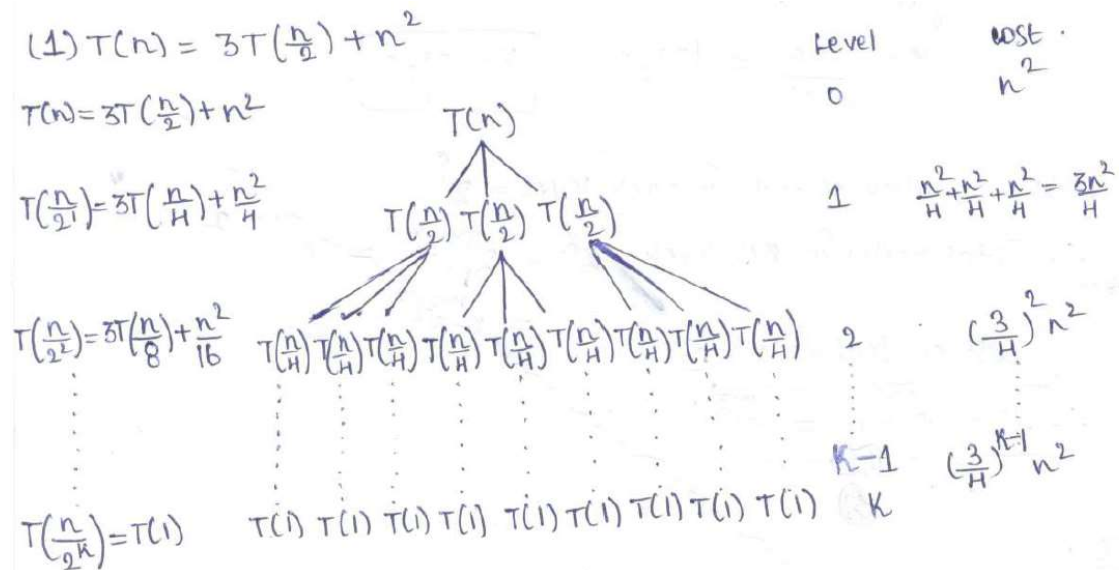


RECURSION TREE METHOD



- * Assume that the problem size reduces to 1 at k th level.
- * So, k th level is the last level with only leaf nodes.
- * Level number 0 to $k-1$ contains internal nodes.

$$\text{TOTAL COST OF TREE (TC)} = \text{COST OF LEAF NODES (LC)} + \text{COST OF INTERNAL NODES (IC)}$$

$LC = \text{cost of leaf node} \times \text{total no. of leaf nodes.}$

$IC = \text{Sum of cost of each level of internal nodes.}$

LC

To find LC, express k in terms of n

$$T\left(\frac{n}{2^k}\right) = T(1)$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log n = K \log 2$$

$$K = \frac{\log n}{\log 2} = \log_2 n$$

$$K = \log_2 n$$

Total number of nodes in each level = 2^i

$$\therefore \text{Total nodes in } K\text{th level} = 2^K = 2^{\log_2 n} = n^{\log_2 2}$$
$$= n^{\log_2 3}$$

Cost of leaf node = 1

$$\therefore LC = n^{\log_2 3} \times 1 = n^{\log_2 3}$$

IC

$$IC = n^2 + \frac{3}{4}n^2 + \left(\frac{3}{4}\right)^2 n^2 + \dots + \left(\frac{3}{4}\right)^{K-1} n^2$$

$$= n^2 \left[\left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2 + \dots + \left(\frac{3}{4}\right)^{K-1} \right]$$

The Sum of GP Series is given by $\frac{1}{1-x}$, where $x = \frac{3}{4}$

$$IC = n^2 \left[\frac{1}{1 - \frac{3}{4}} \right] = \frac{4}{1} \times n^2$$

$$IC = 4n^2$$

$$TC = LC + IC$$

$$= n^{\log_2 3} + 4n^2$$

$$\therefore T(n) = O(n^2)$$