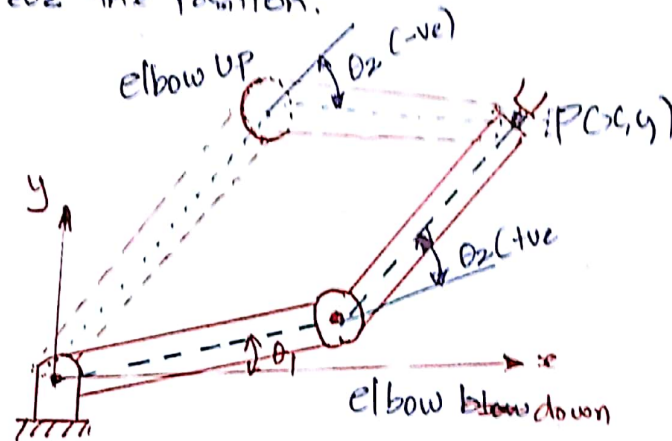


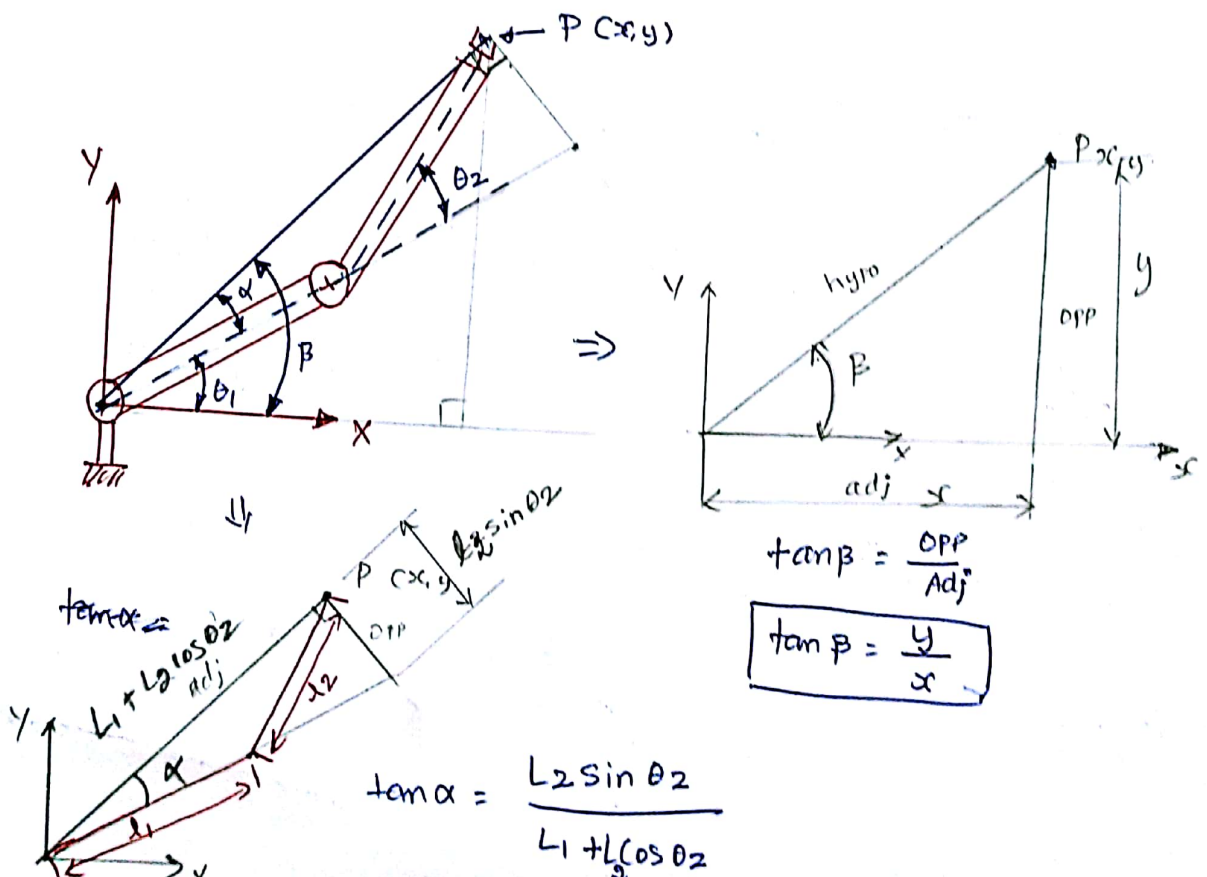
2-link planar Robot

To find θ_1

The two link arm at $P(x, y)$ indicating two possible configuration to achieve the position.



To Solve θ_1 [down elbow]



$$\tan \beta = \frac{y}{x} \quad ; \quad \tan \alpha = \frac{l_2 \sin \theta_2}{l_2 \cos \theta_2 + l_1}$$

Here,

$$\tan(\beta - \alpha) = \tan \theta_1$$

$$\tan(\beta - \alpha) = \tan(\theta_1)$$

$$\left[\because \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \right]$$

$$\tan(\beta - \alpha) = \frac{\tan \beta - \tan \alpha}{1 + \tan \beta \tan \alpha}$$

$$= \frac{\frac{y}{x} - \frac{l_2 \sin \theta_2}{l_2 \cos \theta_2 + l_1}}{1 + \left[\frac{y}{x} \times \frac{l_2 \sin \theta_2}{l_1 + l_2 \cos \theta_2} \right]}$$

$$= \frac{y(l_2 \cos \theta_2 + l_1) - x l_2 \sin \theta_2}{x(l_2 \cos \theta_2 + l_1) + y l_2 \sin \theta_2}$$

$$= \frac{y(l_2 \cos \theta_2 + l_1) - x l_2 \sin \theta_2}{x(l_2 \cos \theta_2 + l_1) + y l_2 \sin \theta_2}$$

$$\tan \theta_1 = \frac{y(l_1 + l_2 \cos \theta_2) - x l_2 \sin \theta_2}{x(l_1 + l_2 \cos \theta_2) + y l_2 \sin \theta_2}$$

$$\boxed{\tan \theta_1 = \frac{y(l_1 + l_2 \cos \theta_2) - x l_2 \sin \theta_2}{x(l_1 + l_2 \cos \theta_2) + y l_2 \sin \theta_2}}$$