School of Computer Science and Engineering

Winter Semester 2023-24

Continuous Assessment Test – 1

SLOT: B2+TB2

Programme Name &Branch: B.Tech
Course Name & code: BCSE304L Theory of Computation
Class Number (s): VL2023240500760, 0763,0766, 0768, 0771, 0774,0784, 0789, 0797, 0845, 0861, 1014, 1015, 1026,1029, 1036, 1037, 1042,1043, 1045, 1047, 1049,1051
Faculty Name (s): Prof.Sathiya Kumar C, Prof.Anand M,Prof.Arumuga Arun R, Prof.Viswanathan P, Prof.Shalini L, Prof. Kannadasan R,Prof. Krishna Rani Samal K,Prof.Navamani T M, , Prof.Rajarajan G,Prof.Madiajagan M, Prof.Saritha Murali, Prof.Vishnupriya,Prof. Mohana CM,Prof.Krishnaraj N, Prof.Kanagaraj R,Prof.Anand Bihari,Prof. Somasundaram S K, Prof.Hussain Ahmed Chowdhury, Prof. Sarwesh P,Prof. Umamaheswari M, Prof. Konatham Sumalatha Prof. Sabyasachi Kamila, Prof. Uma Priya D

Exam Duration: 90 **Maximum Marks: 50**

<table>
<tr><th>Q.No.</th><th>Question</th><th>Max Marks</th><th>CO</th><th>BL</th></tr>
<tr><td>1.</td><td>a) Prove by induction on the following
$(uv)^r = v^r u^r$, where u and v are strings over Σ and r is the reversal operator.
(3 Marks)
b). i) Assume $\Sigma = \{a,b,c\}$, then find Σ^2.
ii) Assume $\Sigma = \{0,1\}$, then find Σ^3.
(4 Marks)
c) Given L in Σ^*, can both L & L^c (c is a complementary operation) be finite? Justify?
(3 Marks)</td><td>10</td><td>CO1</td><td>BL2</td></tr>
<tr><td>2.</td><td>Convert the following NFA with ε to NFA without ε.
<table>
<tr><th></th><th>a</th><th>B</th><th>ε</th></tr>
<tr><td>-> q0</td><td>q1</td><td>∅</td><td>{q0,q2}</td></tr>
<tr><td>q1</td><td>q2</td><td>q1</td><td>q1</td></tr>
<tr><td>*q2</td><td>q2</td><td>∅</td><td>q0</td></tr>
</table>
Starting state : q0 Final State : q2</td><td>10</td><td>CO2</td><td>BL3</td></tr>
<tr><td>3.</td><td>a) Construct a DFA that accepts all the strings over $\Sigma = \{a,b\}$ whose length is divisible by 6.
(5 Marks)
b) Convert the following NFA to DFA.
<table>
<tr><th></th><th>0</th><th>1</th></tr>
<tr><td>-> q0</td><td>{q1,q3}</td><td>q1</td></tr>
<tr><td>*q1</td><td>q2</td><td>{q1,q2}</td></tr>
<tr><td>q2</td><td>q3</td><td>q0</td></tr>
</table></td><td>10</td><td>CO2</td><td>BL3</td></tr>
</table>

<table>
<tr>
<td></td>
<td>
<table>
<tr><td>*q3</td><td>∅</td><td>q0</td></tr>
</table>
Starting state : q0
Final States : {q1,q3}
(5 Marks)
</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>
i) Minimize the given DFA transition table.
<table>
<tr><td></td><td>A</td><td>B</td></tr>
<tr><td>-> q0</td><td>q1</td><td>q2</td></tr>
<tr><td>q1</td><td>q2</td><td>q4</td></tr>
<tr><td>*q2</td><td>q3</td><td>q2</td></tr>
<tr><td>*q3</td><td>q4</td><td>q4</td></tr>
<tr><td>*q4</td><td>q4</td><td>q5</td></tr>
<tr><td>q5</td><td>q5</td><td>q5</td></tr>
</table>
Starting state : q0
Final States : {q2,q3,q4}
(5Marks)

ii) Convert the given Regular Expression into Finite Automata.
$\mathbf{((ab)^{*}.(ab+a^{+}b))}$
(5Marks)
</td>
<td>10</td>
<td>CO2</td>
<td>BL3</td>
</tr>
<tr>
<td>5.</td>
<td>
Convert the Finite Automata (whose transition table is given below) to the equivalent Regular Expression.
<table>
<tr><td></td><td>0</td><td>1</td></tr>
<tr><td>-> q0</td><td>q0</td><td>q1</td></tr>
<tr><td>q1</td><td>q2</td><td>q3</td></tr>
<tr><td>q2</td><td>q2</td><td>q3</td></tr>
<tr><td>*q3</td><td>q2</td><td>q3</td></tr>
</table>
Starting state : q0
Final State : q3
</td>
<td>10</td>
<td>CO2</td>
<td>BL3</td>
</tr>
</table>

Theory of Computation

B2+TB2.

1.a) $(uv)^r = v^r u^r$

(i) when u is an arbitary string with length '0' $\Rightarrow u = \varepsilon$

$(uv)^r = (\varepsilon v)^r = v^r = v^r \varepsilon = v^r \varepsilon^r = v^r u^r$

ii) Let u be an orbitary length with $n > 0$

$u = ay.$

$(uv)^r = ((ay)v)^r = (a.(yv))^r$

$= (yv)^r . a^r$

$= v^r y^r . a^r$

$= v^r (y^r a^r)$

$= v^r u^r$

Hence $\forall u$ $(uv)^r = v^r u^r.$

1. b. i) $\Sigma = \{a, b, c\}$

$\Sigma^2 = \{$ i/p String with length of 2 $\}$

$= \{a, b, c\} . \{a, b, c\}$

$= \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$

(ii) $\Sigma = \{0, 1\}$

$\Sigma^3 = \{0, 1\}\{0, 1\}\{0, 1\}$

$= \{000, 001, 010, 011, 100, 101, 110, 111\}$

(iii) $L \in \Sigma^*$, can L and L^c be finite?

Let $\Sigma = \{0, 1\}$

$\Sigma^* = \{$all ~~strings~~ strings with 0, 1 of any length including 0$\} = U$

finite $\Leftarrow$ $L = \{$Set of strings with Specific Pattern$\}$

infinite. $\Leftarrow$ $L^c = U - L = \{$set of strings of all excluding $L\}$.

2.

	a	b	ε
$\rightarrow q_0$	q_1	ϕ	$\{q_0, q_2\}$
q_1	q_2	q_1	q_1
$* q_2$	q_2	ϕ	q_0.

Soln:

$$\varepsilon\text{-closure}(q_0) = \{q_0, q_2\}$$
$$\varepsilon\text{-closure}(q_1) = \{q_1\}$$
$$\varepsilon\text{-closure}(q_2) = \{q_0, q_2\}$$

<u>State q_0:</u>

$$\delta_N(q_0, a) = \varepsilon\text{-closure}(\hat{\delta}(\delta(q_0, \varepsilon), a))$$
$$= \varepsilon\text{-closure}(\hat{\delta}(q_0, q_2), a)$$
$$= \varepsilon\text{-closure}(q_1, q_2)$$
$$= \{q_0, q_1, q_2\}$$

$$\delta(q_0, b) = \varepsilon\text{-closure}(\hat{\delta}(\delta(q_0, \varepsilon), b))$$
$$= \varepsilon\text{-closure}(\hat{\delta}(q_0, q_2), b)$$
$$= \varepsilon\text{-closure}(\phi \cup \phi)$$
$$= \varepsilon\text{-closure}(\phi) = \phi$$

<u>State q_1:</u>

$$\delta_N(q_1, a) = \varepsilon\text{-cl}(\hat{\delta}(\delta(q_1, \varepsilon), a)]$$
$$= \varepsilon\text{-cl}(\hat{\delta}(q_1, a))$$
$$= \varepsilon\text{-cl}(q_2) = \{q_0, q_2\}$$

$$\delta_N(q_1, b) = \varepsilon\text{-cl}(\hat{\delta}(\delta(q_1, \varepsilon), b))$$
$$= \varepsilon\text{-cl}(\delta(q_1, b))$$
$$= \varepsilon\text{-cl}(q_1) = \{q_1\}$$

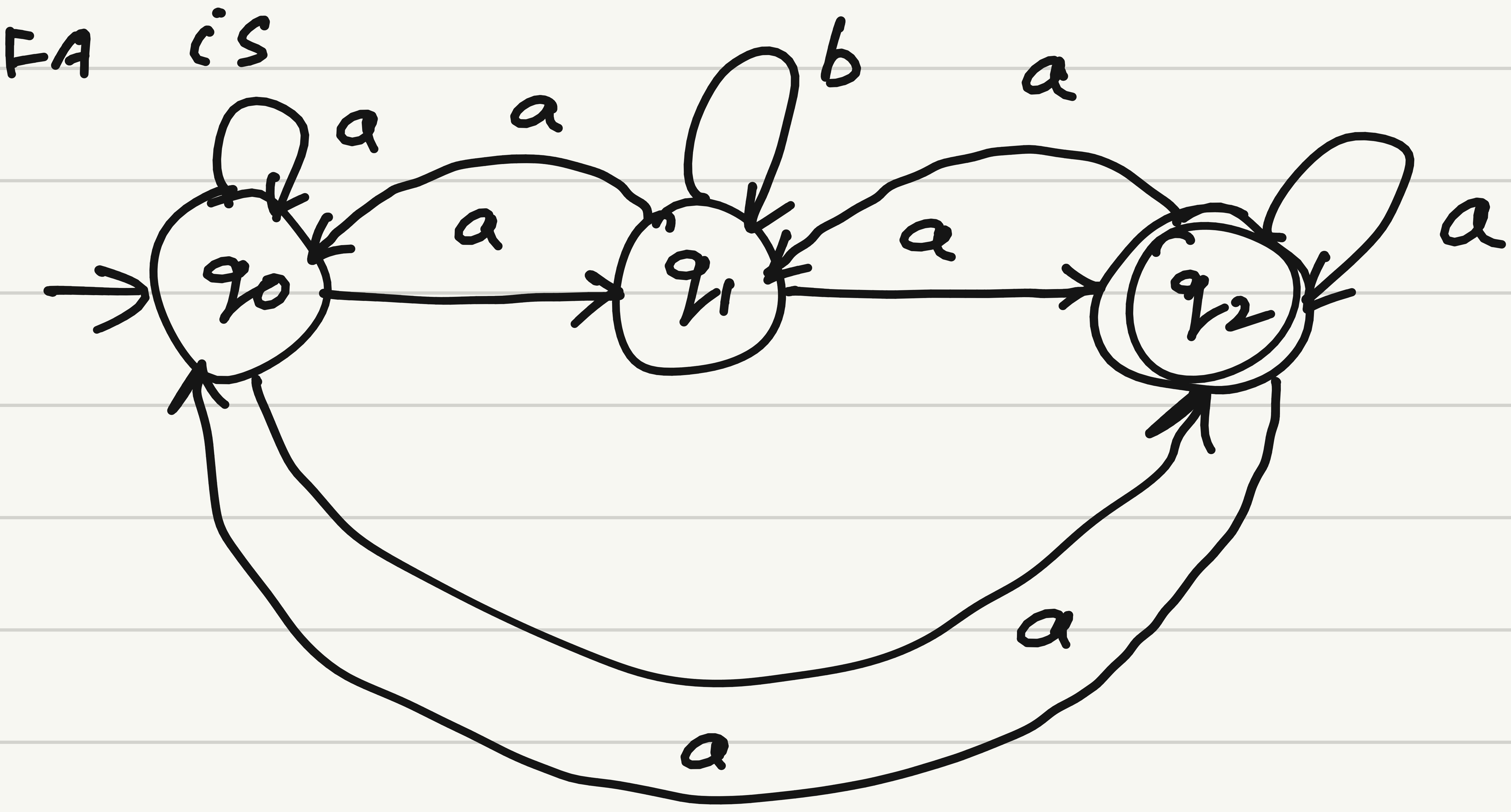
<u>State q_2:</u>

$$\delta_N(q_2, a) = \varepsilon\text{-cl}(\hat{\delta}(\delta(q_2, \varepsilon), a) = \varepsilon\text{-cl}(\hat{\delta}(q_0, q_2, a))$$
$$= \varepsilon\text{-cl}(q_1 \cup q_2) = \{q_0, q_1, q_2\}$$

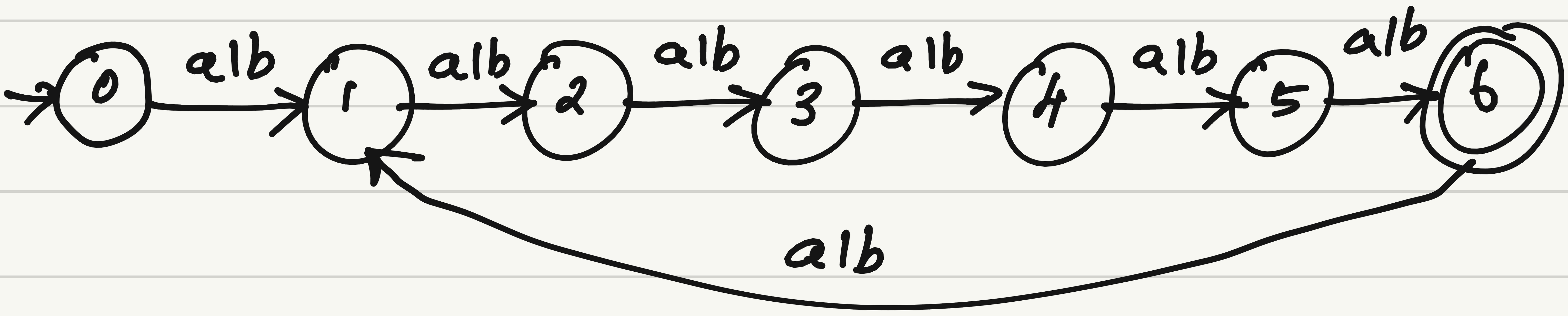
$$\delta_N(q_2, b) = \varepsilon\text{-cl}(\hat{\delta}(\delta(q_2, \varepsilon), b) = \varepsilon\text{-cl}(\hat{\delta}(q_0, q_2, b))$$
$$= \varepsilon\text{-cl}((\phi \cup \phi)) = \phi$$

	a	b
$\rightarrow q_0$	$\{q_0, q_1, q_2\}$	ϕ
q_1	$\{q_0, q_2\}$	$\{q_1\}$
$* q_2$	$\{q_0, q_1, q_2\}$	$\{\phi\}$

NFA is



3.a) All strings over $\Sigma = \{a, b\}$ whose length is divisible by 6. $|w| \% 6 = 0$.



3. b)

	0	1
$\rightarrow q_0$	$\{q_1, q_3\}$	$\{q_1\}$
$* q_1$	q_2	$\{q_1, q_2\}$
q_2	q_3	q_0
q_3	ϕ	q_0.

initial state of DFA $= [q_0] = A$.

State A

$\delta_D(A, 0) = \delta_N(q_0, a)$
$= \{q_1, q_3\}$
$\delta_D(A, 0] = [q_1, q_3] = B$

$\delta_D(A, 1) = \delta_N(\{q_0\}, b))$
$= \{q_1\}$
$\delta_D(A, 1) = [q_1] \Rightarrow C$

State B

$$\delta_D(B,0) = \delta_N(\{q_1, q_3\}, 0) = \{q_2\} \cup \phi = \{q_2\}$$
$$\delta_D(B,0) = [q_2] = D$$
$$\delta_D(B,1) = \delta_N(\{q_1, q_3\}, 1) = \{q_1, q_2\} \cup \{q_0\}$$
$$= \{q_0, q_1, q_2\}$$
$$\delta_D(B,1) = [q_0, q_1, q_2] \Rightarrow E$$

State C

$$\delta_D(C,0) = \delta_N(q_1, 0) = \{q_2\}$$
$$\delta_D(C,0) = [q_2] \Rightarrow D$$

$$\delta_D(C,1) = \delta_N(q_1, 1) = \{q_1, q_2\}$$
$$\delta_D(C,1) = [q_1, q_2] \Rightarrow F$$

State D

$$\delta_D(D,0) = \delta_N(q_2, 0) = \{q_3\} \Rightarrow G$$
$$\delta_D(D,1) = \delta_N(q_2, 1) = \{q_0\} \Rightarrow A$$

State E

$$\delta_D(E,0) = \delta_N(\{q_0, q_1, q_2\}, 0) = \{q_1, q_3, q_2\} = H$$
$$\delta_D(E,1) = \delta_N(\{q_0, q_1, q_2\}, 1) = \{q_1, q_2, q_0\} = E$$

State G

$$\delta_D(F,0) = \delta_N(q_3, 0) = \phi$$
$$\delta_D(F,1) = \delta_N(q_3, 1) = \{q_0\} = A$$

State F

$$\delta_D(G,0) = \delta_N(\{q_1, q_2\}, 0) = \{q_2, q_3\} = I$$
$$\delta_D(G,1) = \delta_N(\{q_1, q_2\}, 1) = \{q_1, q_2, q_0\} \Rightarrow E$$

State H

$$\delta_D(H,0) = \delta_N(\{q_1, q_2, q_3\}, 0) = \{q_2, q_3\} = I$$
$$\delta_D(H,1) = \delta_N(\{q_1, q_2, q_3\}, 1) = \{q_1, q_2, q_0\} \Rightarrow E$$

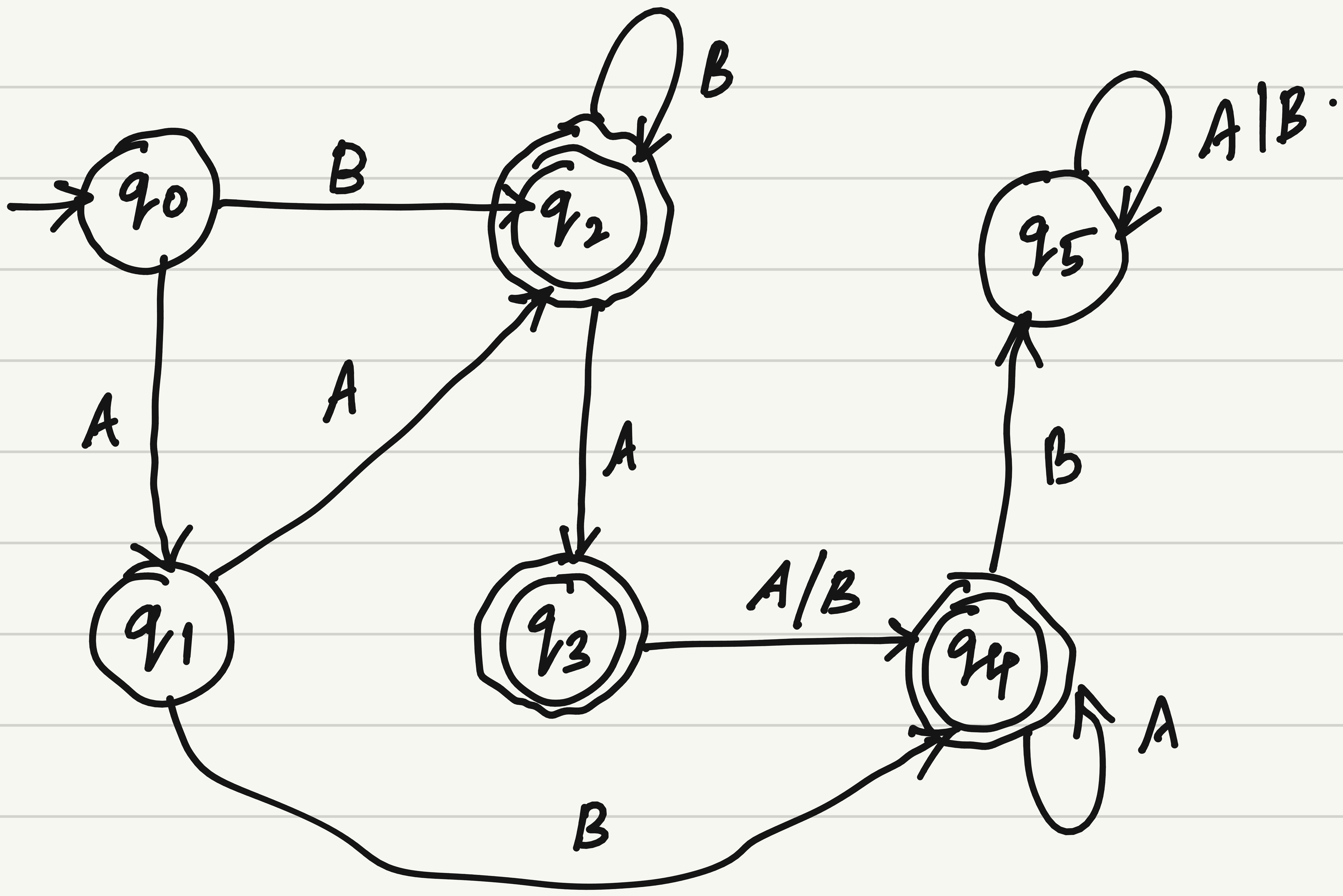
State I

$$\delta_D(I,0) = \delta_N(\{q_2, q_3\}, 0) = \{q_3\} = G$$
$$\delta_D(I,1) = \delta_N(\{q_2, q_3\}, 1) = \{q_0\} = A$$

	0	1
→ A	B	C
* B	D	E
* C	D	F
D	G	A
* E	H	E
* F	I	E
* G	ϕ	A
* H	I	E
* I	G	A

	0	1
q_0	$\{q_1, q_3\}$	$\{q_1\}$
q_1, q_3	$\{q_2\}$	$\{q_0, q_1, q_2\}$
q_1	q_2	q_1, q_2
q_2	q_3	q_0
q_0, q_1, q_2	q_1, q_2, q_3	q_0, q_1, q_2
q_1, q_2	q_2, q_3	q_0, q_1, q_2
q_3	ϕ	q_0
q_1, q_2, q_3	q_2, q_3	q_0, q_1, q_2
q_2, q_3	q_3	q_0

4) i)



	0	1	2	3	4
1	✓				
2	✓	✓			
3	✓	✓	✓		
4	✓	✓	✓	✓	
5	✓	✓	✓	✓	✓

(1,0)
$\delta(1,A) = 2$
$\delta(0,A) = 1$ ✓

3,2
$\delta(3,A) = 4$
$\delta(2,A) = 3$
$\delta(3,B) = 4$
$\delta(2,B) = 2$ ✓

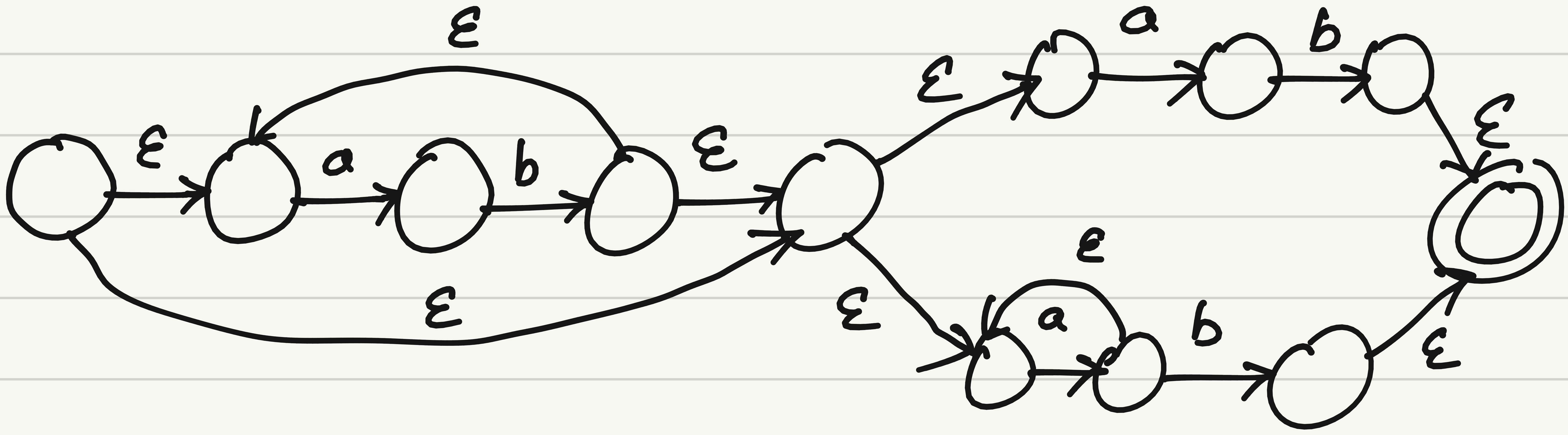
4,2.
$\delta(4,A) = 4$
$\delta(2,A) = 3$
$\delta(4,B) = 5$
$\delta(2,B) = 2$ ✓

$\delta(4,3)$
$\delta(4,A) = 4$
$\delta(3,A) = 4$
$\delta(4,B) = 5$
$\delta(3,B) = 4$ ✓

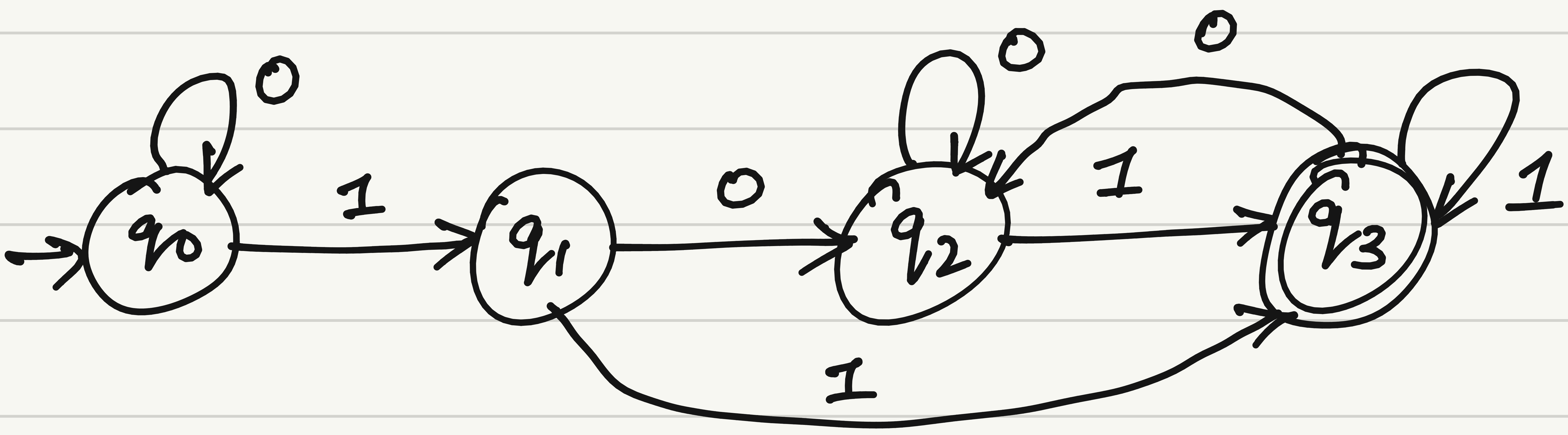
5,1
$\delta(5,A) = 5$
$\delta(1,A) = 2$ ✓

5,2
$\delta(5,A) = 5$
$\delta(2,A) = 3$

4) ii) $(ab)^*(ab+a^+b))$



⑤



$q_0 = \varepsilon + q_0 \cdot 0$ —— (1)

$q_1 = q_0 \cdot 1$ —— (2)

$q_2 = q_1 \cdot 0 + q_2 \cdot 0 + q_3 \cdot 0$ —— (3)

$q_3 = q_1 \cdot 1 + q_2 \cdot 1 + q_3 \cdot 1$ —— (4)

eqn 1 becomes

$q_0 = 0^*$

2 becomes

$q_1 = 0^* 1.$

eqn (3)

$q_2 = q_1 \cdot 0 + q_3 \cdot 0 + q_2 \cdot 0$

$= 0^* 1 0 + q_3 \cdot 0 + q_2 \cdot 0$

$q_2 = (0^* 1 0 + q_3 \cdot 0) \cdot 0^*$

$q_2 = 0^* 1 0 0^* + q_3 \cdot 0 0^*$ —— (5)

Sub (5) in (4)

$$q_3 = q_1 \cdot 1 + q_2 \cdot 1 + q_3 \cdot 1$$

$$= q_1 \cdot 1 + (0^* 1 0 0^* + q_3 \cdot 0 0^*) 1 + q_3 \cdot 1$$

$$= q_1 \cdot 1 + 0^* 1 0 0^* 1 + q_3 0 0^* 1 + q_3 \cdot 1$$

$$= 0^* 1 1 + 0^* 1 0 0^* 1 + q_3 (0 0^* 1 + 1)$$

$$q_3 = 0^* 1 1 + 0^* 1 0 0^* 1 (0 0^* 1 + 1)^*$$

$$= 0^* 1 (1 + 0 0^* 1)(0 0^* 1 + 1)^*$$