Unit –II

AC circuits:

- Alternating voltages and currents , AC values,
- Single Phase RL, RC, RLC Series circuits,
- Power in AC circuits –Power Factor - Three Phase Systems – Star and Delta Connection –
- Three Phase Power Measurement – Electrical Safety – Fuses and Earthing,
- Residential wiring.

DC STEADY STATE

The transient terms in the expressions for currents and voltages in RLC circuits decay to zero with time. (An exception is LC circuits having no resistance.) For dc sources, the steady-state currents and voltages are also constant.

Consider the equation for current through a capacitance:

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

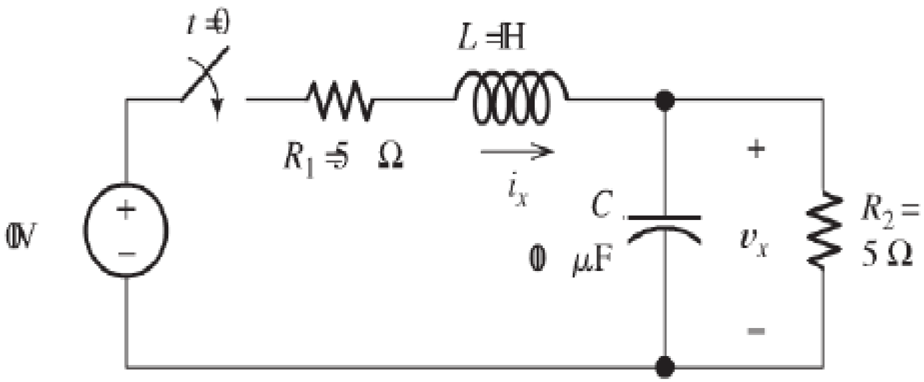
If the voltage $v_C(t)$ is constant, the current is zero. In other words, the capacitance behaves as an open circuit. Thus, we conclude that *for steady-state conditions with dc sources, capacitances behave as open circuits.*

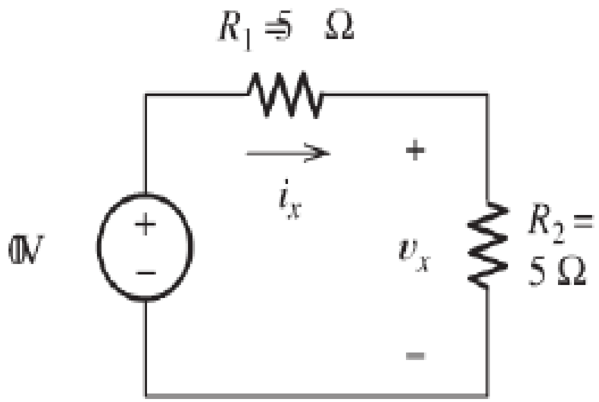
Similarly, for an inductance, we have

$$v_L(t) = L\frac{di_L(t)}{dt}$$

When the current is constant, the voltage is zero. Thus, we conclude that *for steady-state conditions with dc sources, inductances behave as short circuits.*

Find v_x and i_x for the circuit shown in Figure





conditions. We start our analysis by replacing the inductor by a short circuit and the capacitor by an open circuit. The equivalent circuit is shown in Figure 4.5(b).

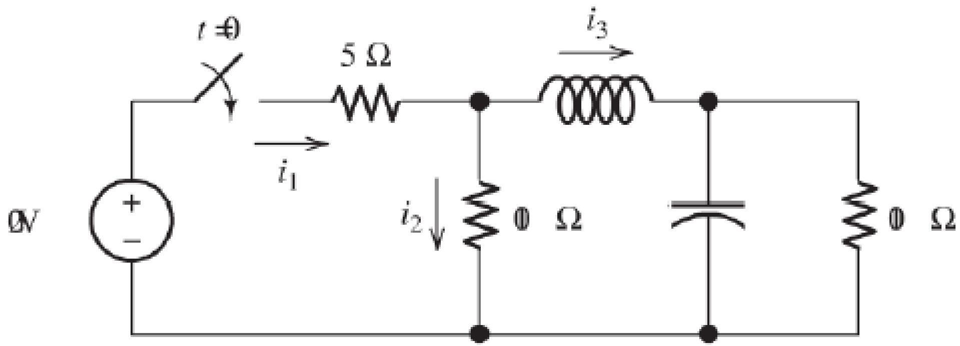
This resistive circuit is readily solved. The resistances R_1 and R_2 are in series. Thus, we have

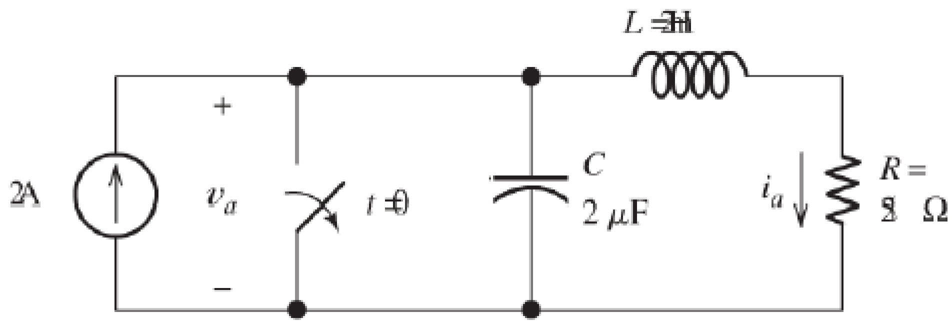
$$i_x = \frac{10}{R_1 + R_2} = 1 \text{ A}$$

and

$$v_x = R_2 i_x = 5 \text{ V}$$

Examples



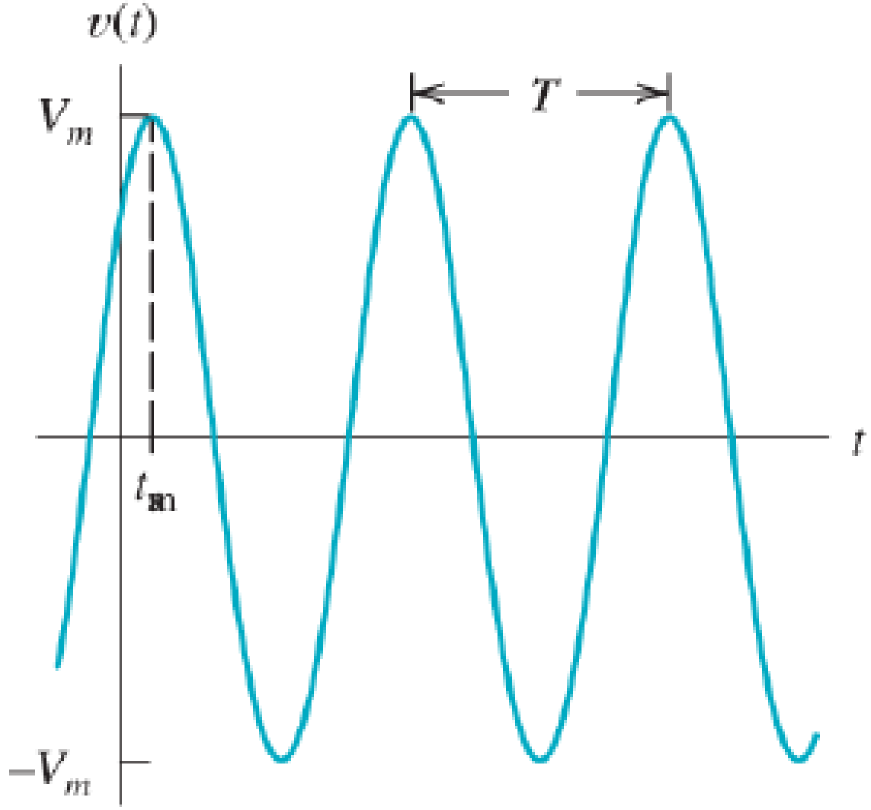


Steady-State Sinusoidal Analysis

Study of this chapter will enable you to:

- Identify the frequency, angular frequency, peak value, rms value, and phase of a sinusoidal signal.
- Determine the root-mean-square (rms) value of any periodic current or voltage.
- Solve steady-state ac circuits, using phasors and complex impedances.
- Compute power for steady-state ac circuits.

SINUSOIDAL CURRENTS AND VOLTAGES



A sinusoidal voltage is shown in Figure 5.1 and is given by

$$v(t) = V_m \cos(\omega t + \theta) \tag{5.1}$$

where V_m is the **peak value** of the voltage, ω is the **angular frequency** in radians per second, and θ is the **phase angle**.

Sinusoidal signals are periodic, repeating the same pattern of values in each **period** T. Because the cosine (or sine) function completes one cycle when the angle increases by 2π radians, we get

$$\omega T = 2\pi \tag{5.2}$$

The **frequency** of a periodic signal is the number of cycles completed in one second. Thus, we obtain

$$f = \frac{1}{T} \tag{5.3}$$

The units of frequency are hertz (Hz). (Actually, the physical units of hertz are equivalent to inverse seconds.) Solving Equation 5.2 for the angular frequency, we have

$$\omega = \frac{2\pi}{T} \tag{5.4}$$

Using Equation 5.3 to substitute for T, we find that

$$\omega = 2\pi f \tag{5.5}$$

Root-Mean-Square Values

Consider applying a periodic voltage $v(t)$ with period T to a resistance R. The power delivered to the resistance is given by

$$p(t) = \frac{v^2(t)}{R} \tag{5.7}$$

Furthermore, the energy delivered in one period is given by

$$E_T = \int_0^T p(t)\, dt \tag{5.8}$$

The average power P_{avg} delivered to the resistance is the energy delivered in one cycle divided by the period. Thus,

$$P_{avg} = \frac{E_T}{T} = \frac{1}{T}\int_0^T p(t)\, dt \tag{5.9}$$

Using Equation 5.7 to substitute into Equation 5.9, we obtain

$$P_{avg} = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\, dt \tag{5.10}$$

This can be rearranged as

$$P_{\text{avg}} = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}\right]^2}{R} \tag{5.11}$$

Now, we define the **root-mean-square** (rms) value of the periodic voltage $v(t)$ as

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \tag{5.12}$$

Using this equation to substitute into Equation 5.11, we get

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} \tag{5.13}$$

Similarly for a periodic current $i(t)$, we define the rms value as

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

and the average power delivered if $i(t)$ flows through a resistance is given by

$$P_{\text{avg}} = I_{\text{rms}}^2 R$$

RMS Value of a Sinusoid

Consider a sinusoidal voltage given by

$$v(t) = V_m \cos(\omega t + \theta) \tag{5.16}$$

To find the rms value, we substitute into Equation 5.12, which yields

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t + \theta)\, dt} \tag{5.17}$$

Next, we use the trigonometric identity

$$\cos^2(z) = \frac{1}{2} + \frac{1}{2}\cos(2z) \tag{5.18}$$

to write Equation 5.17 as

$$V_{\text{rms}} = \sqrt{\frac{V_m^2}{2T}\int_0^T [1 + \cos(2\omega t + 2\theta)]\, dt} \tag{5.19}$$

Integrating, we get

$$V_{\text{rms}} = \sqrt{\frac{V_m^2}{2T}\left[t + \frac{1}{2\omega}\sin(2\omega t + 2\theta)\right]_0^T} \tag{5.20}$$

Evaluating, we have

$$V_{\text{rms}} = \sqrt{\frac{V_m^2}{2T}\left[T + \frac{1}{2\omega}\sin(2\omega T + 2\theta) - \frac{1}{2\omega}\sin(2\theta)\right]} \tag{5.21}$$

Referring to Equation 5.2, we see that $\omega T = 2\pi$. Thus, we obtain

$$\begin{aligned}\frac{1}{2\omega}\sin(2\omega T + 2\theta) - \frac{1}{2\omega}\sin(2\theta) &= \frac{1}{2\omega}\sin(4\pi + 2\theta) - \frac{1}{2\omega}\sin(2\theta)\\ &= \frac{1}{2\omega}\sin(2\theta) - \frac{1}{2\omega}\sin(2\theta)\\ &= 0\end{aligned}$$

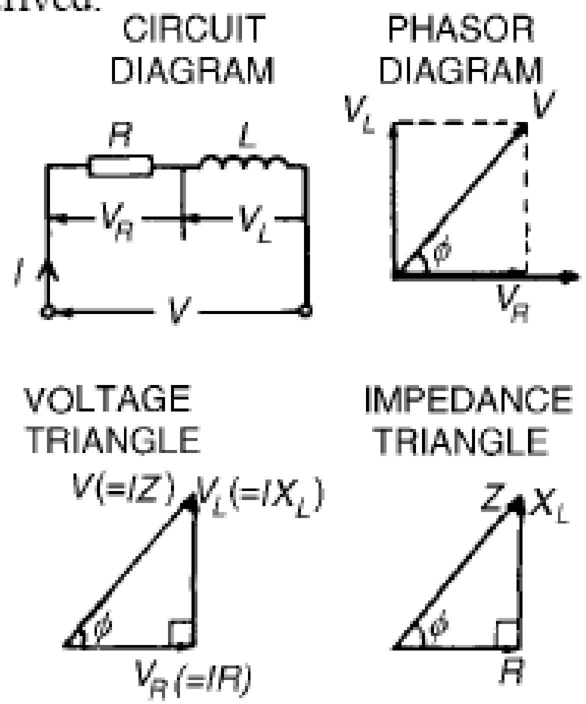
Therefore, Equation 5.21 reduces to

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \tag{5.22}$$

R–*L* series a.c. circuit

In an a.c. circuit containing inductance L and resistance R, the applied voltage V is the phasor sum of V_R and V_L , and thus the current I lags the applied voltage V by an angle lying between 0° and 90° (depending on the values of V_R and V_L), shown as angle φ. In any a.c. series circuit the current is common to each component and is thus taken as the reference phasor.

From the phasor diagram of Figure the '**voltage triangle**' is derived.

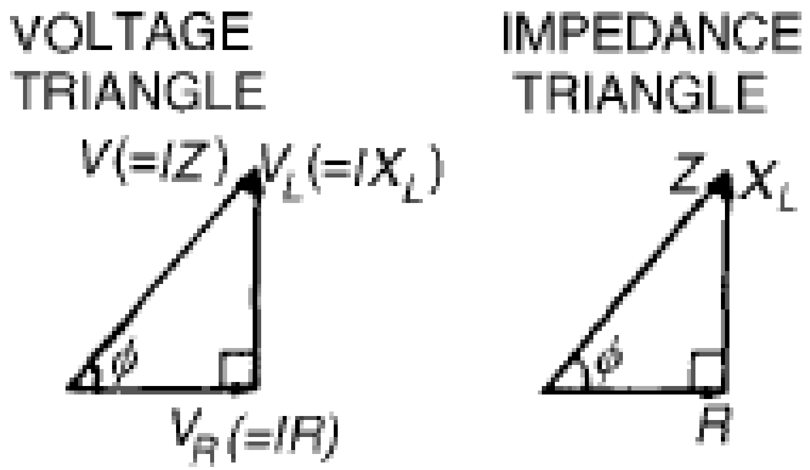


For the *R*–*L* circuit: $V = \sqrt{(V_R^2 + V_L^2)}$ (by Pythagoras' theorem)

and $\tan\phi = \dfrac{V_L}{V_R}$ (by trigonometric ratios)

In an a.c. circuit, the ratio $\dfrac{\text{applied voltage } V}{\text{current } I}$ is called the **impedance *Z***, i.e.

$$Z = \frac{V}{I}\,\Omega$$

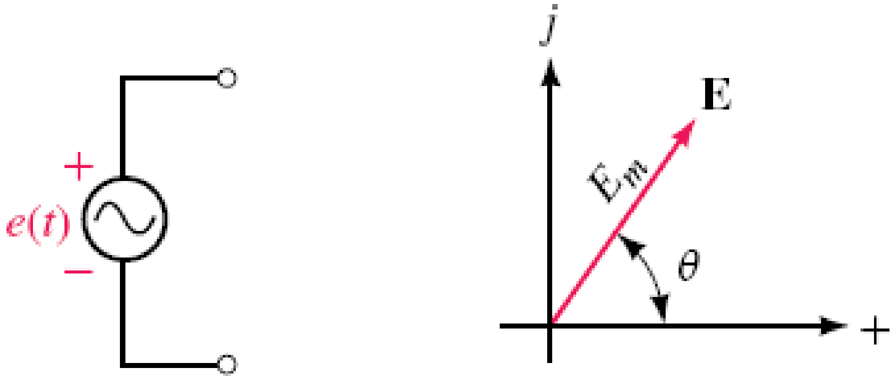


If each side of the voltage triangle in Figure is divided by current I then the **'impedance triangle'** is derived.

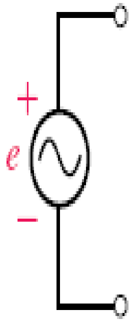
For the R–L circuit: $Z = \sqrt{(R^2 + X_L^2)}$

$$\tan\phi = \frac{X_L}{R}, \ \sin\phi = \frac{X_L}{Z} \text{ and } \cos\phi = \frac{R}{Z}$$

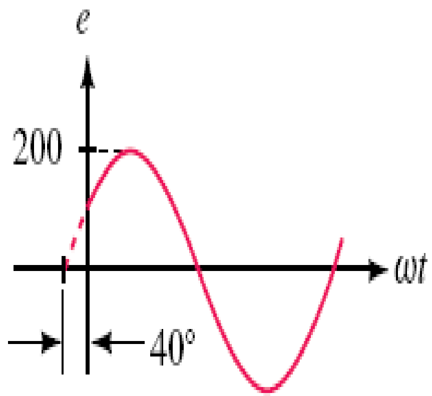
$$\mathbf{E} = E_m \angle \theta$$



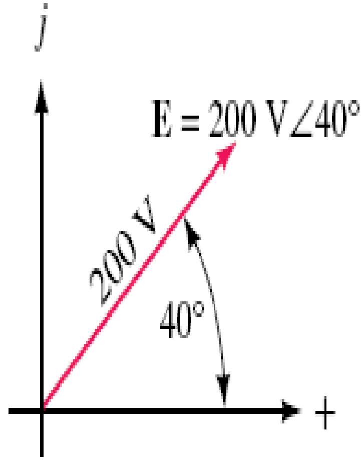
(a) $e(t) = E_m \sin(\omega t + \theta)$ (b) $\mathbf{E} = E_m \angle \theta$



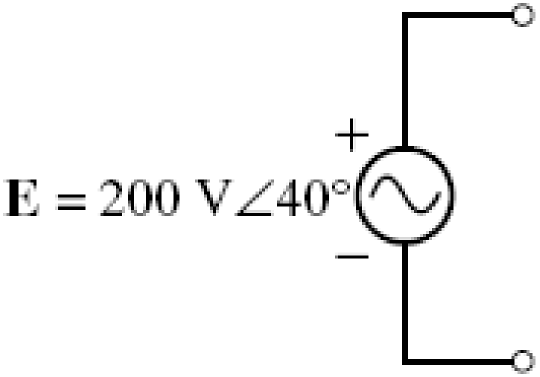
(a) $e = 200 \sin(\omega t + 40°)$ V



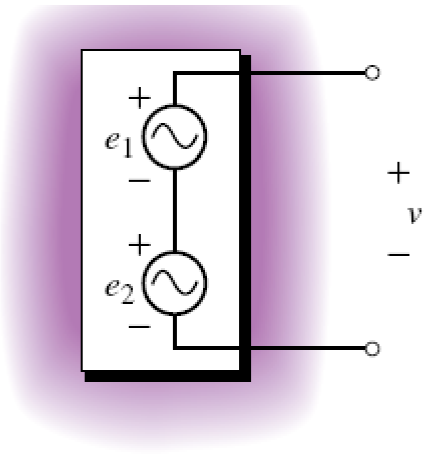
(b) Waveform



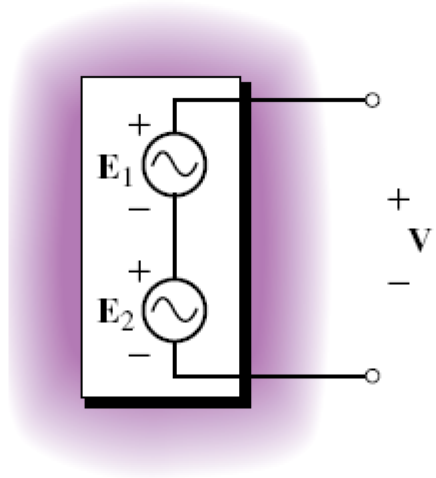
(c) Phasor equivalent



Transformed source



(a) Original network.
$v(t) = e_1(t) + e_2(t)$



(b) Transformed network.
$\mathbf{V} = \mathbf{E}_1 + \mathbf{E}_2$

EXAMPLE Given $e_1 = 10 \sin \omega t$ V and $e_2 = 15 \sin(\omega t + 60°)$ V determine v and sketch it.

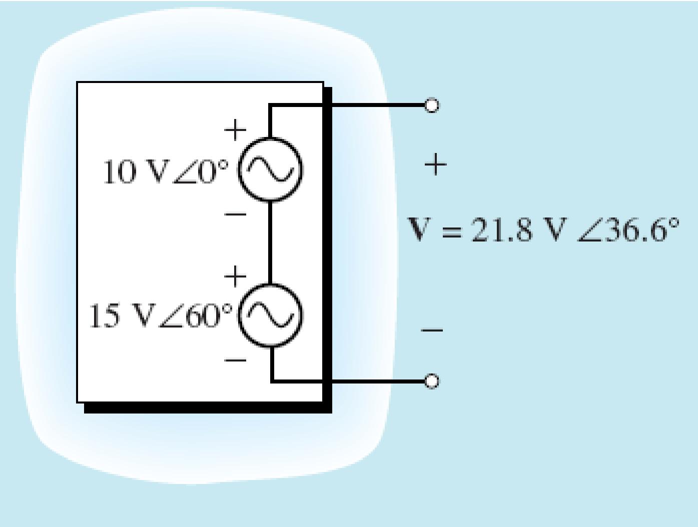
Solution $e_1 = 10 \sin \omega t$ V. Thus, $\mathbf{E}_1 = 10\ \text{V}\angle 0°$.

$e_2 = 15 \sin(\omega t + 60°)$ V. Thus, $\mathbf{E}_2 = 15\ \text{V}\angle 60°$.

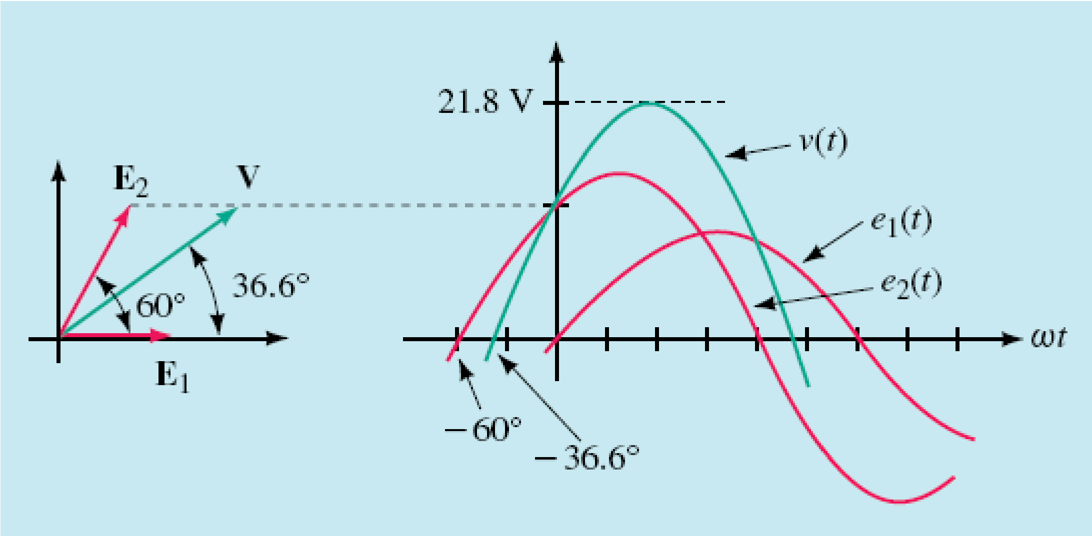
Transformed sources are shown in Figure (a) and phasors in (b).

$$\mathbf{V} = \mathbf{E}_1 + \mathbf{E}_2 = 10\angle 0° + 15\angle 60° = (10 + j0) + (7.5 + j13)$$
$$= (17.5 + j13) = 21.8\ \text{V}\angle 36.6°$$

Thus, $v = 21.8 \sin(\omega t + 36.6°)$ V



(a) Phasor summation

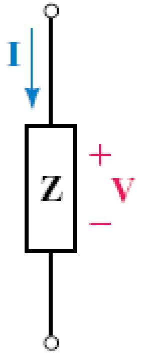


(b) Phasors

(c) Waveforms

Impedance

The opposition that a circuit element presents to current in the phasor domain is defined as its **impedance.**

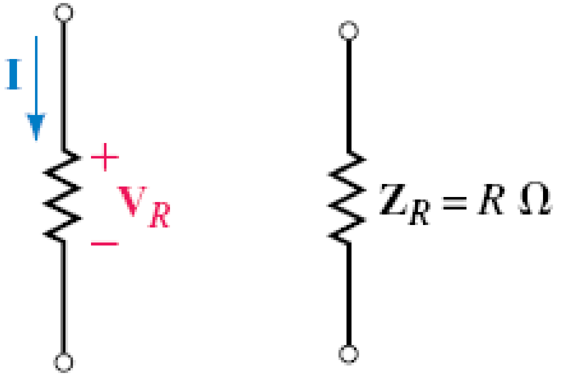


$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \text{ ohms}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V}{I}\angle\theta$$

$$\mathbf{Z} = Z\angle\theta$$

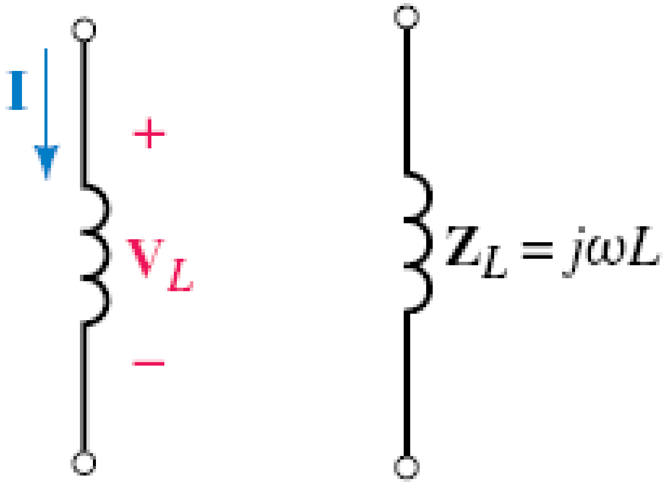
Purely resistive a.c. circuit



(a) Voltage and current (b) Impedance

$$\mathbf{Z}_R = \frac{\mathbf{V}_R}{\mathbf{I}} = \frac{V_R\angle\theta}{I\angle\theta} = \frac{V_R}{I}\angle 0^\circ = R$$

Purely inductive a.c. circuit

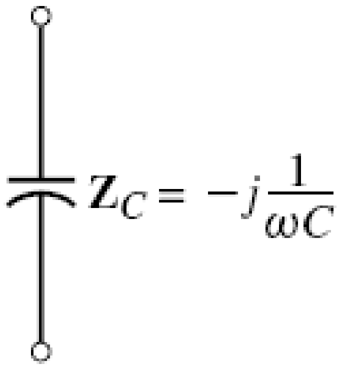


(a) Voltage and current (b) Impedance

$$\mathbf{Z}_L = j\omega L = jX_L$$

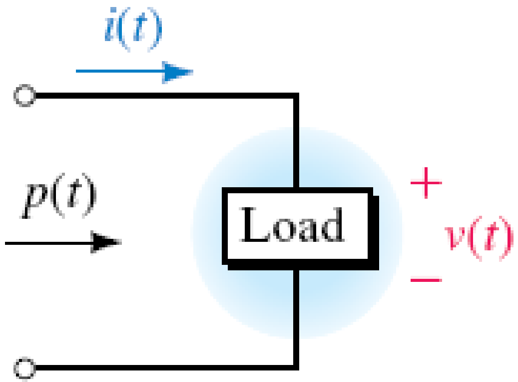
$$\mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}} = \frac{V_L\angle 0^\circ}{I\angle -90^\circ} = \frac{V_L}{I}\angle 90^\circ = \omega L\angle 90^\circ = j\omega L$$

Purely capacitive a.c. circuit



$$\mathbf{Z}_C = -j\frac{1}{\omega C} = -jX_C \quad \text{(ohms)}$$

$$\mathbf{Z}_C = \frac{\mathbf{V}_C}{\mathbf{I}} = \frac{V_C\angle 0^\circ}{I\angle 90^\circ} = \frac{V_C}{I}\angle -90^\circ = \frac{1}{\omega C}\angle -90^\circ = -j\frac{1}{\omega C} \quad \text{(ohms)}$$



$p = vi$

$$p = vi \quad \text{(watts)}$$

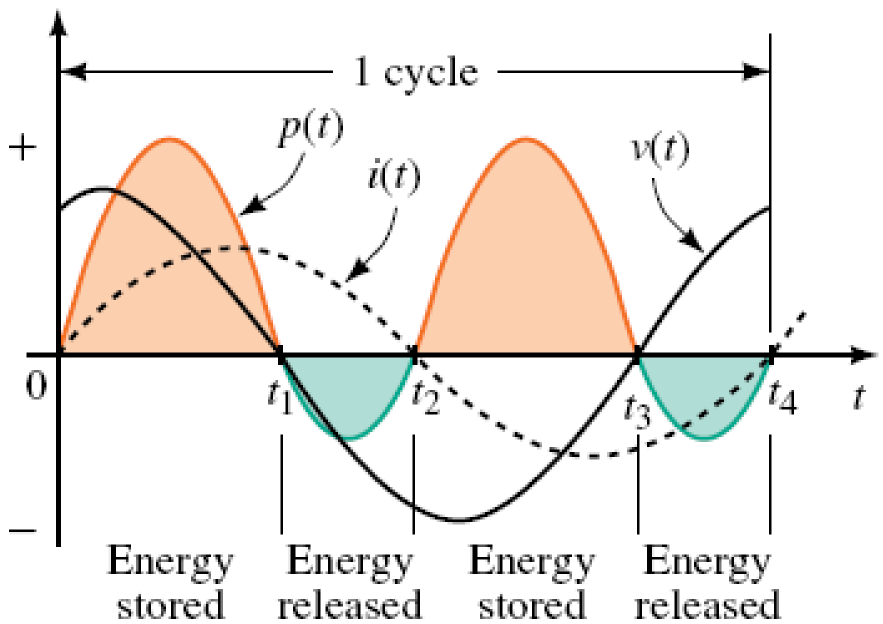


TABLE 9.1 Sinusoid-phasor transformation.

Time-domain representation	Phasor-domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

TABLE 9.2 Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

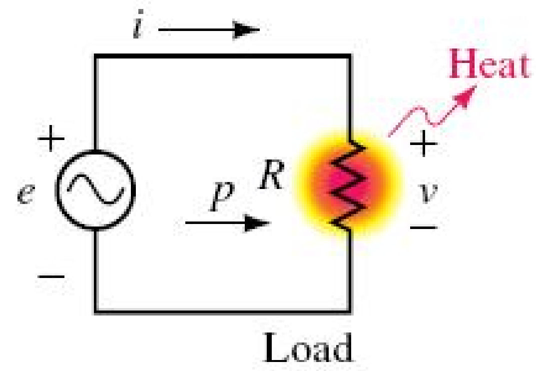
TABLE 9.3 Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \dfrac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \dfrac{1}{j\omega L}$
C	$\mathbf{Z} = \dfrac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

ACTIVE POWER

active power is the average value of the instantaneous power, and the terms real power, active power, and average power mean the same

$$p = vi = (V_m \sin \omega t)(I_m \sin \omega t) = V_m I_m \sin^2 \omega t$$

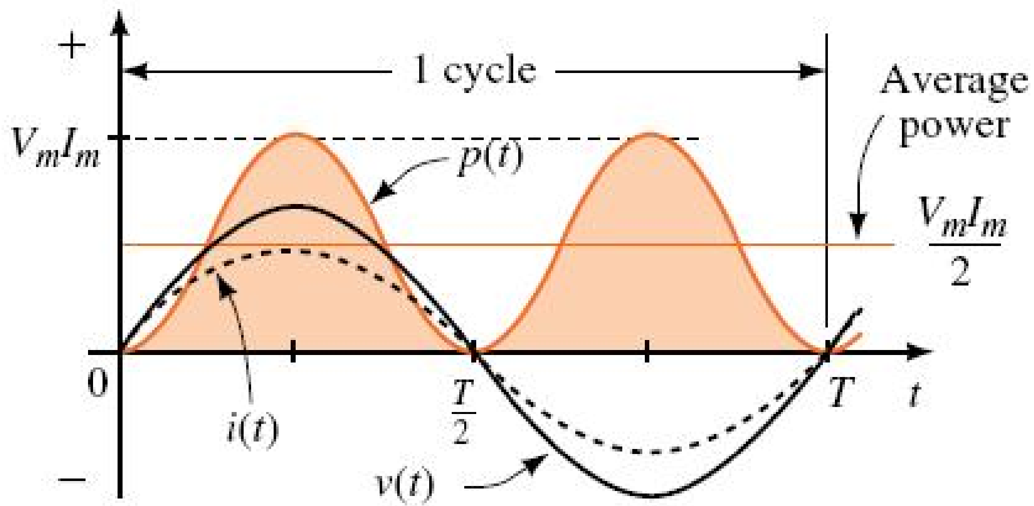


(a)

$$p = \frac{V_m I_m}{2}(1 - \cos 2\,\omega t)$$

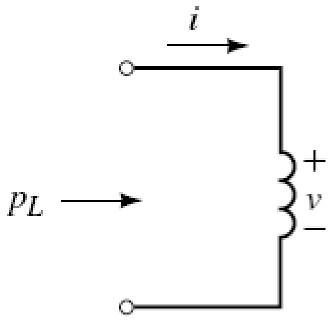
$$P = V_m I_m / 2$$

$$P = VI \quad \text{(watts)}$$

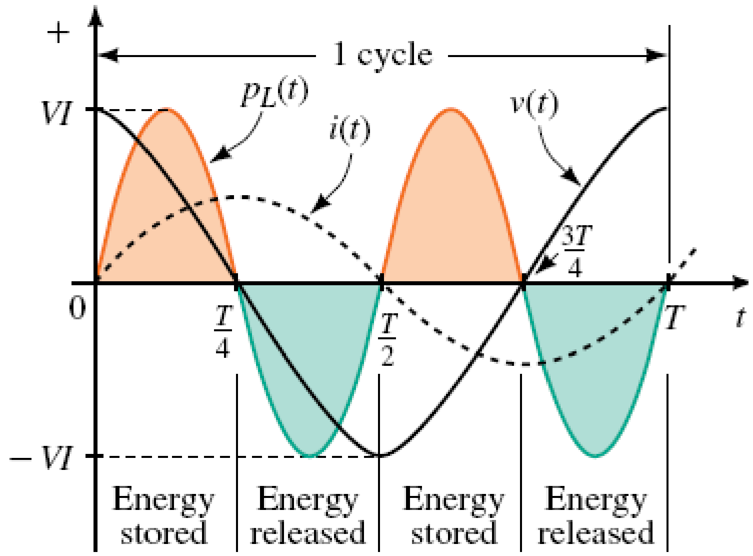


(b)

Purely inductive a.c. circuit



(a) Let $i = I_m \sin \omega t$
$v = V_m \sin(\omega t + 90°)$



(b)

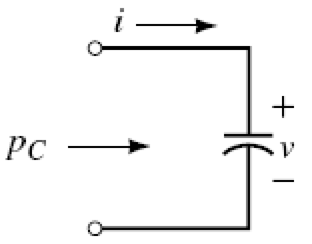
$$p_L = V_m I_m \sin(\omega t + 90°) \sin \omega t$$

$$p_L = VI \sin 2\omega t$$

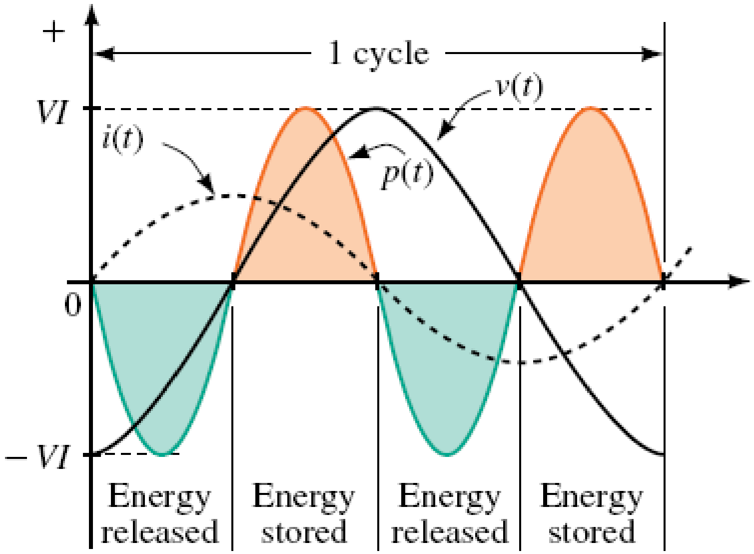
$$Q_L = VI \quad \text{(VAR)}$$

$$Q_L = I^2 X_L = \frac{V^2}{X_L} \quad \text{(VAR)}$$

Purely capacitive a.c. circuit



(a) Let $i = I_m \sin \omega t$
$v = V_m \sin(\omega t - 90°)$



(b)

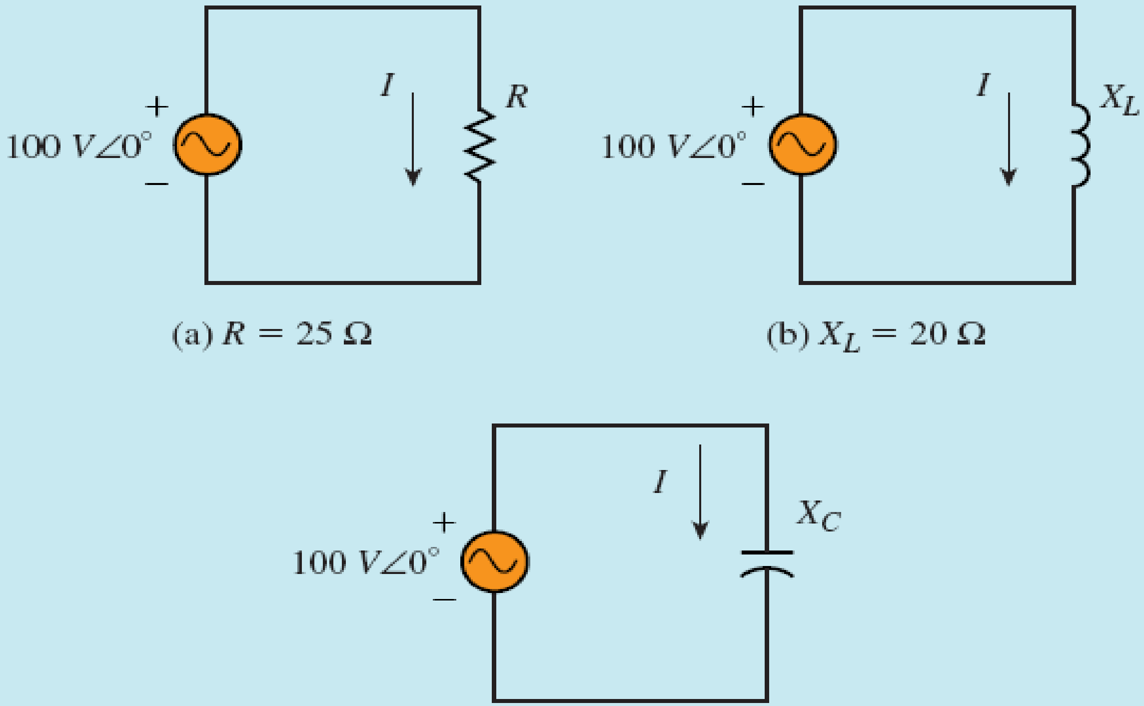
$$p_C = vi = V_m I_m \sin \omega t \sin(\omega t - 90°)$$

$$p_C = -VI \sin 2\omega t$$

$$Q_C = VI \quad \text{(VAR)}$$

$$Q_C = I^2 X_C = \frac{V^2}{X_C} \quad \text{(VAR)}$$

EXAMPLE For each circuit of Figure determine real and reactive power.



(a) $R = 25\ \Omega$ (b) $X_L = 20\ \Omega$

(c) $X_C = 40\ \Omega$

Solution Only voltage and current magnitudes are needed.

a. $I = 100\ \text{V}/25\ \Omega = 4\ \text{A}$. $P = VI = (100\ \text{V})(4\ \text{A}) = 400\ \text{W}$. $Q = 0\ \text{VAR}$

b. $I = 100\ \text{V}/20\ \Omega = 5\ \text{A}$. $Q = VI = (100\ \text{V})(5\ \text{A}) = 500\ \text{VAR}$ (ind.). $P = 0\ \text{W}$

c. $I = 100\ \text{V}/40\ \Omega = 2.5\ \text{A}$. $Q = VI = (100\ \text{V})(2.5\ \text{A}) = 250\ \text{VAR}$ (cap.). $P = 0\ \text{W}$

The answer for (c) can also be expressed as $Q = -250\ \text{VAR}$.

Problem 14. A capacitor C is connected in series with a 40 Ω resistor across a supply of frequency 60 Hz. A current of 3 A flows and the circuit impedance is 50 Ω. Calculate: (a) the value of capacitance, C, (b) the supply voltage, (c) the phase angle between the supply voltage and current, (d) the p.d. across the resistor, and (e) the p.d. across the capacitor. Draw the phasor diagram.

(a) Impedance $Z = \sqrt{(R^2 + X_C^2)}$

Hence $X_C = \sqrt{(Z^2 - R^2)} = \sqrt{(50^2 - 40^2)} = 30\ \Omega$

$$X_C = \frac{1}{2\pi f C} \text{ hence } C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)30}\ \text{F}$$

$$= \mathbf{88.42\ \mu F}$$

(b) Since $Z = \dfrac{V}{I}$ then $V = IZ = (3)(50) = \mathbf{150\ V}$

(c) Phase angle, $\alpha = \arctan\dfrac{X_C}{R} = \arctan\left(\dfrac{30}{40}\right) = 36.87°$

$= \mathbf{36°52'\ leading}$

(d) P.d. across resistor, $V_R = IR = (3)(40) = \mathbf{120\ V}$

(e) P.d. across capacitor, $V_C = IX_C = (3)(30) = \mathbf{90\ V}$

The phasor diagram is shown in Figure 15.11, where the supply voltage V is the phasor sum of V_R and V_C.

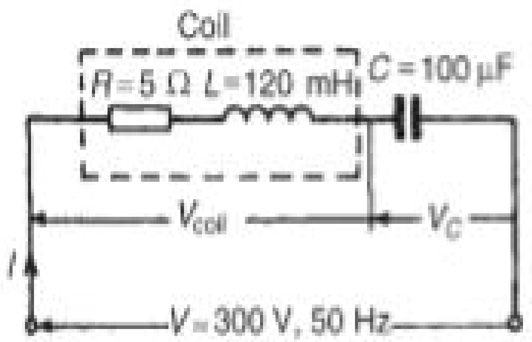


Figure 15.13

Problem 15. A coil of resistance 5 Ω and inductance 120 mH in series with a 100 μF capacitor, is connected to a 300 V, 50 Hz supply. Calculate (a) the current flowing, (b) the phase difference between the supply voltage and current, (c) the voltage across the coil and (d) the voltage across the capacitor.

The circuit diagram is shown in Figure 15.13

$$X_L = 2\pi f L = 2\pi(50)(120 \times 10^{-3}) = \mathbf{37.70\ \Omega}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(100 \times 10^{-6})} = \mathbf{31.83\ \Omega}$$

Since X_L is greater than X_C the circuit is inductive.

$X_L - X_C = 37.70 - 31.83 = 5.87\ \Omega$

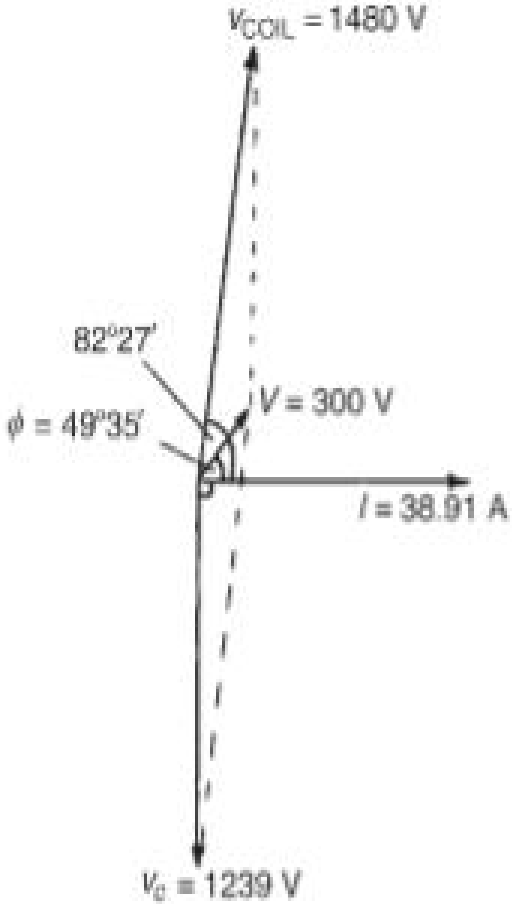


Figure 15.14

Impedance $Z = \sqrt{[R^2 + (X_L - X_C)^2]} = \sqrt{[(5)^2 + (5.87)^2]} = 7.71\ \Omega$

(a) Current $I = \dfrac{V}{Z} = \dfrac{300}{7.71} = \mathbf{38.91\ A}$

(b) Phase angle $\phi = \arctan\left(\dfrac{X_L - X_C}{R}\right) = \arctan\dfrac{5.87}{5} = 49.58^\circ$

$= \mathbf{49^\circ 35'}$

(c) Impedance of coil $Z_{COIL} = \sqrt{(R^2 + X_L^2)} = \sqrt{[(5)^2 + (37.70)^2]}$

$= 38.03\ \Omega$

Voltage across coil $V_{COIL} = IZ_{COIL} = (38.91)(38.03) = \mathbf{1480\ V}$

Phase angle of coil $= \arctan\dfrac{X_L}{R} = \arctan\left(\dfrac{37.70}{5}\right) = 82.45^\circ$

$= 82^\circ 27'$ lagging

(d) Voltage across capacitor $V_C = IX_C = (38.91)(31.83) = \mathbf{1239\ V}$

The phasor diagram is shown in Figure 15.14. The supply voltage V is the phasor sum of V_{COIL} and V_C

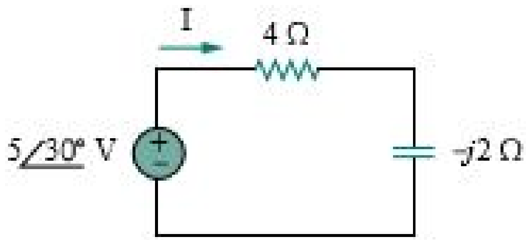


Figure 11.3 For Example 11.3.

For the circuit shown in Fig. 11.3, find the average power supplied by the source and the average power absorbed by the resistor.

Solution:

The current **I** is given by

$$\mathbf{I} = \frac{5\angle 30^\circ}{4 - j2} = \frac{5\angle 30^\circ}{4.472\angle -26.57^\circ} = 1.118\angle 56.57^\circ \text{ A}$$

The average power supplied by the voltage source is

$$P = \frac{1}{2}(5)(1.118)\cos(30^\circ - 56.57^\circ) = 2.5 \text{ W}$$

The current through the resistor is

$$\mathbf{I} = \mathbf{I}_R = 1.118\angle 56.57^\circ \text{ A}$$

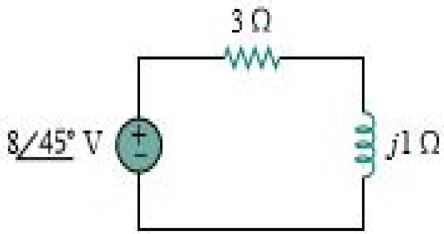
and the voltage across it is

$$\mathbf{V}_R = 4\mathbf{I}_R = 4.472\angle 56.57^\circ \text{ V}$$

The average power absorbed by the resistor is

$$P = \frac{1}{2}(4.472)(1.118) = 2.5 \text{ W}$$

which is the same as the average power supplied. Zero average power is absorbed by the capacitor.



In the circuit of Fig. 11.4, calculate the average power absorbed by the resistor and inductor. Find the average power supplied by the voltage source.

Answer: 9.6 W, 0 W, 9.6 W.

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70\ \Omega$ when a voltage $\mathbf{V} = 120\underline{/0^\circ}$ is applied across it.

Solution:

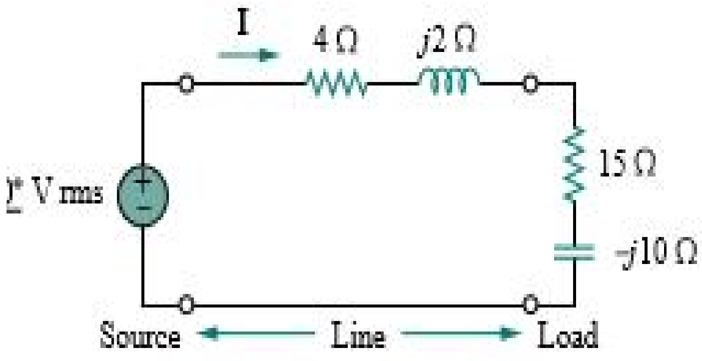
The current through the impedance is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{120\underline{/0^\circ}}{30 - j70} = \frac{120\underline{/0^\circ}}{76.16\underline{/-66.8^\circ}} = 1.576\underline{/66.8^\circ}\ \text{A}$$

The average power is

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576)\cos(0 - 66.8^\circ) = 37.24\ \text{W}$$

Figure 11.24 shows a load being fed by a voltage source through a transmission line. The impedance of the line is represented by the $(4 + j2)\ \Omega$ impedance and a return path. Find the real power and reactive power absorbed by: (a) the source, (b) the line, and (c) the load.



Solution:

The total impedance is

$$\mathbf{Z} = (4 + j2) + (15 - j10) = 19 - j8 = 20.62\angle -22.83^\circ\ \Omega$$

The current through the circuit is

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{220\angle 0^\circ}{20.62\angle -22.83^\circ} = 10.67\angle 22.83^\circ \text{ A rms}$$

(a) For the source, the complex power is

$$\mathbf{S}_s = \mathbf{V}_s\mathbf{I}^* = (220\angle 0^\circ)(10.67\angle -22.83^\circ)$$
$$= 2347.4\angle -22.83^\circ = (2163.5 - j910.8) \text{ VA}$$

From this, we obtain the real power as 2163.5 W and the reactive power as 910.8 VAR (leading).

(b) For the line, the voltage is

$$\mathbf{V}_{\text{line}} = (4 + j2)\mathbf{I} = (4.472\angle 26.57^\circ)(10.67\angle 22.83^\circ)$$
$$= 47.72\angle 49.4^\circ \text{ V rms}$$

The complex power absorbed by the line is

$$\mathbf{S}_{\text{line}} = \mathbf{V}_{\text{line}}\mathbf{I}^* = (47.72\angle 49.4^\circ)(10.67\angle -22.83^\circ)$$
$$= 509.2\angle 26.57^\circ = 455.4 + j227.7 \text{ VA}$$

or

$$\mathbf{S}_{\text{line}} = |\mathbf{I}|^2\mathbf{Z}_{\text{line}} = (10.67)^2(4 + j2) = 455.4 + j227.7 \text{ VA}$$

That is, the real power is 455.4 W and the reactive power is 227.76 VAR (lagging).

(c) For the load, the voltage is

$$\begin{aligned}\mathbf{V}_L = (15 - j10)\mathbf{I} &= (18.03\underline{/-33.7^\circ})(10.67\underline{/22.83^\circ}) \\ &= 192.38\underline{/-10.87^\circ} \text{ V rms}\end{aligned}$$

The complex power absorbed by the load is

$$\begin{aligned}\mathbf{S}_L = \mathbf{V}_L\mathbf{I}^* &= (192.38\underline{/-10.87^\circ})(10.67\underline{/-22.83^\circ}) \\ &= 2053\underline{/-33.7^\circ} = (1708 - j1139) \text{ VA}\end{aligned}$$