Y/K/TX

Reg. No: 23BDS0051

Final Assessment Test – November 2024

VIT
Vellore Institute of Technology

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 2489 / 2491 / 2492 / 2493 / 2494 / 2495 / 2496 / 2497 / 2498 / 2499 / 2500 / 2501 / 2503 / 2504 / 2505 / 2506 / 2524

Slot: A1+TA1+TAA1

Time: Three Hours

Max. Marks: 100

- **KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

Answer ALL Questions

(10 X 10 = 100 Marks)

1.a) Verify that the families of curves $u = c_1$ and $v = c_2$ cut orthogonally, when $w = u + iv = z^3$. [10]

OR

1.b) Find v such that $w = u + iv$ is an analytic function of z, given that $u = e^{x^2-y^2}\cos 2xy$. Hence find w. [10]

2. Find the image of the following regions under the transformation $w = \frac{1}{z}$. [10]

(a) the half plane $x > c$, when $c > 0$.

(b) the half plane $y > c$, when $c < 0$, also show the corresponding regions graphically.

3. Show that the transformation $w = \frac{z-i}{1-iz}$ maps (i) the interior of the circle $|z| = 1$ onto the lower half of the plane and (ii) the upper half of the $z-$plane onto the interior of the circle $|w| = 1$. [10]

4. Find the Laurent's expansion of the function $f(z) = \frac{1}{z(1-z)}$ valid in the region [10]

(i) $|z+1| < 1$, (ii) $1 < |z+1| < 2$.

5. Evaluate $\int_0^{2\pi} \frac{\sin^2\theta}{5 - 3\cos\theta}\, d\theta$, using contour integration. [10]

6. Find the basis and dimension of row, column and null space of the matrix A, [10]

where $A = \begin{bmatrix} 1 & -2 & 5 & -3 \\ 2 & 3 & 1 & -4 \\ 3 & 8 & -3 & -5 \end{bmatrix}$.

7.a) Let $F: R^4 \to R^3$ be the linear mapping defined by [10]

$F(x,y,z,t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$

(i) Find a basis and the dimension of the image of F.

(ii) Find a basis and the dimension of the kernel of the map F.

OR

7.b) Let $\beta = \{v_1, v_2, v_3\}$ be a basis for the 3-space $\mathbb{R}^3$ consisting of $v_1 = (1,1,0), v_2 = (1,0,1), v_3 = (0,1,1)$. Let T be the linear transformation on $\mathbb{R}^3$ given by the matrix $[T]_\beta = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \\ -1 & 1 & 1 \end{bmatrix}$. Let $\alpha = \{e_1, e_2, e_3\}$ be the standard basis. Find the basis change matrix $[id]_\beta^\alpha$ and $[T]_\alpha$. [10]

8. Apply the Gram Schmidt orthogonalization process to find an orthonormal basis for the subspace U of R^4 spanned by $u_1 = (1,1,1,1)$, $u_2 = (1,2,4,5)$, $u_3 = (1,-3,-4,-2)$. [10]

9. Express the following system of equations in matrix form and solve then by the Gauss elimination method. [10]

$$2x_1 + x_2 + 2x_3 + x_4 = 6$$
$$6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$$
$$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$$
$$2x_1 + 2x_2 - x_3 + x_4 = 10.$$

10. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, Verify Cayley Hamilton theorem and hence prove that: [10]

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}.$$

⇔⇔⇔Y/K/TX⇔⇔⇔