Z/K/TY

Reg. No:

# Final Assessment Test – November 2025

VIT
Vellore Institute of Technology

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 0675 / 0723 / 0726 / 0739 / 0740

Slot: D2+TD2+TDD2

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

| COs | CO Statements |
|---|---|
| CO1 | Construct analytic functions and find complex potential of fluid flow and electric fields. |
| CO2 | Find the image of straight lines by elementary transformations and express analytic functions in power series. |
| CO3 | Evaluate real integrals using techniques of contour integration. |
| CO4 | Use the power of inner product and norm for analysis. |
| CO5 | Use matrices and transformation for solving engineering problems. |

BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)

**Answer ALL Questions**
**(10 X 10 = 100 Marks)**

1. The complex potential of a fluid flow is given by $\Omega(z) = V_0\{z + (a^2/z)\}$ where $V_0$ and $a$ are positive constants. — CO1 BL3
   (a) Obtain equations for the streamlines and equipotential lines, represent them graphically, and interpret physically.
   (b) Show that we can interpret the flow as that around a circular obstacle of radius $a$.
   (c) Find the velocity at any point and determine its value far from the obstacle.

2. a) If $f(z) = z^3$, show that $u$ and $v$ satisfy the $C - R$ equations. Also prove that the families of curves $u = C_1$ and $v = C_2$ are orthogonal to each other. [5] — CO1 BL2
   b) Show that the function $u(x,y) = 3x^2y + x^2 - y^3 - y^2$ is a harmonic function. Find the harmonic conjugate $v(x,y)$ such that $u + iv$ is an analytic function. [5]

3. Let the rectangular region $\mathcal{R}$ in the $z$ plane be bounded by $x = 0,\ y = 0,$ $x = 2,\ y = 1$. Determine the region $\mathcal{R}'$ of the $w$ plane into which $\mathcal{R}$ is mapped under the transformations: — CO2 BL3
   (a) $w = z + (1 - 2i)$
   (b) $w = \sqrt{2}e^{\pi i/4}z + (1 - 2i)$.

4. Find the bilinear transformation which maps $z = 1, i, -1$ respectively onto $w = i, 0, -i$. Hence find the fixed points. — CO2 BL2

5.a) Evaluate $\int_0^\infty \frac{dx}{x^4+a^4}, a>0$. CO3 BL3

OR

5.b) Expand $f(z) = 1/(z-1)(z-2)$ in a Laurent's series valid in the region (i) $|z-1|>1$, (ii) $0<|z-2|<1$, (iii) $|z|>2$, (iv) $0<|z-1|<1$. CO3 BL3

6. Find a basis and dimension for the row space of $A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$ consisting entirely the row vectors of $A$. Also find the nullity of $A$. CO4 BL2

7.a) Let the transformation T be a multiplication by the matrix $A$ where CO4 BL3

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 5 & 13 \\ -2 & -1 & -4 \end{bmatrix}$$

(i) Find a basis for the range of $T$.
(ii) Find a basis for the kernel of $T$.
(iii) Find the rank and nullity of $T$.
(iv) Verify the dimension theorem.

OR

7.b) Let $T: R^3 \to R^3$ be the linear operator defined by the formula CO4 BL3
$T(X_1, X_2, X_3) = (X_1 - X + X_3, 2X_2 - X_3, 2X_1 + 3X_2)$.
Determine whether T is one-to-one. If so, find $T^{-1}(X_1, X_2, X_3)$.

8. Use the Gram–Schmidt process to transform the basis $\{1, x, x^2\}$ of $P_2$ into an orthonormal basis if CO4 BL3
(i) $\langle p, q\rangle = p(0)\,q(0) + p(1)\,q(1) + p(2)\,q(2)$
(ii) $\langle p,q\rangle = \int_0^2 p(x)q(x)\,dx$

9. Find the Eigenvalues, Eigenvectors and the Eigen spaces of the matrix CO5 BL2

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}.$$

10. Sodium hydroxide ($NaOH$) reacts with sulphuric acid ($H_2SO_4$) to yield sodium sulphate ($Na_2SO_4$) and water ($H_2O$), CO5 BL3

$$NaOH + H_2SO_4 \rightarrow Na_2SO_4 + H_2O$$

Balance the equation by using the Gauss-Jordan Method.

⇔⇔⇔ Z/K/TY ⇔⇔⇔