



VIT
Vellore Institute of Technology
11th and 12th Floors, Vellore, Tamil Nadu, India

DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
FALL SEMESTER 2023-2024

CONTINUOUS ASSESSMENT TEST – II

Programme Name & Branch : B.Tech
Course Code : BMAT101L
Course Name : Calculus
Slot : E2+TE2
Date of the Examination : 19-10-2023
Duration : 90 minutes
Max. Marks : 50

Handwritten notes:
 $\frac{y}{x} - \frac{z}{x} + \frac{z}{y} - \frac{xy}{z} + \frac{x}{z} - \frac{y}{z}$
 $\rightarrow y^2z - z^2y + x^2z - xz^2 + x^2y - xy^2$
 $\frac{z}{x} - \frac{z}{y} = \frac{y-z}{xy}$

General instruction(s): Students are permitted to bring any number of text books and hand written note books (class notes)

| Q. No | Question | Marks |
|-------|---|-------|
| 1. | Find whether the following functions are functionally dependent. If so, find the relation between them. $u = \frac{x}{y-z}, v = \frac{y}{z-x}$ and $w = \frac{z}{x-y}$ | 10 |
| 2. | Find the second order Taylor expansion of $f(x, y) = \sqrt{1+x+y^2}$ about (1, 0). | 10 |
| 3. | A Car manufacturer estimates a production function to be $f(x, y) = 100x^{3/4}y^{1/4}$, where x represents the units of labour (at Rs. 150 per unit) and y represents the units of capital (at Rs. 250 per unit). The total cost of labour and capital is equal to Rs. 50,000 (i.e., $150x + 250y = 50000$). Find the maximum production level for this manufacturer. | 10 |
| 4. | Evaluate $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3) dy dx$ by changing the order of integration. | 10 |
| 5. | Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2 \sqrt{x^2+y^2} dy dx$ by changing into polar coordinates. | 10 |

Handwritten solution for Q3:
 $100x^{3/4} \cdot y^{1/4} + \lambda(150x + 250y - 50000)$
 $\frac{\partial}{\partial x} = 100 \cdot \frac{3}{4} x^{-1/4} y^{1/4} + \lambda 150 = 0$
 $\frac{\partial}{\partial y} = 100x^{3/4} \cdot \frac{1}{4} y^{-3/4} + \lambda 250 = 0$
 $\frac{\partial}{\partial \lambda} = 150x + 250y - 50000 = 0$
 $2\lambda = -\left(\frac{3}{4}\right) \frac{100}{x^{1/4}} y^{1/4}$
 $10\lambda = -\left(\frac{3}{4}\right) \frac{100}{x^{1/4}} y^{1/4}$
 $5\lambda = -\left(\frac{3}{4}\right) \frac{100}{x^{1/4}} y^{1/4}$
 $5y = 3x$