



KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

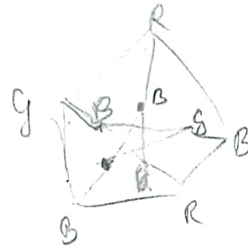
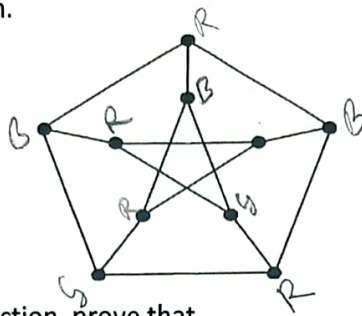
Answer any TEN Questions

(10 X 10 = 100 Marks)

1. ✓ How many numbers from 1 to 2000 (both inclusive) are divisible by 2, 3, 5 or 7?
2. Which of the following is an equivalence relation?
 - (i) The relation R on Z defined by aRb if $a^2 - b^2 \leq 7$.
 - (ii) The relation R on Z defined by aRb if $a^2 - b^2 = 0$.
3. ✓ If * is the binary operation on the set R of real numbers defined by $a * b = a + b + 2ab$,
 - (i) Find if $\{R, *\}$ is a semigroup. Is it commutative?
 - (ii) Find the identity element, if exists.
4. ✓ a) How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, and 7 if no digit is repeated? no [5]
b) A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has
 - (i) at least one boy and one girl all
 - (ii) at least three girls all
5. ✓ a) Solve the recurrence relation $a_n = a_{n-1} + n$ with initial term $a_0 = 4$. [5]
b) Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$, where $a_0 = 1$ and $a_1 = 4$. [5]
6. a) Draw a graph with the specified properties or give reason to show that no such graph exists. [6]
 - (i) A graph with four vertices of degree 1, 1, 2 and 3
 - (ii) A graph with four vertices of degree 1, 1, 3 and 3
 - (iii) A simple graph with four vertices of degree 1, 1, 3 and 3
b) Draw a graph with following adjacency matrix. [4]
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$
7. ✓ Define Eulerian circuit and Hamiltonian circuit. For each of the following cases, give an example with justifications for the graphs that contain
 - (i) a Eulerian circuit that is also a Hamiltonian circuit
 - (ii) a Eulerian circuit and a Hamiltonian circuit that are distinct
 - (iii) a Eulerian circuit, but not a Hamiltonian circuit
 - (iv) a Hamiltonian circuit, but not a Eulerian circuit
 - (v) neither a Eulerian circuit nor a Hamiltonian circuit.

8. a) Define a planar graph. Is it possible for a planar graph to have 6 vertices 10 edges and 5 faces? [5]

- b) Find the chromatic number, the clique number and the independence number of the following graph. [5]



9. Using mathematical induction, prove that

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

10. a) Check whether the statement $p \rightarrow (q \rightarrow p)$ is a tautology or a contradiction without using the truth table. [5]

- b) Check whether $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are logically equivalent. [5]

11. a) Check whether $p \rightarrow (\neg q \vee r)$ and $\neg p \vee (\neg q \vee r)$ are logically equivalent using truth table. [5]

- b) Write the converse, inverse, and contrapositive of each of the following implication. [5]

"If x and y are numbers such that $x = y$, then $x^2 = y^2$ ".

12. Find the sum-of-products and product-of-sums expansion for the function
/ $F(x, y, z) = (x + y)\bar{z}$.

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