# RECURSION TREE METHOD

(1) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

| Level | cost |
|---|---|
| 0 | $n^2$ |
| 1 | $\frac{n^2}{4}+\frac{n^2}{4}+\frac{n^2}{4} = \frac{3n^2}{4}$ |
| 2 | $\left(\frac{3}{4}\right)^2 n^2$ |
| ⋮ | ⋮ |
| $k-1$ | $\left(\frac{3}{4}\right)^{k-1} n^2$ |
| $k$ | |

$T(n) = 3T\left(\frac{n}{2}\right) + n^2$ $\qquad T(n)$

$T\left(\frac{n}{2^1}\right) = 3T\left(\frac{n}{4}\right) + \frac{n^2}{4}$ $\qquad T\left(\frac{n}{2}\right)\ T\left(\frac{n}{2}\right)\ T\left(\frac{n}{2}\right)$

$T\left(\frac{n}{2^2}\right) = 3T\left(\frac{n}{8}\right) + \frac{n^2}{16}$ $\qquad T\left(\frac{n}{4}\right) T\left(\frac{n}{4}\right) T\left(\frac{n}{4}\right) T\left(\frac{n}{4}\right) T\left(\frac{n}{4}\right) T\left(\frac{n}{4}\right) T\left(\frac{n}{4}\right) T\left(\frac{n}{4}\right) T\left(\frac{n}{4}\right)$

⋮

$T\left(\frac{n}{2^k}\right) = T(1)$ $\qquad T(1)\ T(1)\ T(1)\ T(1)\ T(1)\ T(1)\ T(1)\ T(1)\ T(1)$

* Assume that the problem size reduces to 1 at Kth level.
* So, Kth level is the last level with only leaf nodes.
* Level number 0 to K-1 contains internal nodes.

TOTAL COST OF TREE (TC) = COST OF LEAF NODES (LC) + COST OF INTERNAL NODES (IC).

LC = cost of leaf node × total no. of leaf nodes.

IC = Sum of cost of each level of internal nodes.

**LC**

To find LC, express K in terms of n

$$T\left(\frac{n}{2^k}\right) = T(1)$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$\log n = K \log 2$

$K = \frac{\log n}{\log 2} = \log_2 n$ $\quad \boxed{K = \log_2 n}$

Total number of nodes in each level $= 3^i$

$\therefore$ Total nodes in Kth level $= 3^K = 3^{\log_2 n} = n^{\log_2 3}$

$= n^{\log_2 3}$

cost of leaf node $= 1$

$\therefore \boxed{LC = n^{\log_2 3} \times 1 = n^{\log_2 3}}$

IC

$IC = n^2 + \frac{3}{4} n^2 + \left(\frac{3}{4}\right)^2 n^2 \cdots\cdots + \left(\frac{3}{4}\right)^{K-1} n^2$

$= n^2 \left[ \left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2 + \cdots\cdots + \left(\frac{3}{4}\right)^{K-1} \right]$

The sum of GP series is given by $\frac{1}{1-x}$, where $x = \frac{3}{4}$

$IC = n^2 \left[ \frac{1}{1 - \frac{3}{4}} \right] = \frac{4}{1} \times n^2$

$\boxed{IC = 4n^2}$

$TC = LC + IC$

$= n^{\log_2 3} + 4n^2$

$\therefore \boxed{T(n) = O(n^2)}$