Test for Single Mean :-

Q :- A random sample of 200 tins of coconut oil gave an average weight of 4.95 kg, with a standard deviation of 0.21 kg. Do we accept that net weight is 5 kg per tin at 5% level of significance.

Sol^n.-

$n = 200$, $\bar{x}$ = sample mean   s = sample S.D

$\bar{x} = 4.95$ $s = 0.21$

μ = population mean

$H_0 : \mu = 5$ kg

$H_1 : \mu \neq 5$ (two tailed test)

$\alpha = 5\% = .05$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

as σ is unknown

so $$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$Z = 3.36$

$Z_\alpha = 1.96$

as $|Z_{cal}| > |Z_\alpha|$

ie H_0 is rejected. ie we donot accept that net weight is 5 kg per tin at 5% level of significance.

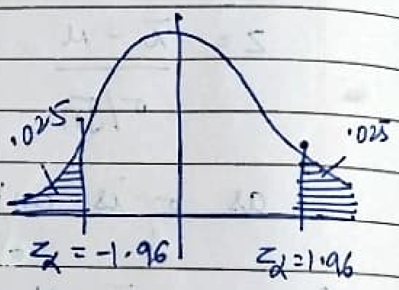
(2) A manufacturer of ball pens claims that a certain pen ~~be~~ ~~mon~~ has a mean writing life of 400 pages, with a S.D of 20 pages. A purchasing agent selects a sampee of 100 pens and put them for test. The mean writing life for the sampee was 390 pages. Should the purchasing agent reject the manufacture's claim at 5% level of significance.

Solⁿ:- $\mu = 400$, $\sigma = 20$ (population proportion)
$n = 100$, $\bar{x} = 390$, $\alpha = .05$

Null Hypothesis:-
$H_0 : \mu = 400$, (The mean ~~writing~~ writing life is 400 pages)

$H_1 : \mu \neq 400$ (2 tailed)

Test Statistic



$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$z = \frac{390 - 400}{20/\sqrt{100}} = 5$$

$Z_\alpha = -1.96$

As $|Z_{cal}| \ngtr |Z_\alpha|$

i.e Null Hypo is ~~accepted~~ rejected.
yes, purchasing agent should reject the manufacturer claim.

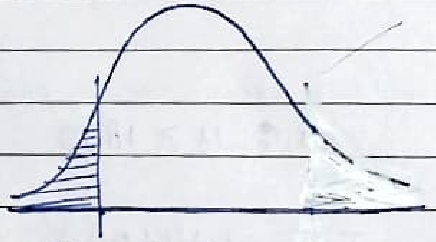
Q:- An insurance agent has claimed that the average age of policy holders who insure through him is less than the average for all agent which is 30.5 years. A random sample of 100 policy holders who had insured through him reveals that the mean and s.d are 28.8 and 6.35 years resp. Test the claim at 5% level of significance.

Solⁿ:- Let $H_0 : \mu = 30.5$ ie the average age of policy holders is 30.5 years.

$H_1 : \mu < 30.5$ (one tailed test) left tailed

$n = 100$, $\bar{x} = 28.8$ $s = 6.35$ (Sample's S.D)

Test Statistic



$Z_\alpha = -1.645$

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$Z = \frac{28.8 - 30.5}{6.35/\sqrt{100}} = 2.6$$

$\alpha = 5\% = .05$

$Z_\alpha = -1.645$

$|Z_{cal}| > |Z_\alpha|$ ie H_0 is rejected.

ie H_1 accepted. ie claim of insurance agent is valid.

Q:- The mean breaking strength of the cables supplied by a manufacturer is 1800 with a S.D of 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cable has increased. In order to test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. can we support the claim at 1% level of significance.

Sol^n: given $\mu = 1800$, $\sigma = 100$

$n = 50$, $\bar{x} = 1850$

$\alpha = .01$

$H_0 : \mu = 1800$, ie mean breaking strength of cables is 1800.

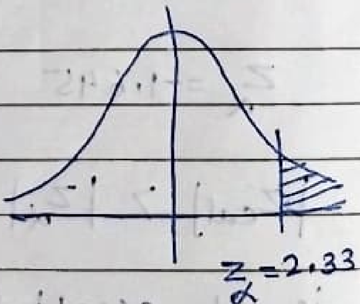
$H_1 : \mu > 1800$ (right tailed)

Test statistic :-

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{1850 - 1800}{.01/\sqrt{50}}$$

$Z = 3.54$

ie $Z_{cal} = 3.54$



$Z_\alpha = 2.33$

ie $|Z_{cal}| > |Z_\alpha|$

ie Null Hypo is rejected

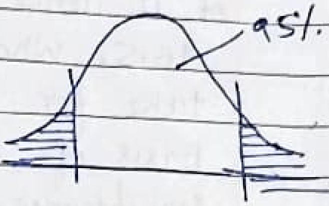
ie H_1 is accepted.

ie manufacturer claim is accepted.

Q:- The mean value of a random sample of 60 items was found to be 145 with a S.D of 40. Find the 95% confidence limit for the population mean.

Sol^n $n = 60$, $\bar{x} = 145$, $s = 40$ (Sample S.D)

for 95% confidence limit



$$|Z| \le 1.96$$

ie $$\left|\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right| \le 1.96$$

as population S.D is not given, we can approximate it by sample's S.D, therefor 95% confidence limit for μ are given by

$$\left|\frac{\bar{x} - \mu}{s/\sqrt{n}}\right| \le 1.96$$

ie $$\bar{x} - 1.96\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + 1.96\frac{s}{\sqrt{n}}$$

$$145 - 1.96 \times \frac{40}{\sqrt{60}} \le \mu \le 145 + 1.96 \times \frac{40}{\sqrt{60}}$$

$$\boxed{134.9 \le \mu \le 155.1}$$

Q:- The mean of a certain production process is known to be 50 with a S.D of 2.5. The production manager may welcome any change in the mean value towards the higher side but would like to safeguard against decreasing values of mean. He takes a sample of 12 items that gives a mean value of 46.5. What inference should the manager take for the population process on the basis of sample result. Use 5% level of significance.

Soln.-

$\mu = 50, \quad \sigma = 25$

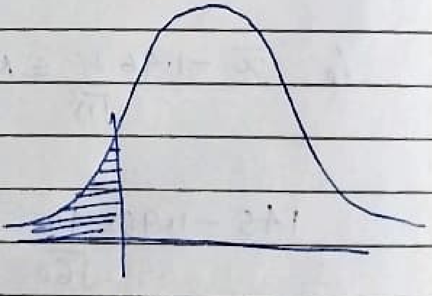
$n = 30, \quad \bar{x} = 46.5$

$\alpha = .05$

Test sh

$H_0 : \mu = 50$

$H_1 : \mu < 50$ (left tailed)



test statistic

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$Z = \frac{46.5 - 50}{2.5/\sqrt{30}} = \frac{-3.5}{.4564} = 7.668.$$

$Z_\alpha = -1.645$

$|Z_{cal}| > |Z_\alpha|$ so null Hypothesis is rejected. ie mean of production process is less than 50.

Q:- An insurance agent has claimed that the average age of policy holders who issue through him is less than the average for all agent which is 30.5 years. A random sample of 100 policy holders who had issued through him gave the following age distribution.

Age	16-20	21-25	26-30	31-35	36-40
No. of Persons	12	22	20	30	16

calculate arithmetic mean and S.D of this distribution and use these values to ~~get~~ test his claim at 5% level of significance.

solution:- $\bar{x} = A + \frac{\Sigma fd}{N}$

age	No. of persons	m	$d = m - A$	fd	fd^2
15.5-20.5	12	18	-10	-120	1200
20.5-25.5	22	23	-5	-110	650
25.5-30.5	20	28	0	0	0
30.5-35.5	30	33	5	150	750
35.5-40.5	16	38	10	160	1600
	N=100			80	

Let A = 28 $\quad \Sigma fd = 80$

$$\bar{x} = A + \frac{\Sigma fd}{N}$$

$$\bar{x} = 28 + \frac{80}{100} = 28.8$$

$$\boxed{\bar{x} = 28.8}$$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = 6.35$$

$$\boxed{\sigma = 6.35}$$

Null Hypothesis $H_0 : \mu = 30.5$ year

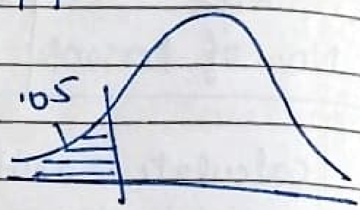
Alternate Hypothesis $H_1 : \mu < 30.5$ (left tailed)

$S = 6.35$ (sample S.D)

$n = 100$

test statistic $z = \dfrac{\bar{x} - \mu}{S/\sqrt{n}}$ (as population S.D is not known)

$$Z = \frac{28.8 - 30.5}{\frac{6.35}{\sqrt{100}}} = -2.677$$



$|Z| = 2.68$

$\alpha = 5\%$

$Z_\alpha = 1.645$

as $|Z_{cal}| > |Z_\alpha|$

ie H_0 rejected and H_1 accepted.

ie $\bar{x}$ and μ differ significantly.

Test of Significance for difference of mean of two large samples:-

Q1: Random samples drawn from two places gave the following data relating to the heights of children.

	Place A	Place B
Mean Height	68.50	68.58
S.D	2.5	3.0
Sample Size	1200	1500

Test at 5% level of significance that the mean height is the same for the children at two places.

Solⁿ:-

$\bar{x}_1 = 68.50$, $\bar{x}_2 = 68.58$

$\sigma_1 = 2.5$ $\sigma_2 = 3.0$

$n_1 = 1200$ $n_2 = 1500$

$\alpha = .05$

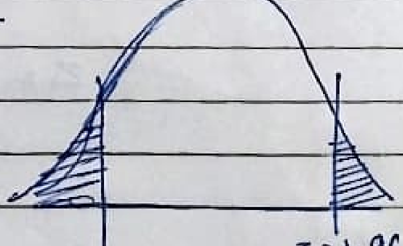
Null Hypothesis: $H_0 : \bar{x}_1 = \bar{x}_2$

Alternate Hypothesis: $H_1 : \bar{x}_1 \neq \bar{x}_2$ (Two tailed)

Test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$Z = -.756$



z_α $z_\alpha = 1.96$

as

$|Z_{cal}| < |Z_\alpha|$

ie Null Hypo. is accepted ie ~~the~~ mean height is same for the children at two places.

If Samples are drawn from two populations with unknown S.D then $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

Q:-

Q:- In a survey of buying habits, 400 women shoppers are chosen at random in super market 'A' located in a certain section of the city. Their average weekly food expenditure is Rs. 250 with S.D of Rs. 40. For 400 women shoppers chosen at random in super market B in another section of the city, the average weekly food expenditure is Rs 220 with a S.D of Rs. 55. Test at 1% level of significance whether the average weekly food expenditure of two populations of shoppers are equal.

Solⁿ:- $n_1 = 400$ $\quad \bar{x}_1 = 250,$ $\quad s_1 = 40$

$n_2 = 400,$ $\quad \bar{x}_2 = 220,$ $\quad s_2 = 55$

H_0: assume average weekly food expenditure of the two populations of shoppers are equal ie

$H_0: \bar{x}_1 = \bar{x}_2$

Alternate Hypo.

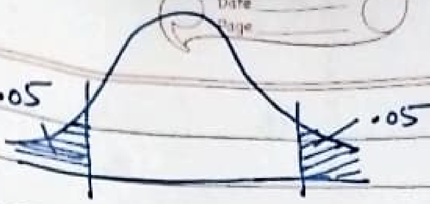
$H_1: \bar{x}_1 \neq \bar{x}_2$

Test Statistic:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

(as Sample's S.D are given)

$$Z = \frac{250 - 220}{\sqrt{\frac{(40)^2}{400} + \frac{(55)^2}{400}}}$$



$$Z = \frac{30}{3.4} = 8.82, \quad Z_\alpha = 2.58$$

as

$|Z_{cal}| > |Z_\alpha|$

ie Ho is rejected, H_1 accepted

ie average weekly food expenditure of two population of shoppers are not equal

Q:- Test the significance of the difference between the means of the samples drawn from two normal populations with same S.D. from the following data:-

	Size.	Mean	S.D
Sample 1	100	61	4
Sample 2.	200	63	6

Solⁿ:-

$n_1 = 100, \quad \bar{x}_1 = 61, \quad s_1 = 4$

$n_2 = 200, \quad \bar{x}_2 = 63, \quad s_2 = 6$

as populations have same S.D

ie $$\sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$$

Let $H_0: \bar{x}_1 = \bar{x}_2$ or $\mu_1 = \mu_2$

$H_1: \bar{x}_1 \neq \bar{x}_2$ or $\mu_1 \neq \mu_2$

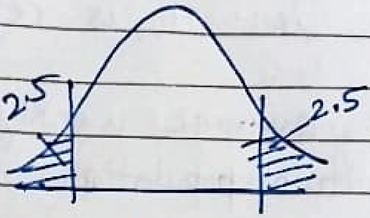
ie

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_2} + \frac{s_2^2}{n_1}}}$$

$$z = \frac{61 - 63}{\sqrt{\frac{16}{200} + \frac{36}{100}}} = -3.02$$

Let $\alpha = 5\%$.

$Z_\alpha = 1.96$



two tailed

as

$|Z_{cal}| > |Z_\alpha|$

ie H_0 is rejected.
and H_1 is accepted

ie average weekly expenditure of two populations are not equal, even though they have same S.D.

The average marks scored by 32 boys is 72 with S.D of 8, while that for 32 girls is 70 with a S.D of 6. Test at 1% level of significance whether the boys perform better than girls.

Solution:- $H_0: \overline{x_1} = \overline{x_2}$ or $(\mu_1 = \mu_2)$

$H_1: \overline{x_1} > \overline{x_2}$ or $(\mu_1 > \mu_2)$

Right tailed test

level of significance $\alpha = 1\%$

$Z_\alpha = 2.33$

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

(The two populations assumed to have S.D $\sigma_1 = s_1$ & $\sigma_2 = s_2$)

$$Z = \frac{72 - 70}{\sqrt{\frac{8^2}{32} + \frac{6^2}{36}}} = 1.15$$

$|Z| < |Z_\alpha|$

ie H_0 is accepted. & H_1 is rejected.

ie statistically we cannot conclude that boys perform better than girls.