The Karnaugh Map

Karnaugh Maps

- Karnaugh maps (K-maps) are *graphical* representations of Boolean functions.
- One map cell corresponds to a row in the truth table.
- Also, one map cell corresponds to a minterm or a maxterm in the Boolean expression
- Multiple-cell areas of the map correspond to standard terms.
- A K-map provides a systematic method for simplifying Boolean expressions and, if properly used, will produce the simplest SOP or POS expression possible, known as the minimum expression.

What is K-Map

- It’s similar to truth table; instead of being organized (i/p and o/p) into columns and rows, the K-map is an array of cells in which each cell represents a binary value of the input variables.
- The cells are arranged in a way so that simplification of a given expression is simply a matter of properly grouping the cells.
- K-maps can be used for expressions with 2, 3, 4, and 5 variables.

Two-Variable Map

$x_1 \backslash x_2$	0	1
0	0 m_0	1 m_1
1	2 m_2	3 m_3

OR

$x_2 \backslash x_1$	0	1
0	0 m_0	2 m_2
1	1 m_1	3 m_3

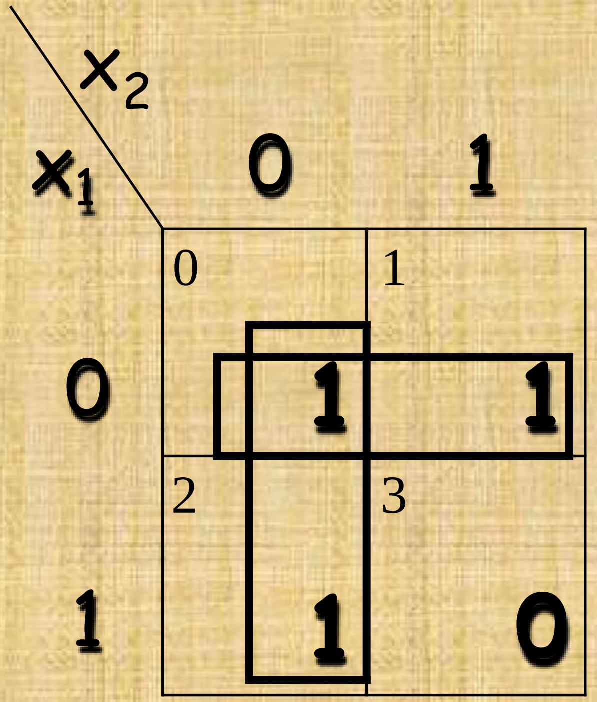
- ❑ ordering of variables is IMPORTANT for $f(x_1,x_2)$, x_1 is the row, x_2 is the column.
- ➢ Cell 0 represents $x_1'x_2'$; Cell 1 represents $x_1'x_2$; etc. If a minterm is present in the function, then a 1 is placed in the corresponding cell.

Two-Variable Map

- Any two adjacent cells in the map differ by ONLY one variable, which appears complemented in one cell and uncomplemented in the other.
- Example:
  m_0 ($=x_1'x_2'$) is adjacent to m_1 ($=x_1'x_2$) and m_2 ($=x_1x_2'$) but NOT m_3 ($=x_1x_2$)

2-Variable Map -- Example

- $f(x_1,x_2) = x_1'x_2' + x_1'x_2 + x_1x_2'$
  $= m_0 + m_1 + m_2$
  $= x_1' + x_2'$
- 1s placed in K-map for specified minterms m_0, m_1, m_2
- Grouping of 1s allows simplification
- What (simpler) function is represented by each dashed rectangle?
  - $x_1' = m_0 + m_1$
  - $x_2' = m_0 + m_2$
- Here m_0 covered twice



The 3 Variable K-Map

- There are 8 cells as shown:

AB \ C	0	1
00	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$
01	$\bar{A}B\bar{C}$	$\bar{A}BC$
11	$AB\bar{C}$	ABC
10	$A\bar{B}\bar{C}$	$A\bar{B}C$

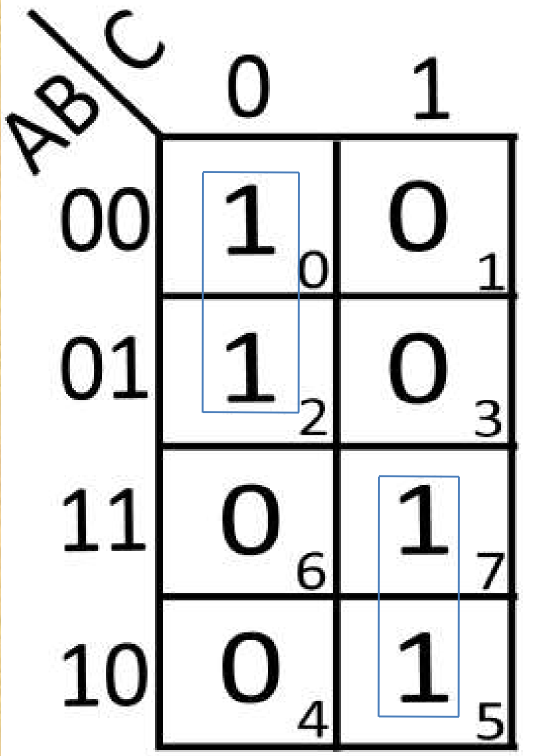
Example 3 var. k-map

- Minimize the following equation using k-map
  $y=\bar{A}\bar{B}\bar{C}+\bar{A}B\bar{C}+A\bar{B}C+ABC$

$\bar{A}\bar{B}\bar{C} = 000 = 0$ $\quad$ $\bar{A}B\bar{C} = 010 = 2$

$A\bar{B}C = 101 = 5$ $\quad$ $ABC = 111 = 7$

Using this fill the k-map



•Grouping – here 2 groups of 2 1's
Is possible

- For upper group A and C are constants and B is varying. Neglect B.A and C both are 0. Hence output of this group is $\bar{A}\bar{C}$
- For upper group A and C are constants and B is varying. Neglect B.A and C both are 0. Hence output of this group is AC

- Thus output Y is given by ,

$$Y=AC+\bar{A}\bar{C}$$
$$=A\odot B$$

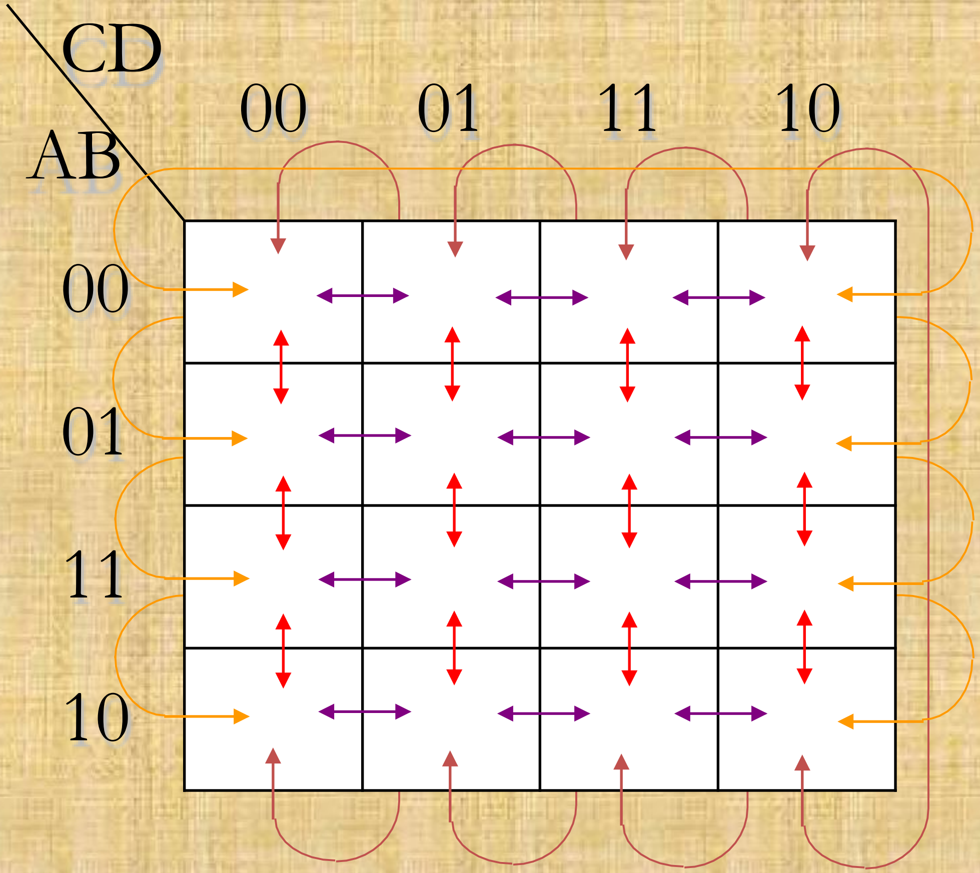
AB \ C	0	1
00	1 (0)	0 (1)
01	1 (2)	0 (3)
11	0 (6)	1 (7)
10	0 (4)	1 (5)

The 4-Variable K-Map

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

AB \ CD	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

Cell Adjacency



- Solve the given k-map

- Step I -grouping
- Step II -output of each group
- Step III -final output

➢ Here answer is ,

$$Y=C\overline{D}+\overline{B}C+B\overline{D}$$

AB \ CD	00	01	11	10
00	0 $_0$	0 $_1$	1 $_3$	1 $_2$
01	1 $_4$	0 $_5$	0 $_7$	1 $_6$
11	1 $_{12}$	0 $_{13}$	0 $_{15}$	1 $_{14}$
10	0 $_8$	0 $_9$	1 $_{11}$	1 $_{10}$

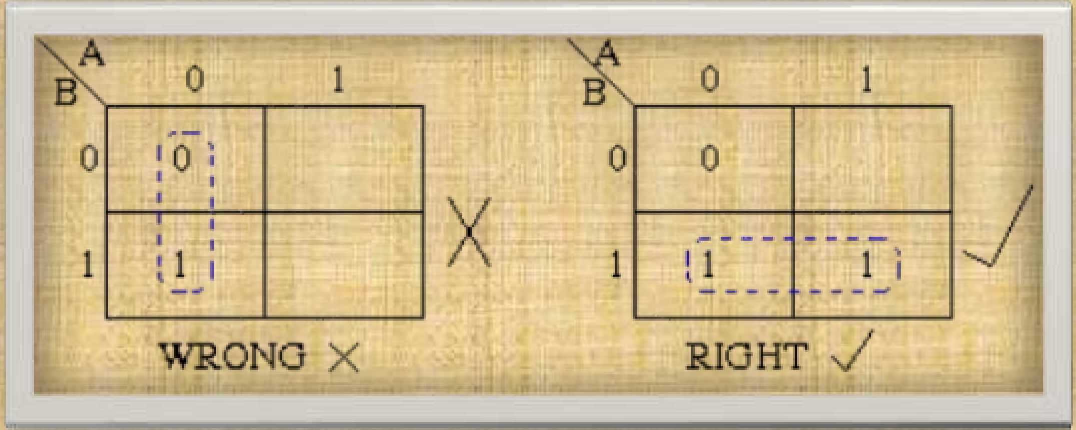
K-Map SOP Minimization

- The K-Map is used for simplifying Boolean expressions to their minimal form.
- A minimized SOP expression contains the fewest possible terms with fewest possible variables per term.
- Generally, a minimum SOP expression can be implemented with fewer logic gates than a standard expression.

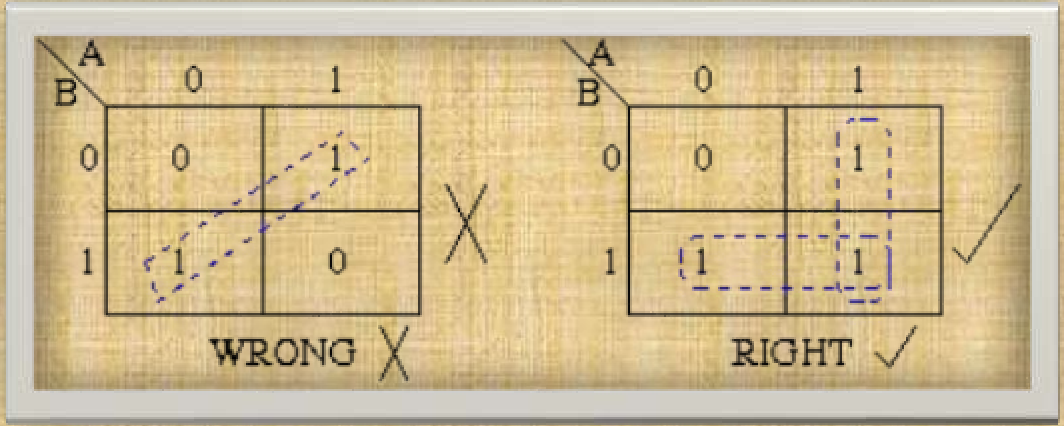
Grouping

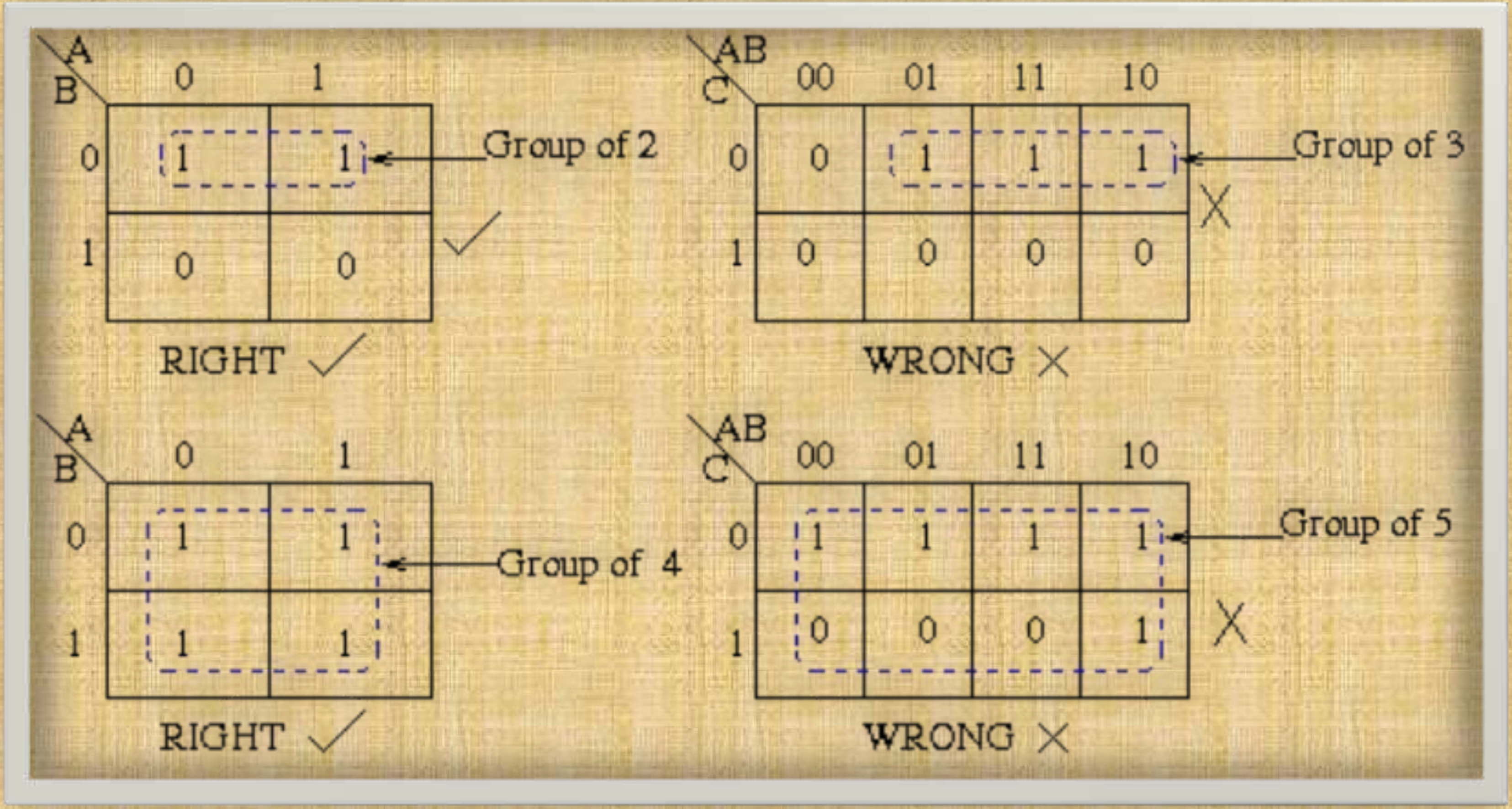
Rules of grouping -

1's & 0's can not be grouped



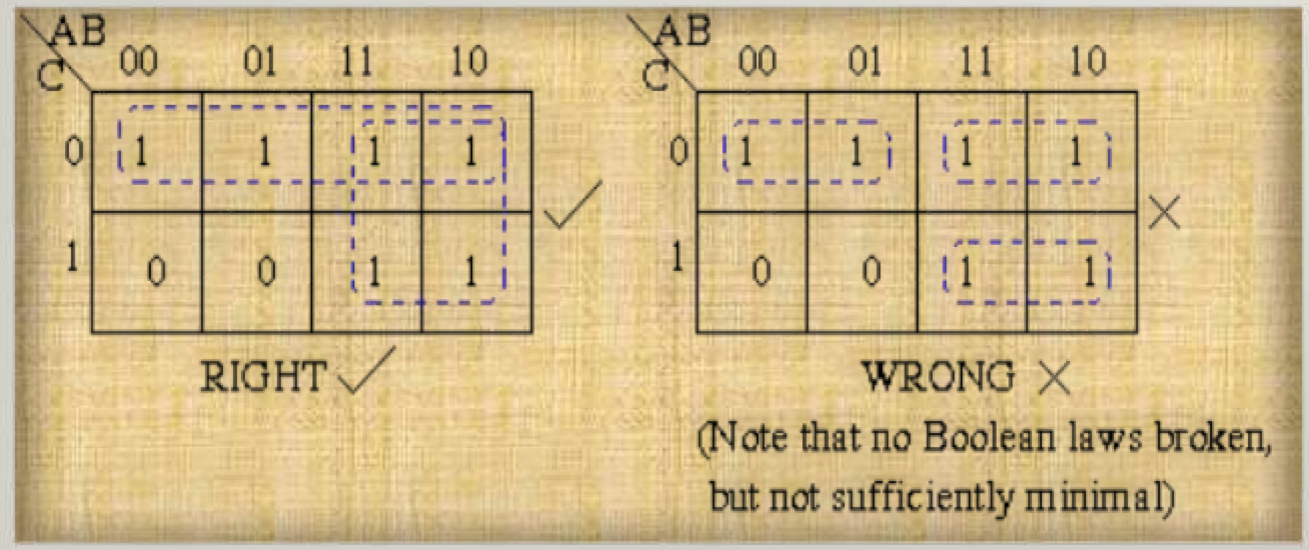
diagonal 1's can not be grouped



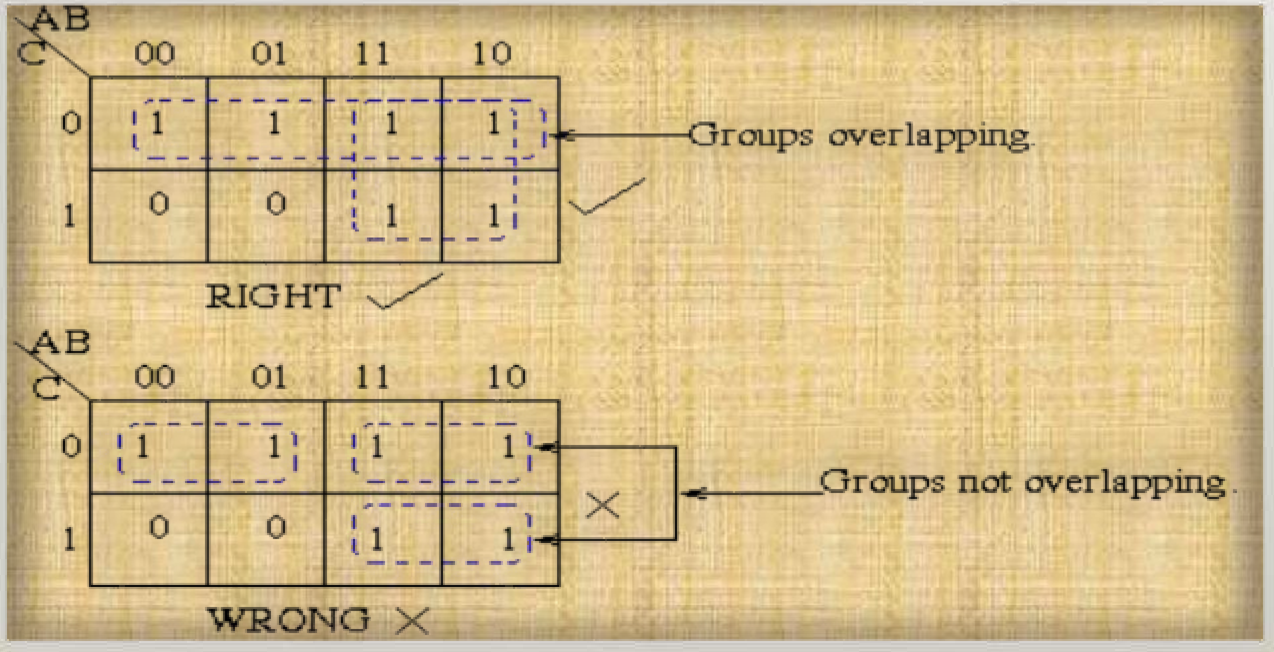


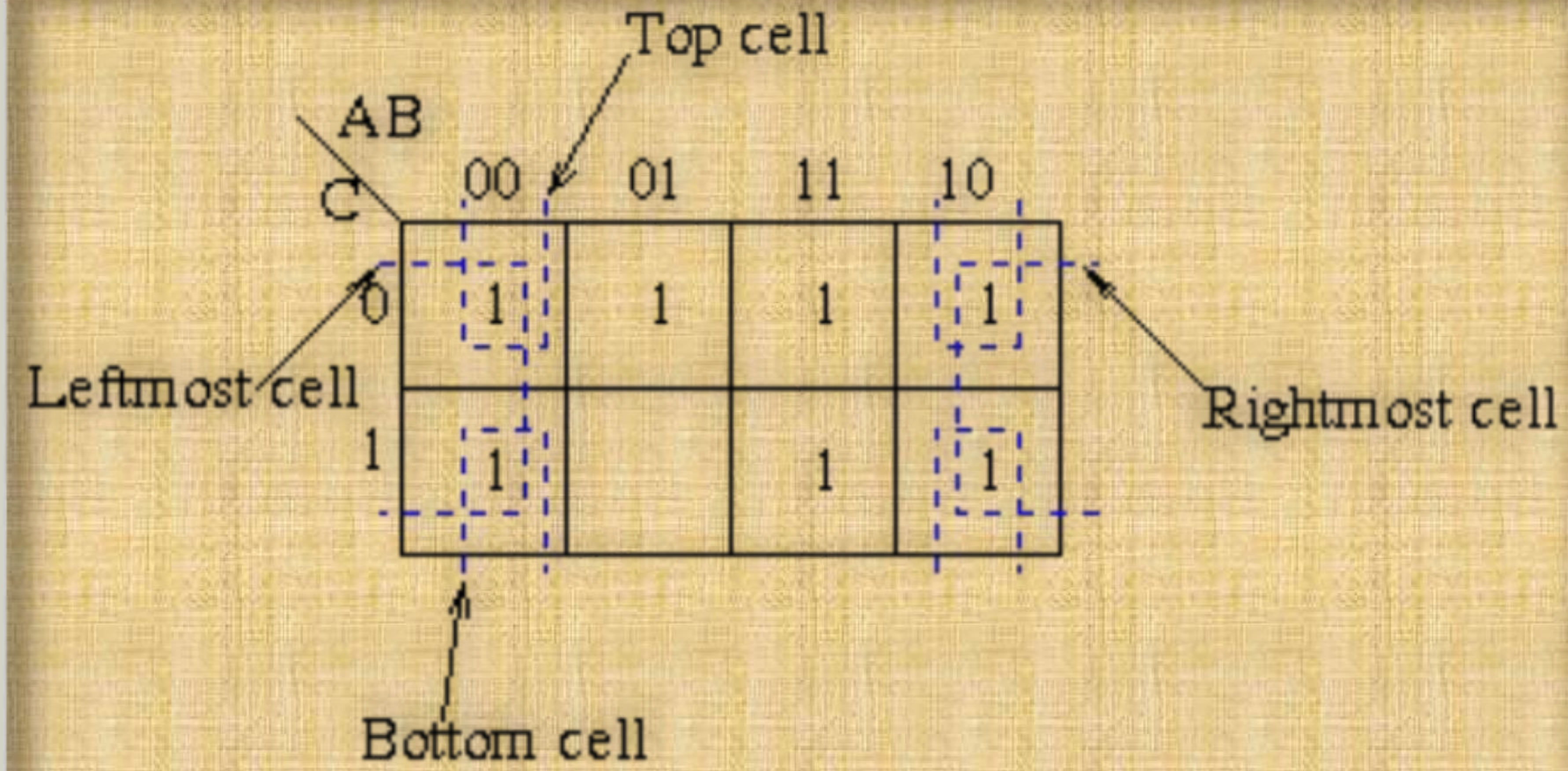
Elements in a group should be 2^n

Minimum Groups should be formed



For above rule group Overlapping is applicable





Top cell
AB
C
00
01
11
10
0
1
1
1
1
1
1
1
1
Leftmost cell
Rightmost cell
Bottom cell

Mapping a Standard SOP Expression

- For an SOP expression in standard form:
  - A 1 is placed on the K-map for each product term in the expression.
  - Each 1 is placed in a cell corresponding to the value of a product term.
  - Example: for the product term , a 1 goes in the 101 cell on a 3-variable map. $A\overline{B}C$

AB \ C	0	1
00	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}\,\overline{B}C$
01	$\overline{A}B\overline{C}$	$\overline{A}BC$
11	$AB\overline{C}$	ABC
10	$A\overline{B}\,\overline{C}$	$A\overline{B}C$

Mapping a Standard SOP Expression

The expression:

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + AB\overline{C} + A\overline{B}\,\overline{C}$$

A

000	001	110	100

Practice:

$ABC + ABC + ABC + ABC$

$\overline{A}BC + A\overline{B}C + A\overline{B}\,\overline{C}$

$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$

Three-Variable K-Maps

$f = \sum(0,4) = \overline{B}\,\overline{C}$

A \ BC	00	01	11	10
0	**1**	**0**	**0**	**0**
1	**1**	**0**	**0**	**0**

$f = \sum(4,5) = A\overline{B}$

A \ BC	00	01	11	10
0	**0**	**0**	**0**	**0**
1	**1**	**1**	**0**	**0**

$f = \sum(0,1,4,5) = \overline{B}$

A \ BC	00	01	11	10
0	**1**	**1**	**0**	**0**
1	**1**	**1**	**0**	**0**

$f = \sum(0,1,2,3) = \overline{A}$

A \ BC	00	01	11	10
0	**1**	**1**	**1**	**1**
1	**0**	**0**	**0**	**0**

$f = \sum(0,4) = \overline{A}C$

A \ BC	00	01	11	10
0	**0**	**1**	**1**	**0**
1	**0**	**0**	**0**	**0**

$f = \sum(4,6) = A\overline{C}$

A \ BC	00	01	11	10
0	**0**	**0**	**0**	**0**
1	**1**	**0**	**0**	**1**

$f = \sum(0,2) = \overline{A}\,\overline{C}$

A \ BC	00	01	11	10
0	**1**	**0**	**0**	**1**
1	**0**	**0**	**0**	**0**

$f = \sum(0,2,4,6) = \overline{C}$

A \ BC	00	01	11	10
0	**1**	**0**	**0**	**1**
1	**1**	**0**	**0**	**1**

Four-Variable K-Maps

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	0	0	0	0
10	1	0	0	0

$$f = \sum(0,8) = \overline{B} \bullet \overline{C} \bullet \overline{D}$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	0	1	0	0
10	0	0	0	0

$$f = \sum(5,13) = B \bullet \overline{C} \bullet D$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	1	1	0
10	0	0	0	0

$$f = \sum(13,15) = A \bullet B \bullet D$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

$$f = \sum(4,6) = \overline{A} \bullet B \bullet \overline{D}$$

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	0	0

$$f = \sum(2,3,6,7) = \overline{A} \bullet C$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$$f = \sum(4,6,12,14) = B \bullet \overline{D}$$

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	0	0	0	0
10	0	0	1	1

$$f = \sum(2,3,10,11) = \overline{B} \bullet C$$

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$$f = \sum(0,2,8,10) = \overline{B} \bullet \overline{D}$$

Four-Variable K-Maps

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$f = \sum(4,5,6,7) = \overline{A} \bullet B$

AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$f = \sum(3,7,11,15) = C \bullet D$

AB \ CD	00	01	11	10
00	1	0	1	0
01	0	1	0	1
11	1	0	1	0
10	0	1	0	1

$f = \sum(0,3,5,6,9,10,12,15)$

$f = A \otimes B \otimes C \otimes D$

AB \ CD	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

$f = \sum(1,2,4,7,8,11,13,14)$

$f = A \oplus B \oplus C \oplus D$

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$f = \sum(1,3,5,7,9,11,13,15)$

$f = D$

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$f = \sum(0,2,4,6,8,10,12,14)$

$f = \overline{D}$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$f = \sum(4,5,6,7,12,13,14,15)$

$f = B$

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

$f = \sum(0,1,2,3,8,9,10,11)$

$f = \overline{B}$

Determining the Minimum SOP Expression from the Map

AB \ CD	00	01	11	10
00			1	1
01	1	1	1	1
11	1	1	1	1
10		1		

$\overline{A}C$

B

$A\overline{C}D$

$$B + \overline{A}C + A\overline{C}D$$

4.5.3 Grouping Cells for Simplification

In the last section we have seen representation of logic function on the Karnaugh map. We have also seen that minterms are marked by 1s and maxterms are marked by 0s. Once the logic function is plotted on the Karnaugh map we have to use grouping technique to simplify the logic function. The grouping is nothing but combining terms in adjacent cells. Two cells are said to be adjacent if they conform the single change rule. The simplification is achieved by grouping adjacent 1s or 0s in groups of 2^i, where $i = 1, 2, \ldots, n$ and n is the number of variables. When adjacent 1s are grouped then we get result in the sum of products form; otherwise we get result in the product of sums form. Let us see the various grouping rules.

Grouping two adjacent ones (Pair)

Fig. 4.11 (a) shows the Karnaugh map for a particular three variable truth table. This K-map contains a pair of 1s that are horizontally adjacent to each other; the first represents $\overline{A}\,\overline{B}C$ and the second represents $\overline{A}$ BC. Note that in these two terms only the B variable appears in both normal and complemented form ($\overline{A}$ and C remain unchanged). Thus these two terms can be combined to give a resultant that eliminates the B variable since it appears in both uncomplemented and complemented form. This is easily proved as follows :

$$\begin{aligned} Y &= \overline{A}\,\overline{B}C + \overline{A}BC \\ &= \overline{A}C(\overline{B} + B) \qquad \text{Rule 6 : } [A + \overline{A} = 1] \\ &= \overline{A}C \end{aligned}$$

This same principle holds true for any pair of vertically or horizontally adjacent 1s. Fig. 4.11 (b) shows an example of two vertically adjacent 1s. These two can be combined to

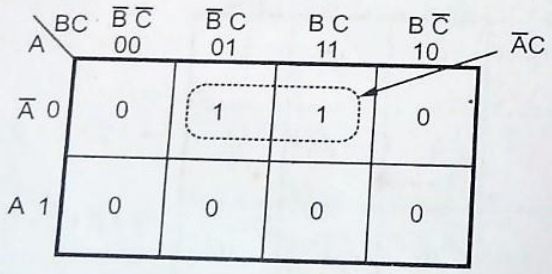


Fig. 4.11 (a)

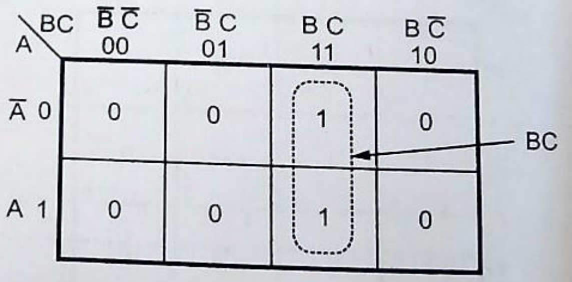


Fig. 4.11 (b)

eliminate A variable since it appears in both its uncomplemented and complemented forms. This gives result

$$Y = \overline{A}BC + ABC = BC$$

A \ BC	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	0	0	0	0
A 1	1	0	0	1

$A\overline{C}$

Fig. 4.11 (c)

In a Karnaugh map the leftmost column and rightmost column are considered to be adjacent. Thus, the two 1s in these columns with a common row can be combined to eliminate one variable. This is illustrated in Fig. 4.11 (c).

Here variable B has appeared in both its complemented and uncomplemented forms and hence eliminated as follows :

$$
\begin{aligned}
Y &= A\,\overline{B}\,\overline{C} + A\,B\,\overline{C} \\
&= A\,\overline{C}(\overline{B}+B) \qquad \text{Rule 6 : } [\,\overline{A} + A = 1] \\
&= A\,\overline{C}
\end{aligned}
$$

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0	1	0	0
$\overline{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\overline{B}$ 10	0	1	0	0

$\overline{B}\,\overline{C}D$

Fig. 4.11 (d)

Let us see another example shown in Fig. 4.11 (d). Here two 1s from top row and bottom row of some column are combined to eliminate variable A, since in a K map the top row and bottom row are considered to be adjacent.

$$
\begin{aligned}
Y &= \overline{A}\,\overline{B}\,\overline{C}D + A\,\overline{B}\,\overline{C}D \\
&= \overline{B}\,\overline{C}D\,(\overline{A} + A) \\
&= \overline{B}\,\overline{C}D
\end{aligned}
$$

Group 1 → $\overline{A}C$

A \ BC	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	0	1	1	0
A 1	0	0	1	0

Group 2 → BC

Fig. 4.11 (e)

Fig. 4.11 (e) shows a Karnaugh map that has two overlapping pairs of 1s. This shows that we can share one term between two pairs.

$$Y = \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + A\,B\,C$$
$$= \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,C + A\,B\,C$$

Rule 5 : $[A + A = A]$

$$= \overline{A}\,C\,(\overline{B} + B) + B\,C\,(\overline{A} + A)$$
$$= \overline{A}\,C + B\,C$$

Fig. 4.11 (f) shows a K-map where three group of pairs can be formed. But only two pairs are enough to include all 1s present in the K-map. In such cases third pair is not required.

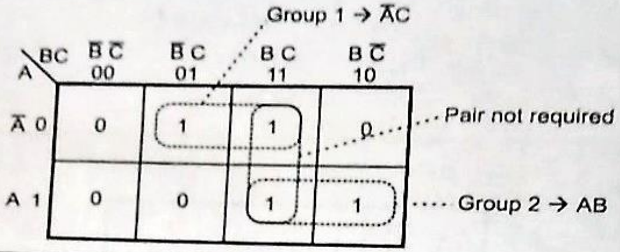


Fig. 4.11 (f) Examples of combining pairs of adjacent ones

$$Y = \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + A\,B\,C + A\,B\,\overline{C}$$
$$= \overline{A}\,C\,(\overline{B} + B) + A\,B\,(C + \overline{C}) \quad \text{Rule 6} : [\overline{A} + A = 1]$$
$$= \overline{A}\,C + A\,B$$

Grouping four Adjacent ones (Quad)

In a Karnaugh map we can group four adjacent 1s. The resultant group is called Quad. Fig. 4.12 shows several examples of quads. Fig. 4.12 (a) shows the four 1s are horizontally adjacent and Fig. 4.12 (b) shows they are vertically adjacent.

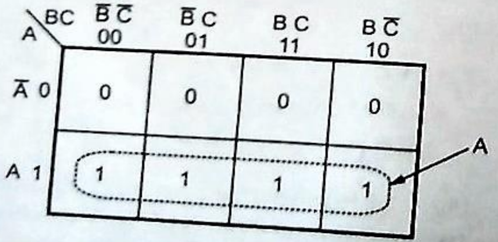


Fig. 4.12 (a) Y = A

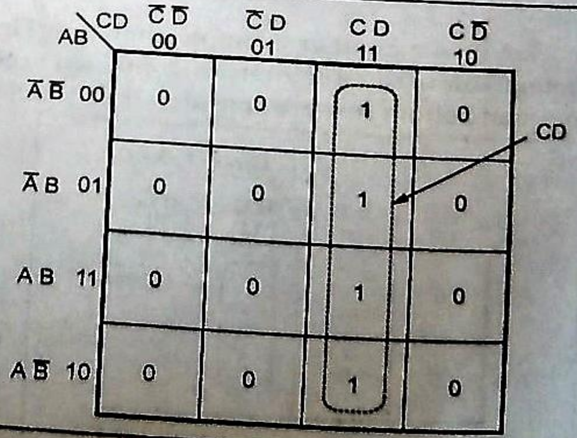


Fig. 4.12 (b) Y = CD

A K-map in Fig. 4.12 (c) contains four 1s in a square, and they are considered adjacent to each other. The four 1s in Fig. 4.12 (d) are also adjacent, as are those in Fig. 4.12 (e) because, as mentioned earlier, the top and bottom rows are considered to be adjacent to each other and the leftmost and rightmost columns are also adjacent to each other.

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	0	0	0

Fig. 4.12 (c) Y = BD

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	0	0	0
AB 11	1	0	0	1
$A\bar{B}$ 10	1	0	0	1

Fig. 4.12 (d) Y = $A\bar{D}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	0	0	1
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	0	0	1

Fig. 4.12 (e) Y = $\bar{B}\bar{D}$

From the above Karnaugh maps we can easily notice that when a quad is combined two variables are eliminated. For example, in Fig. 4.12 (c) we have following terms with 4 variables :

$$
\begin{aligned}
Y &= \bar{A}B\bar{C}D + \bar{A}BCD + AB\bar{C}D + ABCD \\
&= \bar{A}BD(\bar{C}+C) + ABD(\bar{C}+C) \\
&= \bar{A}BD + ABD \\
&= BD(\bar{A}+A) \\
&= BD
\end{aligned}
$$

(only two variables in the result, variables A and C are eliminated)

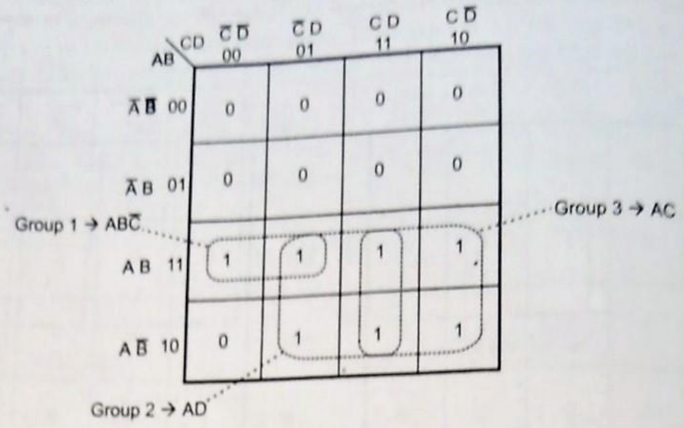


Fig. 4.12 (f) $Y = AB\overline{C} + AD + AC$

Fig. 4.12 Examples of combining quads of adjacent ones

Fig. 4.12 (f) shows overlapping groups. As mentioned earlier one term can be shared between two or more groups.

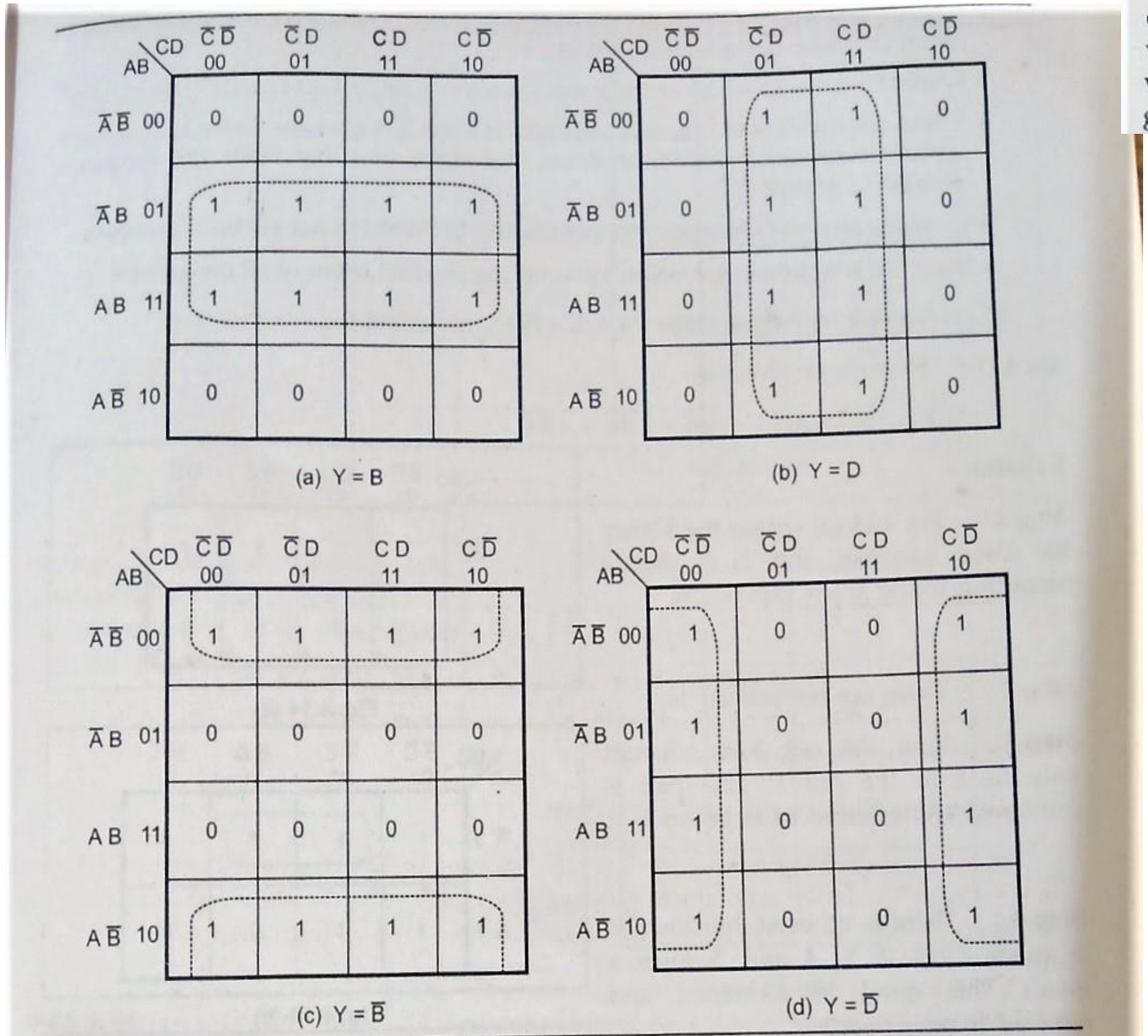
Grouping Eight Adjacent Ones (Octet)

In a Karnaugh map we can group eight adjacent 1s. The resultant group is called as octet. Fig. 4.13 shows several examples of octets. Fig. 4.13 (a) shows the eight 1s are horizontally adjacent and Fig. 4.13 (b) shows they are vertically adjacent.

From the figures 4.13 we can easily observe that when an octet is combined in a four variable map, three of the four variables are eliminated because only one variable remains unchanged. For example, in K-map shown in Fig. 4.13 (a) we have following terms :

$$\begin{aligned}
Y &= \overline{A}B\overline{C}\,\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + \overline{A}BC\overline{D} \\
&\quad + AB\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + ABC\overline{D} \\
&= \overline{A}B\overline{C}(\overline{D} + D) + \overline{A}BC(D + \overline{D}) \\
&\quad + AB\overline{C}(\overline{D} + D) + ABC(D + \overline{D}) \\
&= \overline{A}B\overline{C} + \overline{A}BC + AB\overline{C} + ABC \\
&= \overline{A}B(\overline{C} + C) + AB(\overline{C} + C) \\
&= \overline{A}B + AB \\
&= B(\overline{A} + A) \\
&= B
\end{aligned}$$

Simplification of SOP Expressions



AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	1	1	1	1
AB 11	1	1	1	1
$A\bar{B}$ 10	0	0	0	0

(a) $Y = B$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	1	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	1	1	0

(b) $Y = D$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1	1	1
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	1	1	1

(c) $Y = \bar{B}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	0	0	1
$\bar{A}B$ 01	1	0	0	1
AB 11	1	0	0	1
$A\bar{B}$ 10	1	0	0	1

(d) $Y = \bar{D}$

Fig. 4.13 Examples of combining octets of adjacent ones

4.5.4 Simplification of Sum of Products Expressions

We have seen how combination of pairs, quads and octets on a Karnaugh map can be used to obtain a simplified expression. A pair of 1s eliminates one variable, a quad of 1s eliminates two variables and an octet of 1s eliminates three variables. In general, when a variable appears in both complemented and uncomplemented form within a group, that variable is eliminated from the resultant expression. Variables that are same in all with the group must appear in the final expression.

Generalized procedure to simplify B.E:

1. Plot the K- map and place 1s in those cells corresponding to the 1s in the truth table or sum pf product expression. Place 0s in other cells

2. Check the K-map for adjacent 1s and encircle those 1s which are not adjacent to any other 1s. These are called isolated 1s

3. Check for those 1s which are adjacent to only one other 1 and encircle such pairs

4. Check for quads and octets of ajacent 1s even if it contains some 1s that have already been encircled. While doing this make sure
   that there are minimum number of groups.

5. Combine any pairs necessary to include any 1s that have not yet been grouped.

6. Form the simplified expression by summing product terms of all the groups.

Ex. 4.14 : *Minimize the expression*

$$Y = A\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

Solution :

Step 1 : Fig 4.14 (a) shows the K-map for three variables and it is plotted according to the given expression.

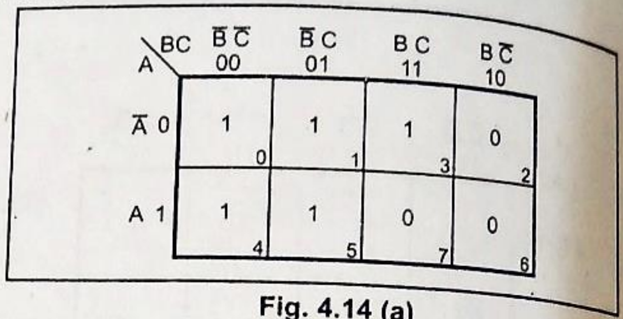


Fig. 4.14 (a)

Step 2 : There are no isolated 1s

Step 3 : 1 in the cell 3 is adjacent only to 1 in the cell 1. This pair is combined and referred to as group 1.

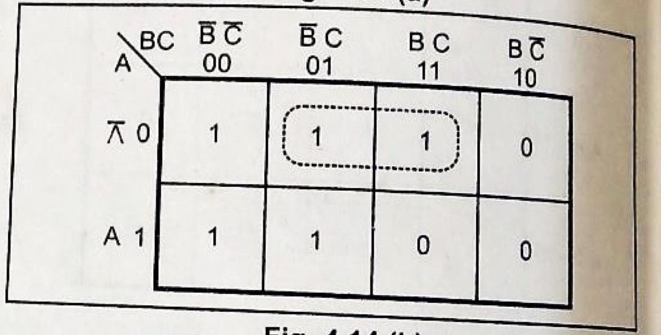


Fig. 4.14 (b)

Step 4 : There is no octet, but there is a quad. Cells 0, 1, 4 and 5 form a quad. This quad is combined and referred to as group 2.

Step 5 : All 1s have already been grouped.

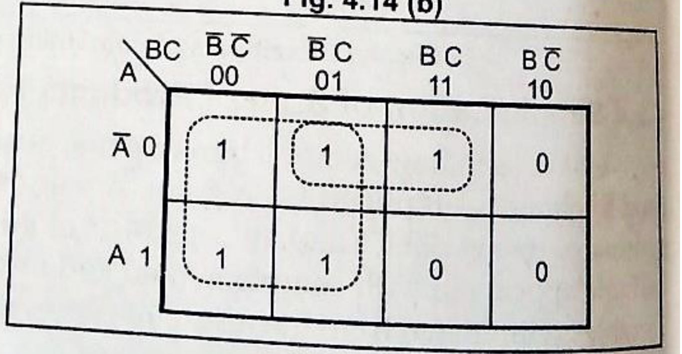


Fig. 4.14 (c)

Step 6 : Each group generates a term in the expression for Y. In group 1 B variable is eliminated and in group 2 variables A and C are eliminated and we get

$$Y = \bar{A}C + \bar{B}$$

Ex. 4.15

Minimize the expression

$$Y = \overline{A}B\overline{C}\,\overline{D} + \overline{A}B\overline{C}D + AB\overline{C}\,\overline{D} + AB\overline{C}D + A\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,C\overline{D}$$

Solution :

Step 1 : Fig. 4.15 (a) shows the K-map for four variables and it is plotted according to the given expression.

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0 [0]	0 [1]	0 [3]	1 [2]
$\overline{A}B$ 01	1 [4]	1 [5]	0 [7]	0 [6]
AB 11	1 [12]	1 [13]	0 [15]	0 [14]
$A\overline{B}$ 10	0 [8]	1 [9]	0 [11]	0 [10]

Fig. 4.15 (a)

Step 2 : Cell 2 is the only cell containing a 1 that is not adjacent to any other 1. It is referred to separately as group 1.

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0	0	0	1
$\overline{A}B$ 01	1	1	0	0
AB 11	1	1	0	0
$A\overline{B}$ 10	0	1	0	0

Fig. 4.15 (b)

Step 3 : 1 in the cell 9 is adjacent only to 1 in the cell 13. This pair is combined and referred to as group 2.

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0	0	0	1
$\overline{A}B$ 01	1	1	0	0
AB 11	1	1	0	0
$A\overline{B}$ 10	0	1	0	0

Fig. 4.15 (c)

Step 4 : There is no octet, but there is quad cells 4, 5, 12 and 13 form a quad. This quad is combined and referred to as group 3.

Step 5 : All 1s have already grouped.

Step 6 : Each group generates a term in the expression for Y. In group 1 variable is not eliminated. In group 2 variable B is eliminated and in group 3 variables A and D are eliminated and we get,

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0	0	0	1
$\overline{A}B$ 01	1	1	0	0
AB 11	1	1	0	0
$A\overline{B}$ 10	0	1	0	0

Fig. 4.15 (d)

$$Y = \overline{A}\,\overline{B}C\overline{D} + A\overline{C}D + B\overline{C}$$

Ex. 4.16 : *Reduce the following four variable function to its minimum sum of products form :*

$$Y = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D} + A\overline{B}CD + A\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D}$$

Solution :

Step 1 : Fig. 4.16 (a) shows the K-map for four variables and it is plotted according to the given expression.

Step 2 : There are no isolated 1s.

Step 3 : There are no such 1s which are adjacent to only one other 1.

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	1 (0)	0 (1)	1 (3)	1 (2)
$\overline{A}B$ 01	0 (4)	0 (5)	0 (7)	0 (6)
AB 11	1 (12)	0 (13)	0 (15)	1 (14)
$A\overline{B}$ 10	1 (8)	0 (9)	1 (11)	1 (10)

Fig. 4.16 (a)

Step 4 : There are three quads formed by cells 0, 2, 8, 10, cells 8, 10, 12, 14 and cells 2, 3, 10, 11. These quads are combined and referred to as group 1, group 2 and group 3 respectively.

Step 5 : All 1s have already been grouped.

Step 6 : Each group generates a term in the expression for Y. In group 1 variables A and C are eliminated, in group 2 variables B and C are eliminated and in group 3 variables A and D are eliminated and we get

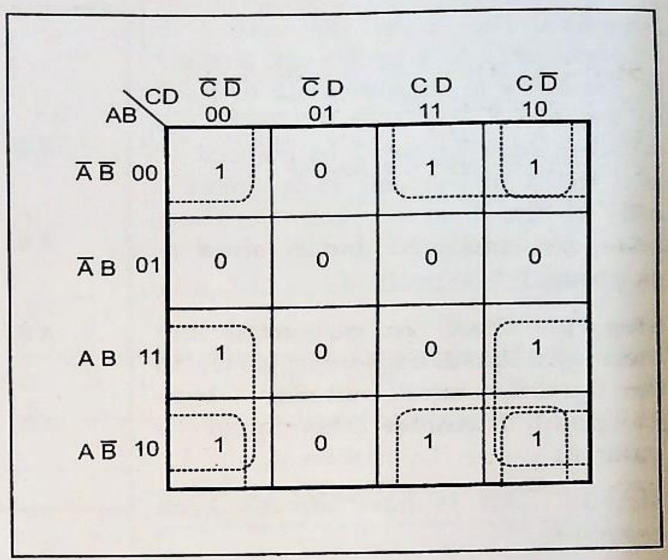


Fig. 4.16 (b)

$$Y = \bar{B}\bar{D} + A\bar{D} + \bar{B}C$$

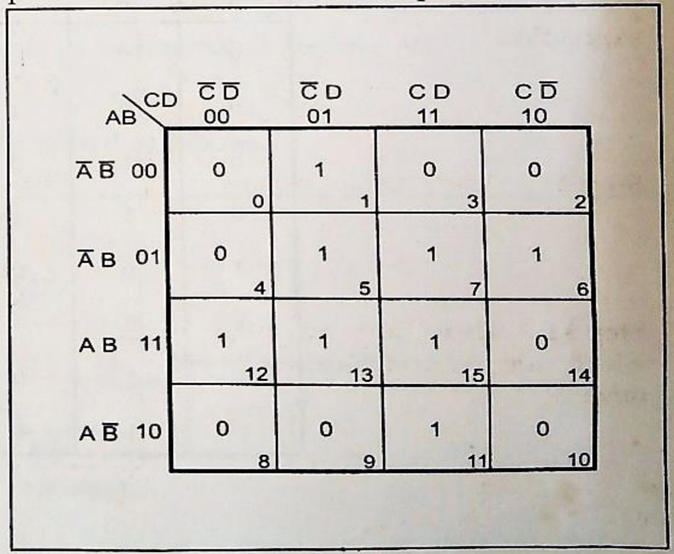
Ex. 4.17 : *Reduce the following function to its minimum sum of products form :*

$$Y = \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + \bar{A}BC\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + ABCD + A\bar{B}CD$$

Solution :

Step 1 : Fig. 4.17 (a) shows the K-map for four variables and it is plotted according to the given expression.

Step 2 : There are no isolated 1s



Step 3 : The 1 in the cell 1 is adjacent only to 1 in the cell 5, the 1 in the cell 6 is adjacent only to the 1 in the cell 7, the 1 in the cell 12 is adjacent only to the 1 in the cell 13 and the 1 in the cell 11 is adjacent only to the 1 in the cell 15. These pairs are combined and referred to as group 1-4 respectively.

Step 4 : There is no octet, but there is a quad. However, all 1s in the quad have already been grouped. Therefore this quad is ignored.

Step 5 : All 1s have already been grouped.

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	0	0
$\bar{A}B$ 01	0	1	1	1
AB 11	1	1	1	0
$A\bar{B}$ 10	0	0	1	0

Fig. 4.17 (b)

Step 6 : Each group generates a term in the expression for Y. In group 1 variable B is eliminated. Similarly, in group 2-4 variables D, D and B are eliminated one in each group. We finally get minimum sum of products form as

$$Y = \bar{A}\bar{C}D + \bar{A}BC + AB\bar{C} + ACD$$