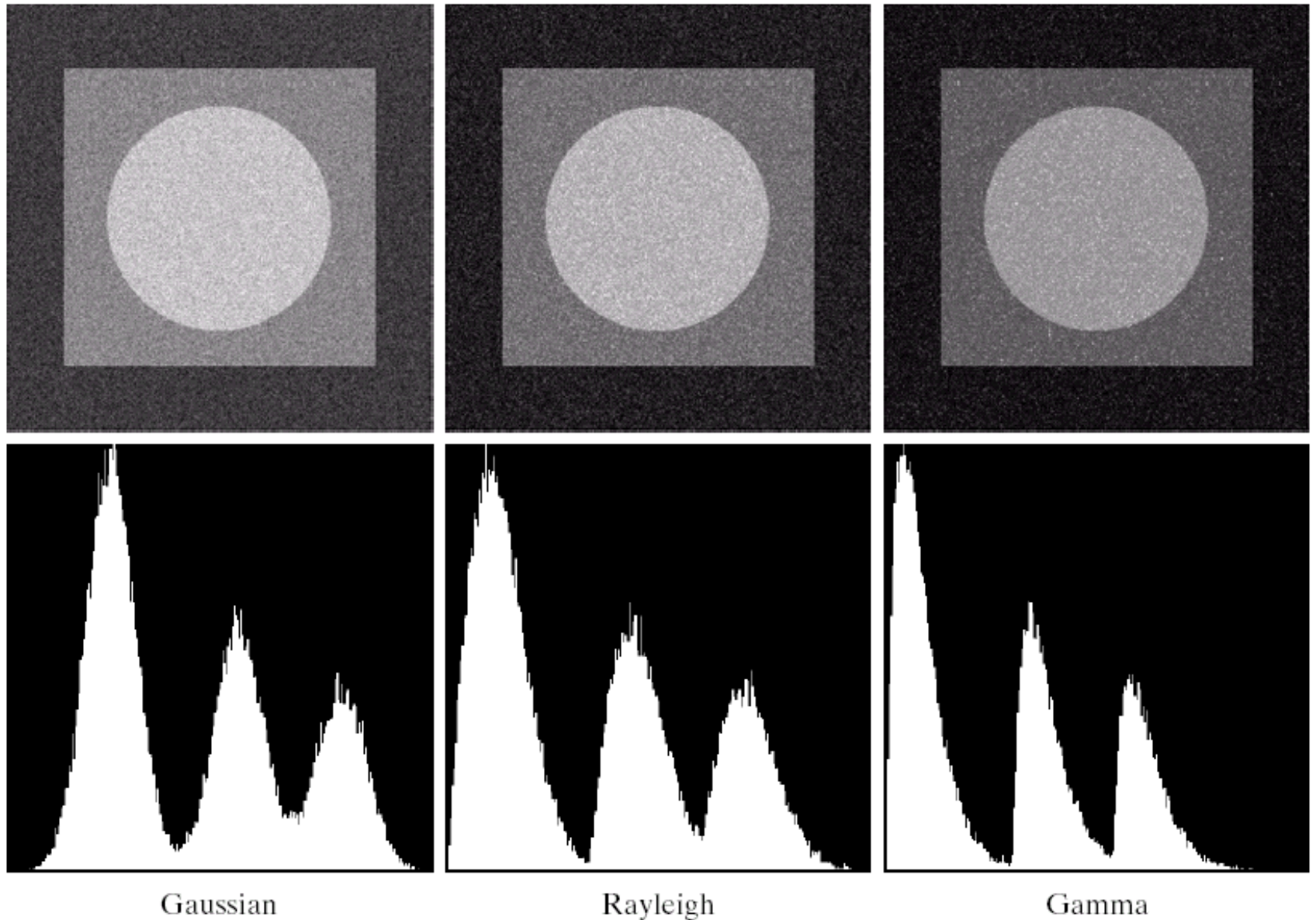
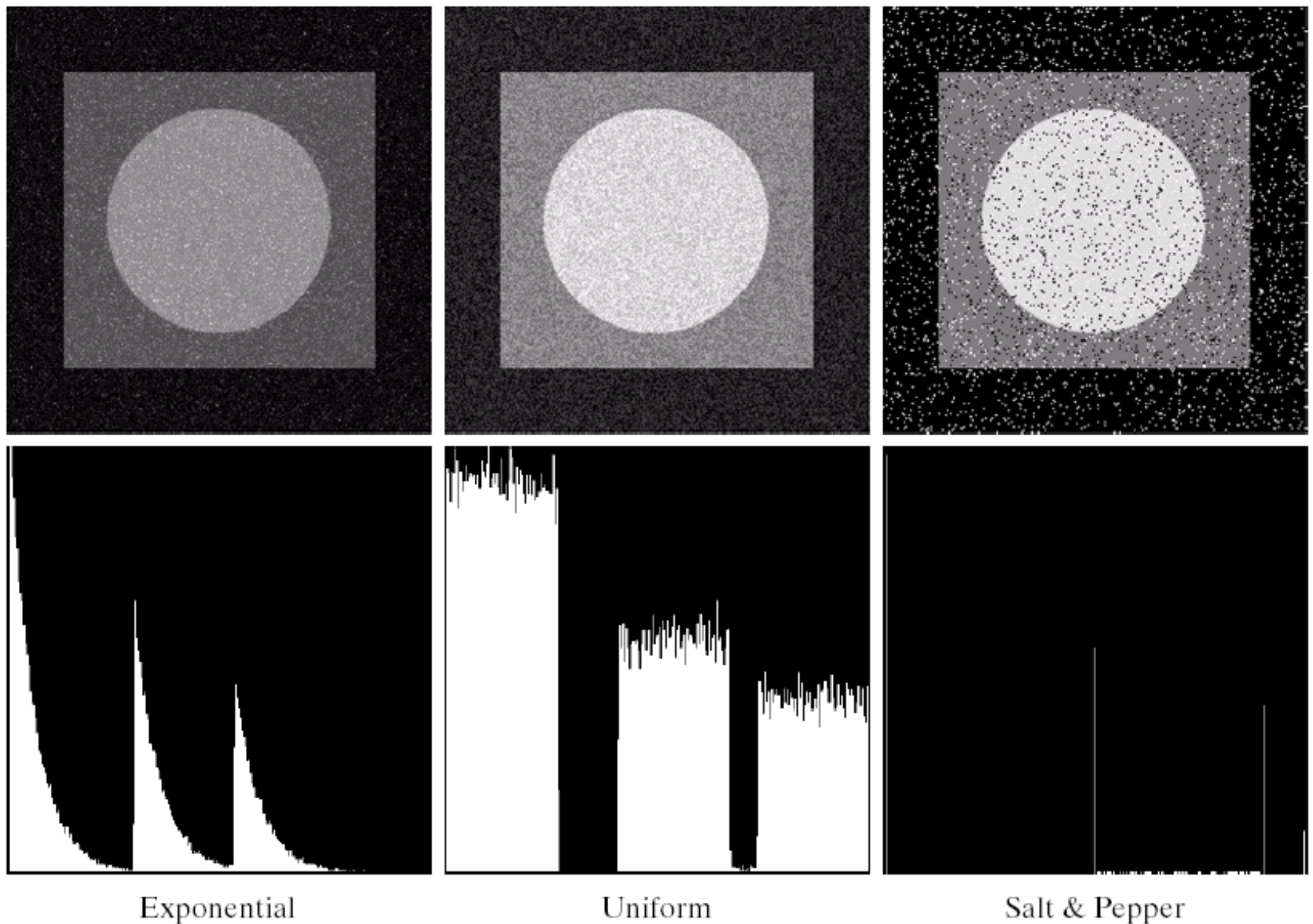


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.



a	b	c
d	e	f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.



g	h	i
j	k	l

FIGURE 5.4 (Continued) Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.



○ Periodic noise

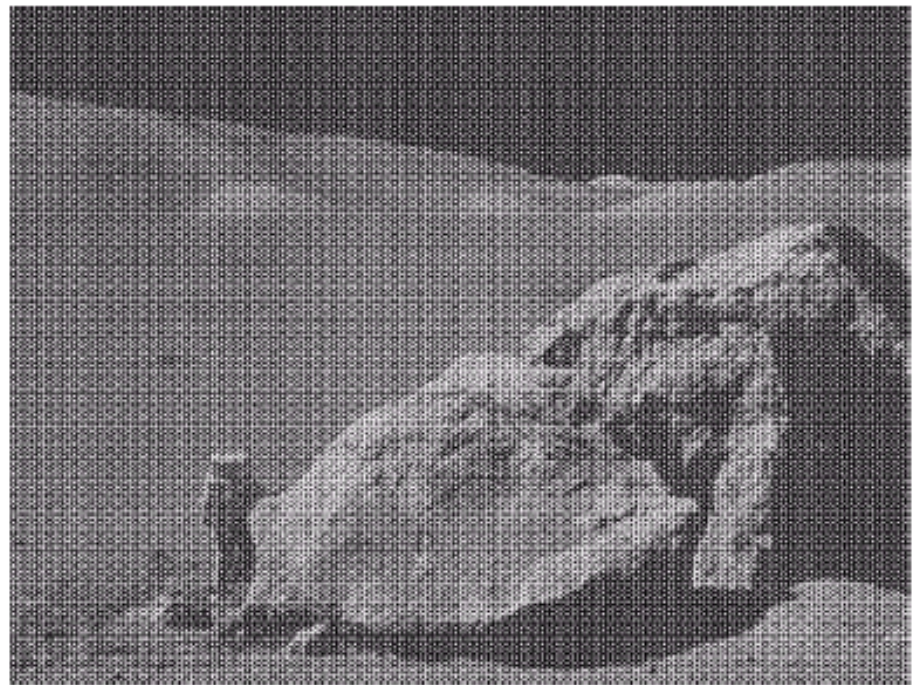
- Arises typically from electrical or electromechanical interference
- Reduced significantly via frequency domain filtering

a

b

FIGURE 5.5

(a) Image corrupted by sinusoidal noise.
(b) Spectrum (each pair of conjugate impulses corresponds to one sine wave).
(Original image courtesy of NASA.)



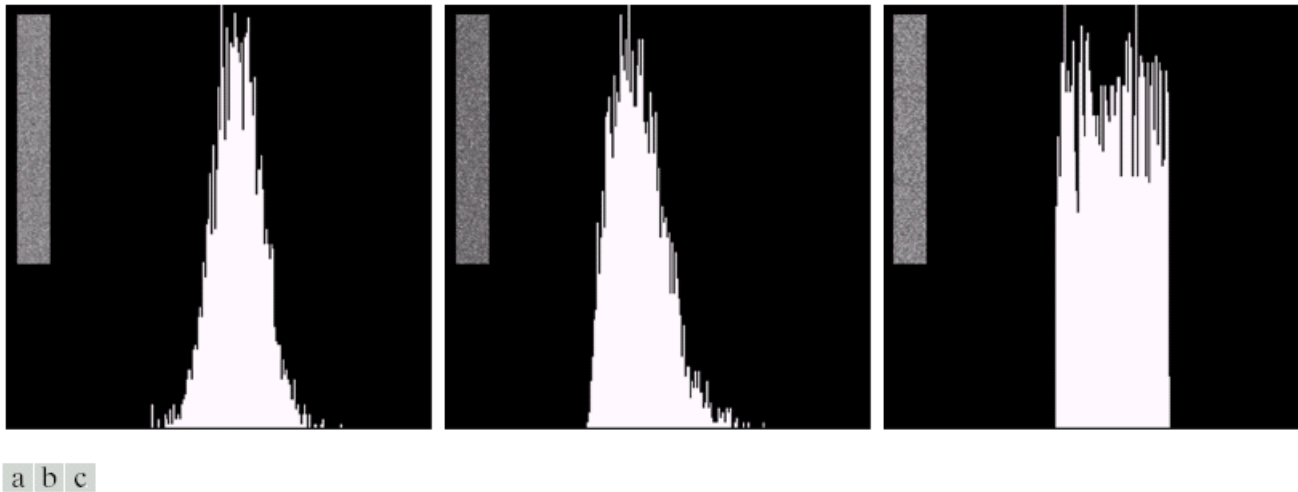


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Restoration in the Presence of Noise

Only-Spatial Filtering

- Degradation

- Spatial domain

$$g(x, y) = f(x, y) + \eta(x, y)$$

- Frequency domain

$$G(u, v) = F(u, v) + N(u, v)$$



○ Mean filters

- Arithmetic mean filter

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

- Geometric mean filter

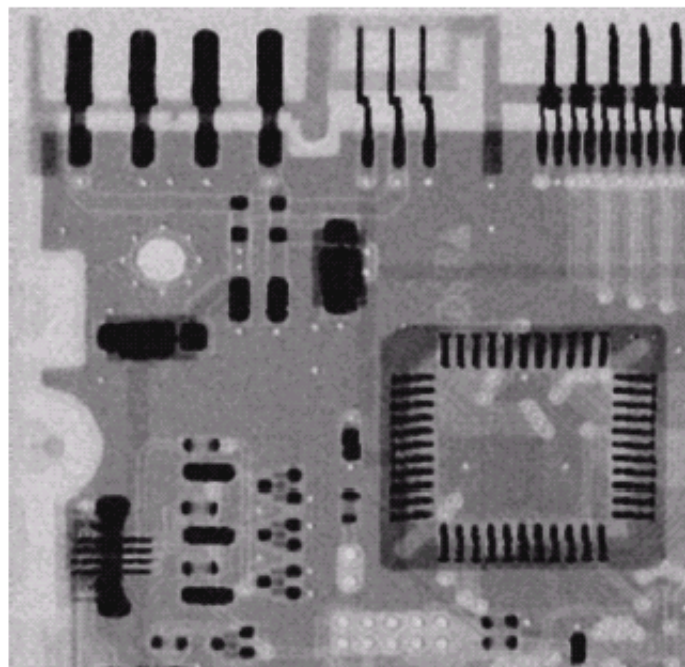
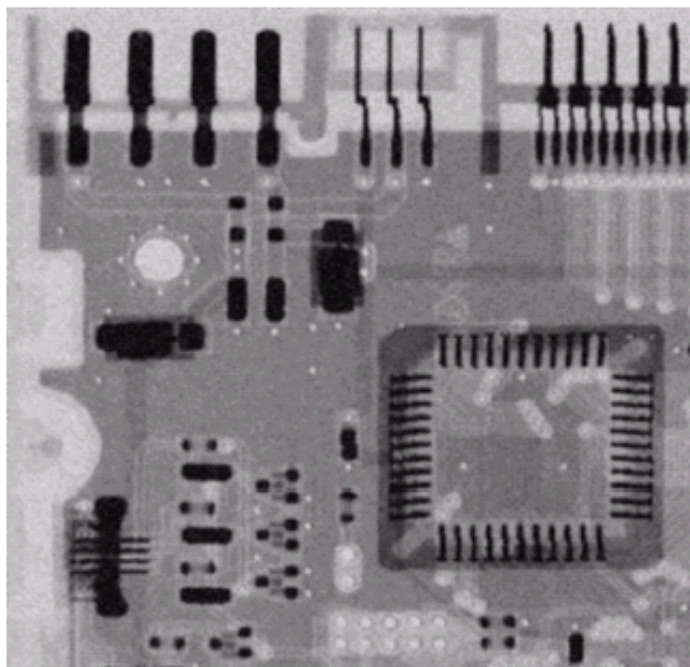
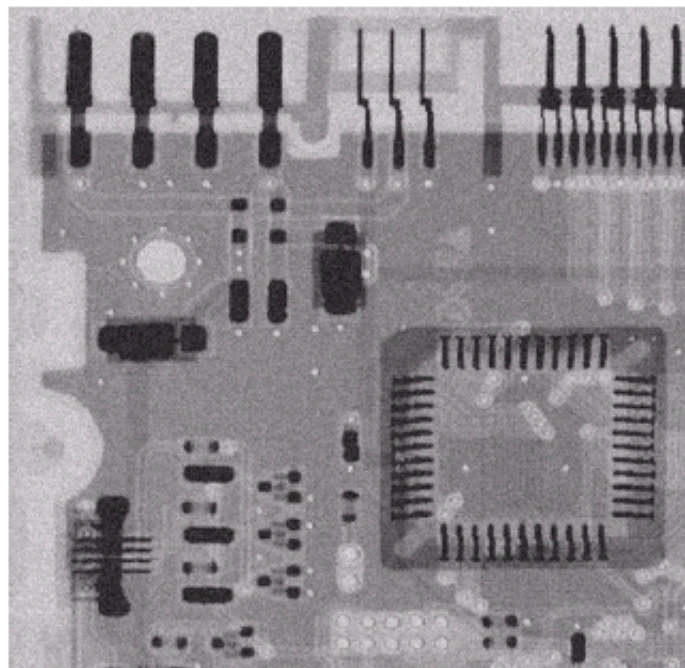
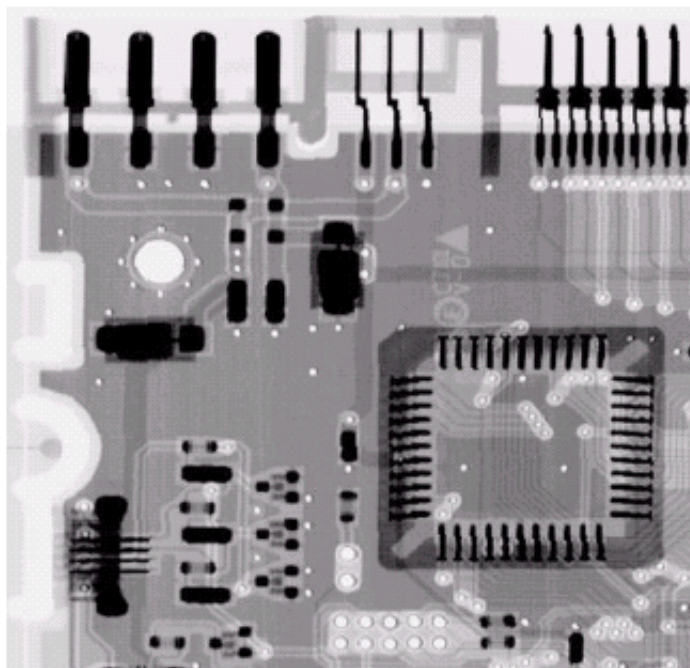
$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$

-
- Harmonic mean filter
 - Works well for salt noise, but fails for pepper noise

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$$

-
- Contraharmonic mean filter
 - $Q > 0$: eliminates pepper noise
 - $Q < 0$: eliminates salt noise

$$\hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q}$$



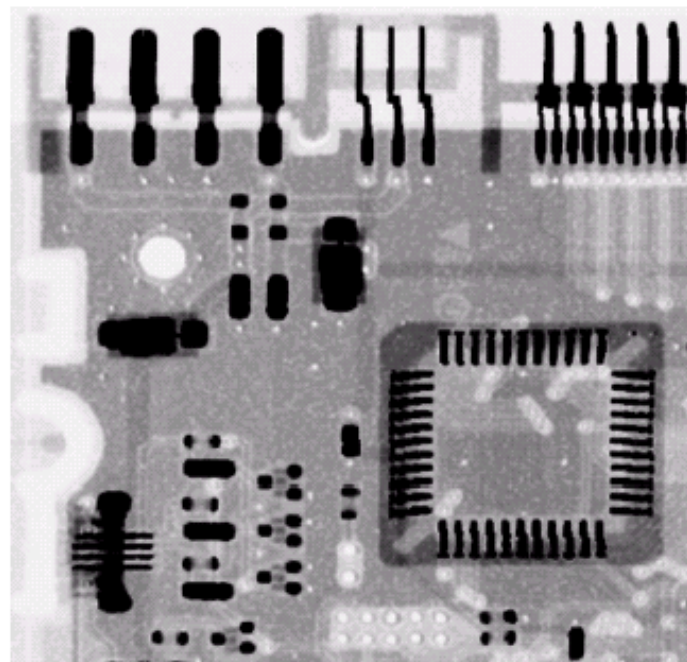
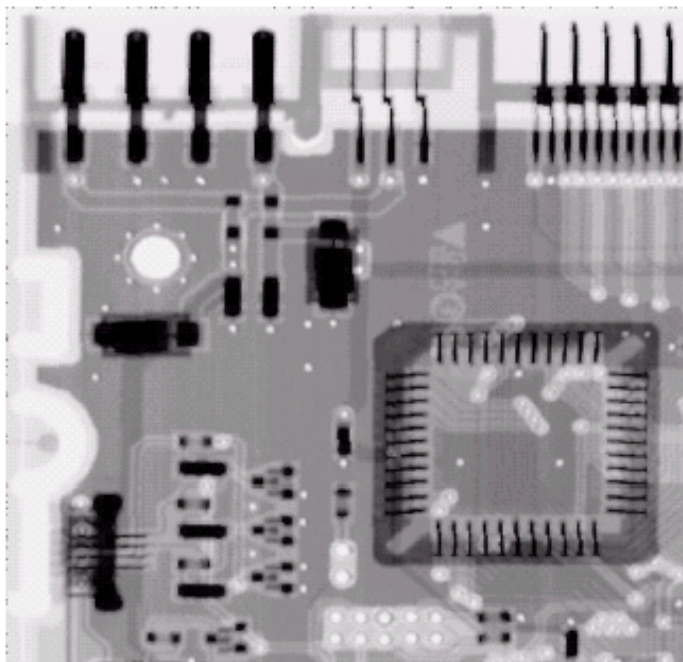
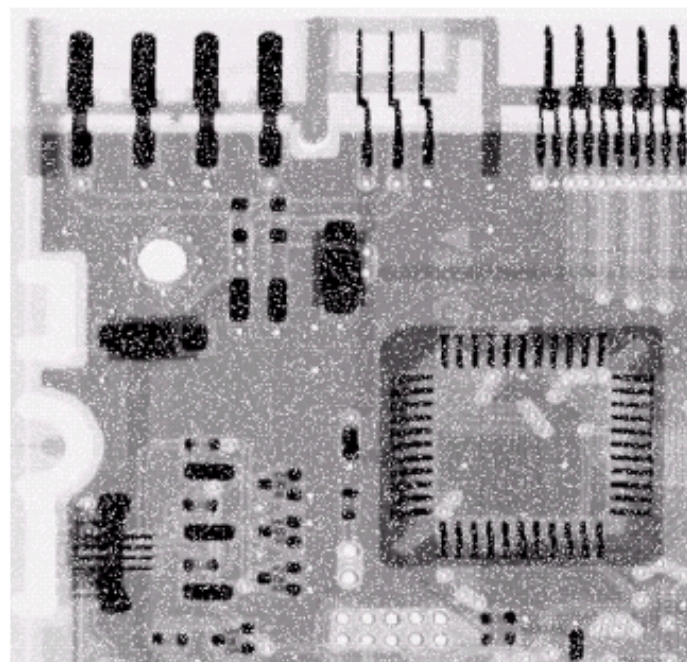
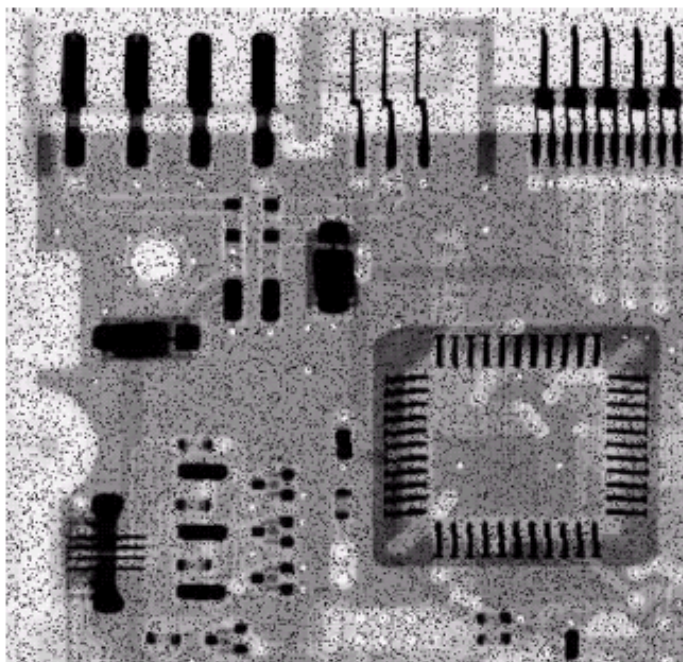
a	b
c	d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3 . (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

a	b
c	d

FIGURE 5.8

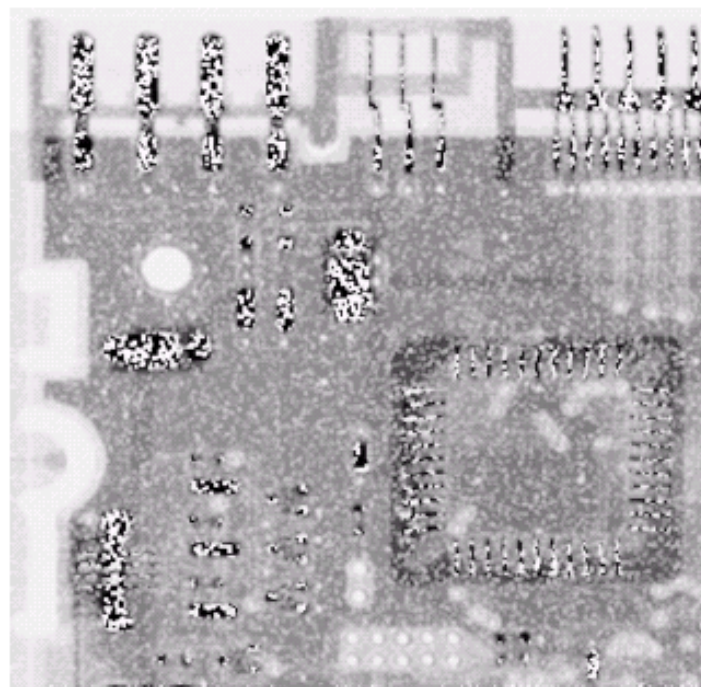
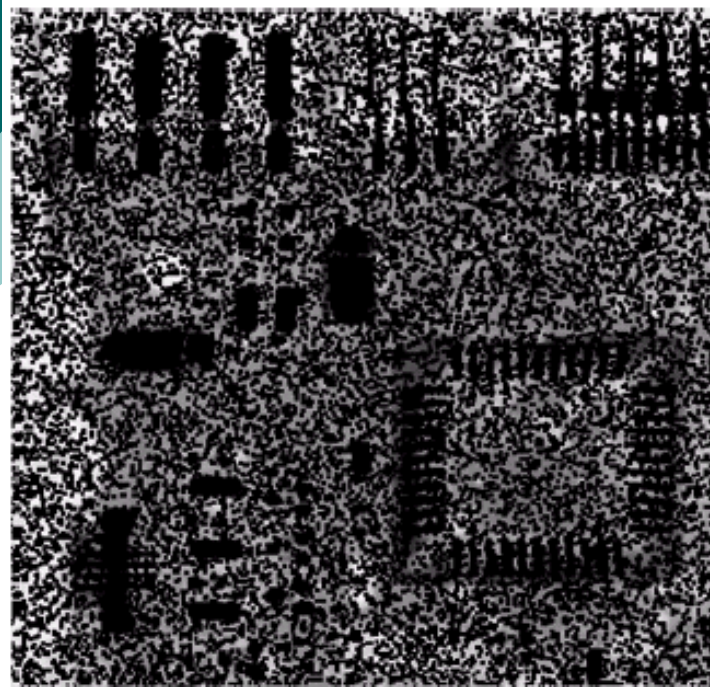
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.





○ Usage

- Arithmetic and geometric mean filters: suited for Gaussian or uniform noise
- Contraharmonic filters: suited for impulse noise



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.



○ Order-statistics filters

- Median filter

- Effective in the presence of both bipolar and unipolar impulse noise

$$\hat{f}(x, y) = \underset{(s,t) \in S_{xy}}{\text{median}} \{g(s, t)\}$$

- Max and min filters

- max filters reduce pepper noise

$$\hat{f}(x, y) = \max_{(s,t) \in S_{xy}} \{g(s, t)\}$$

- min filters salt noise

$$\hat{f}(x, y) = \min_{(s,t) \in S_{xy}} \{g(s, t)\}$$

- Midpoint filter

- Works best for randomly distributed noise, like Gaussian or uniform noise

$$\hat{f}(x, y) = \frac{1}{2} \left[\max_{(s,t) \in S_{xy}} \{g(s, t)\} + \min_{(s,t) \in S_{xy}} \{g(s, t)\} \right]$$

- 
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- Alpha-trimmed mean filter
 - Delete the $d/2$ lowest and the $d/2$ highest gray-level values
 - Useful in situations involving multiple types of noise, such as a combination of salt-and-pepper and Gaussian noise

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s,t) \in S_{xy}} g_r(s, t)$$

a	b
c	d

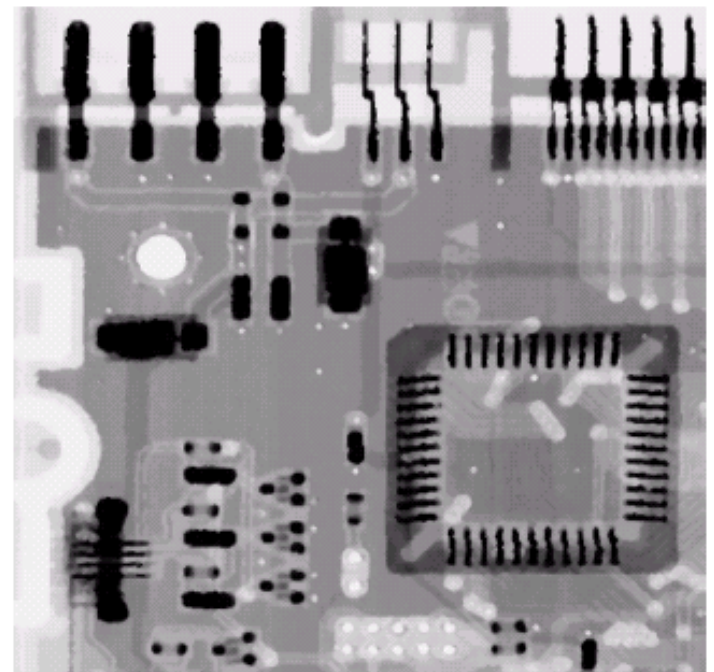
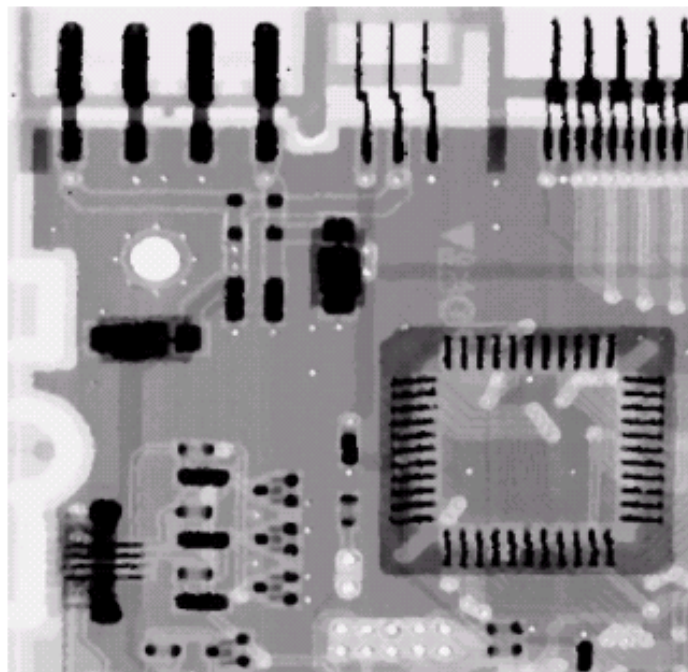
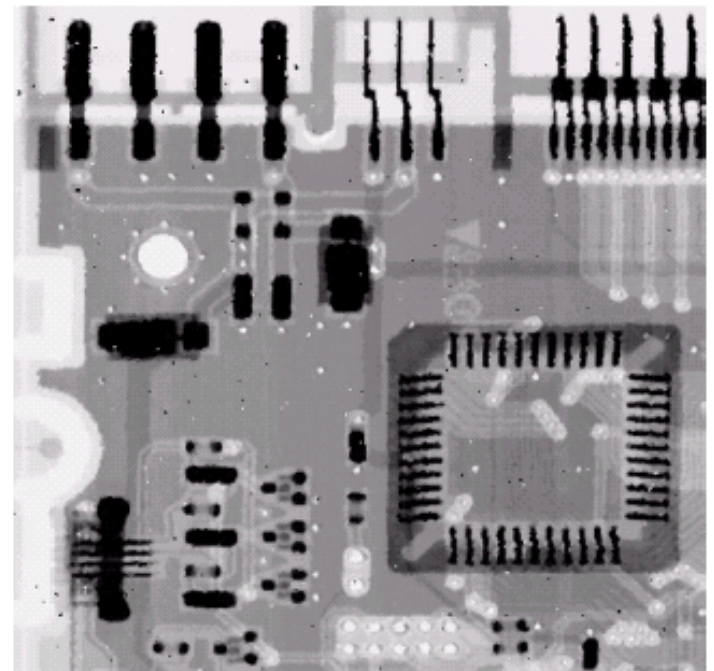
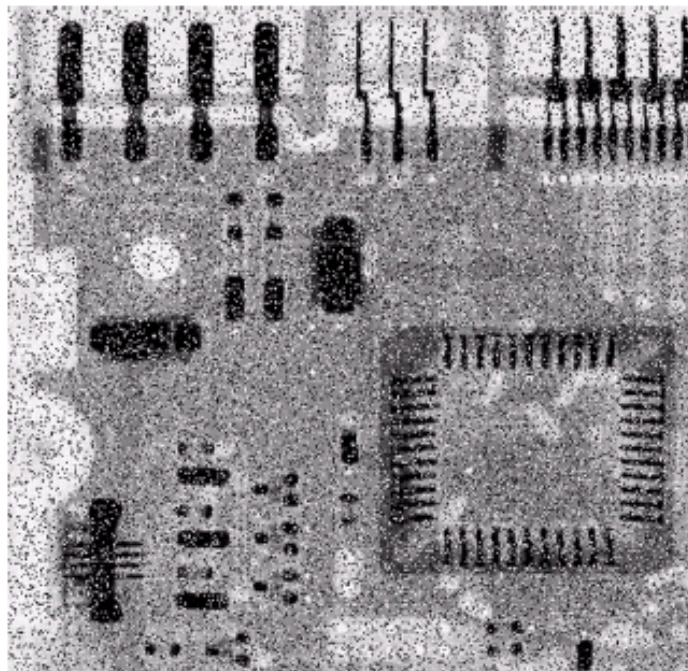
FIGURE 5.10

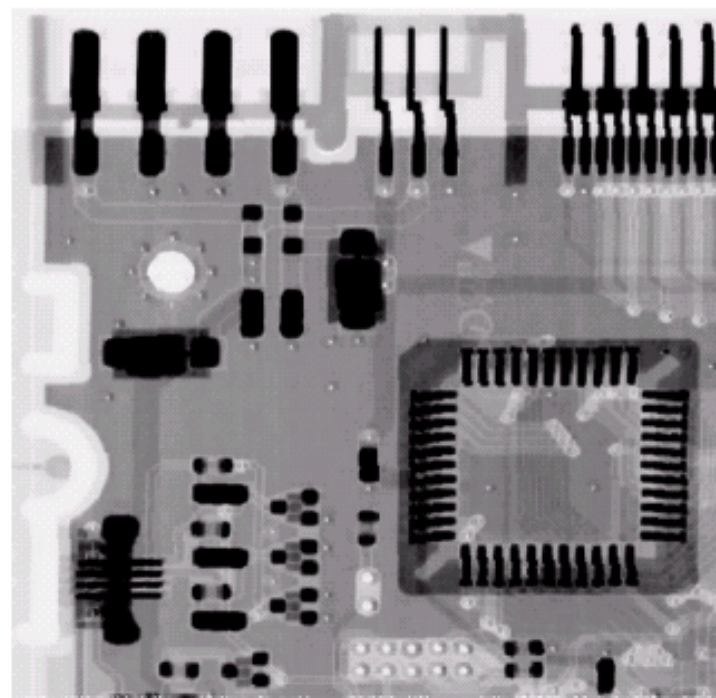
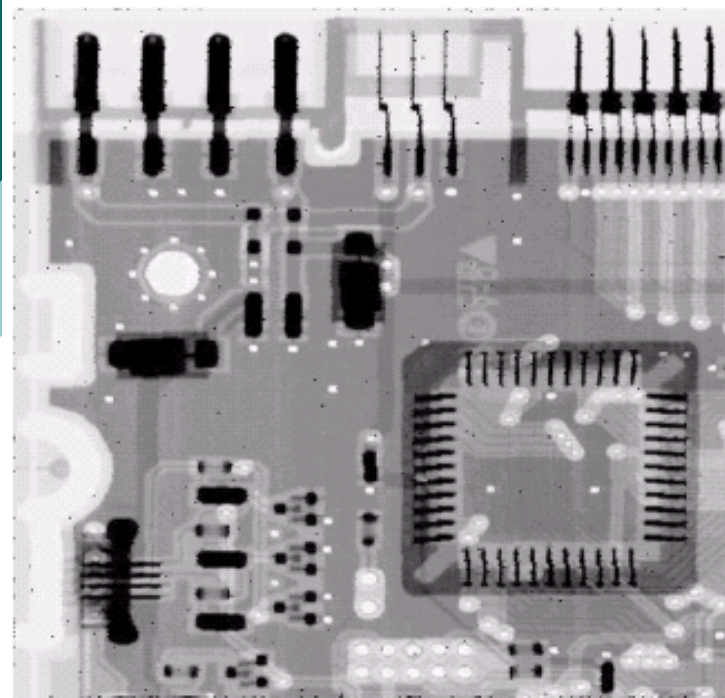
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.

(b) Result of one pass with a median filter of size 3×3 .

(c) Result of processing (b) with this filter.

(d) Result of processing (c) with the same filter.





a b

FIGURE 5.11

(a) Result of filtering

Fig. 5.8(a) with a max filter of size 3×3 . (b) Result of filtering 5.8(b) with a min filter of the same size.

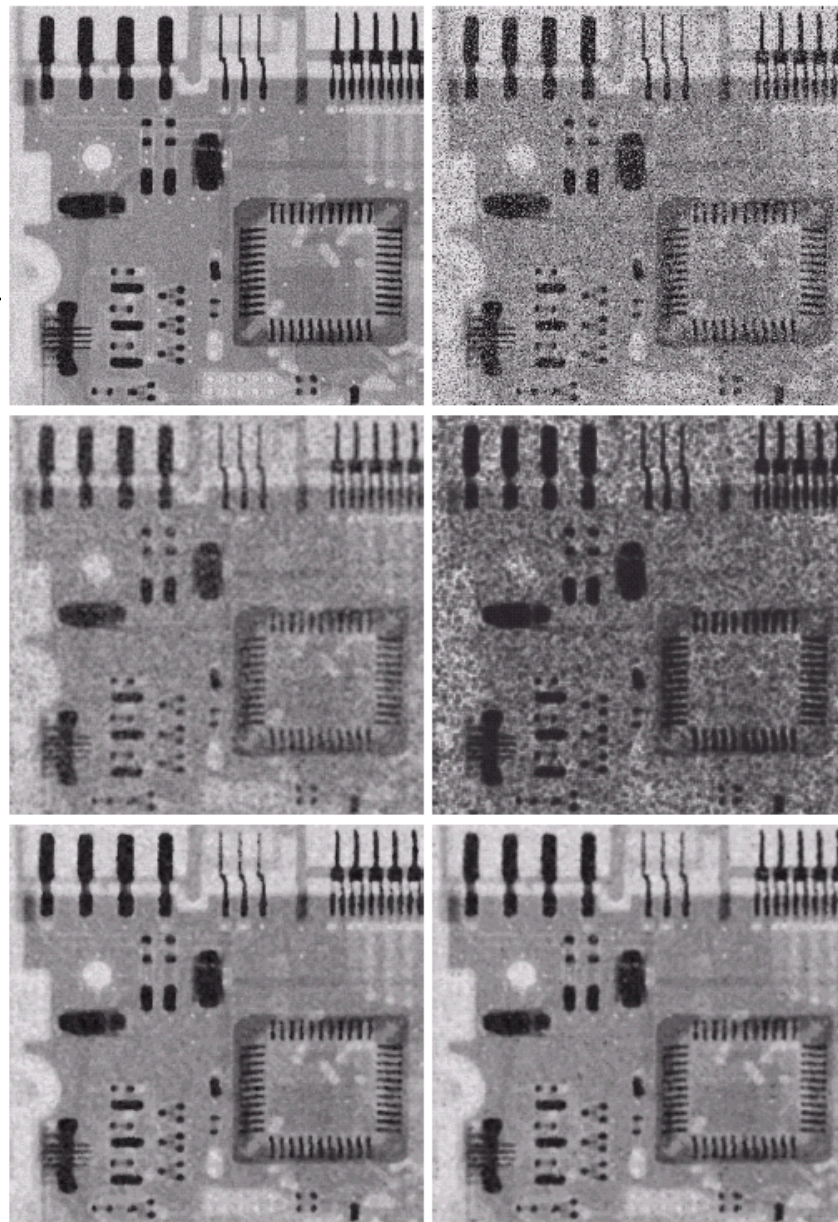


FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5×5 : (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

a	b
c	d

FIGURE 5.13

(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000.
 (b) Result of arithmetic mean filtering.
 (c) Result of geometric mean filtering.
 (d) Result of adaptive noise reduction filtering. All filters were of size 7×7 .

