Engineering Physics

Course Code: BPHY101L; Course Type: Theory Only (TH)

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Module-2: Electromagnetic Waves

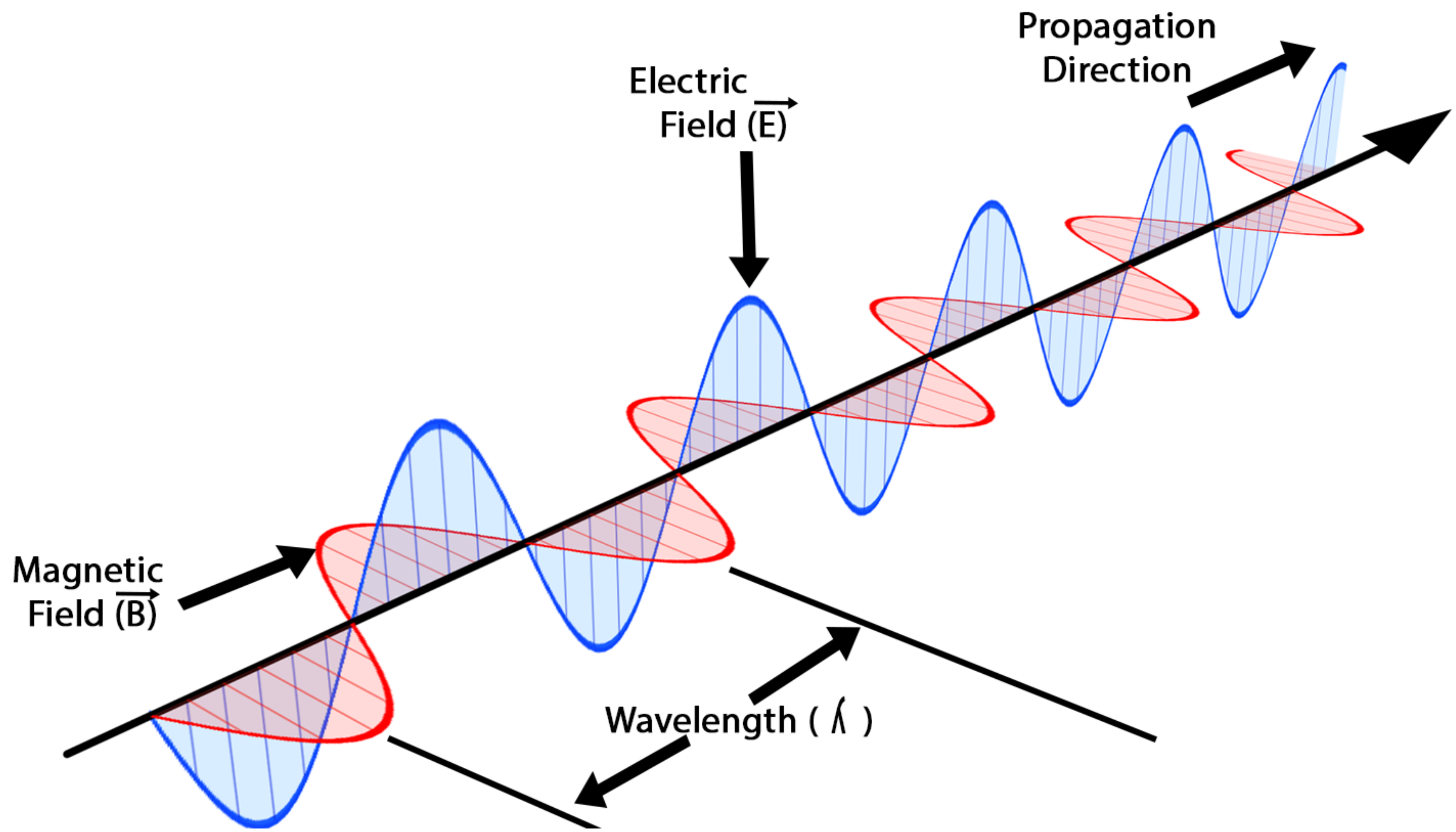
Syllabus

Physics of divergence - gradient and curl – Qualitative understanding of surface and volume integral - Maxwell Equations (Qualitative) - Displacement current - Electromagnetic wave equation in free space - Plane electromagnetic waves in free space - Hertz's experiment.

Reference Book:

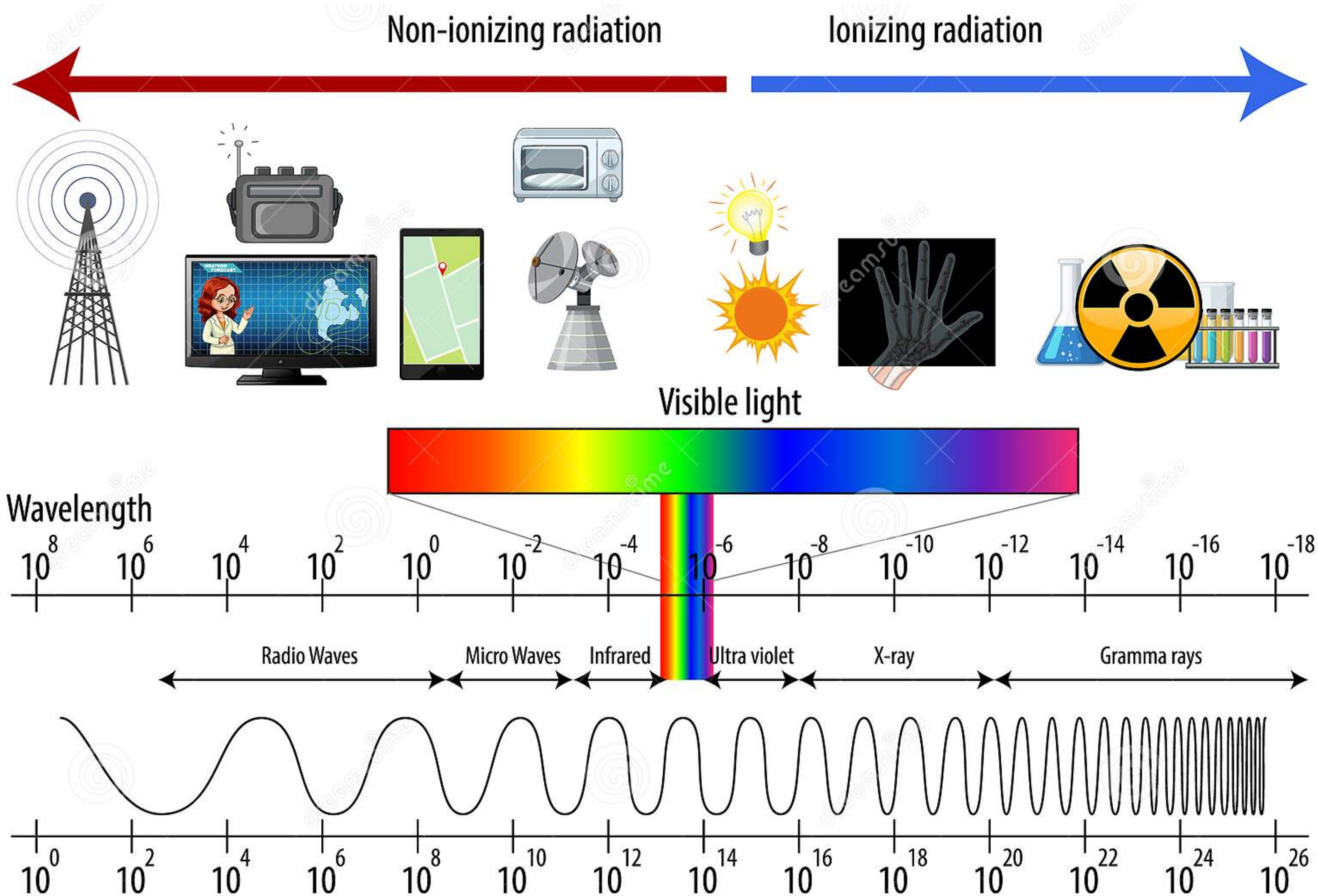
1. Introduction to Electrodynamics, D. J. Griffith, 4th Edition (2015)
2. University Physics with Modern Physics by H. D. Young and R. A. Freedman)

Electromagnetic Waves



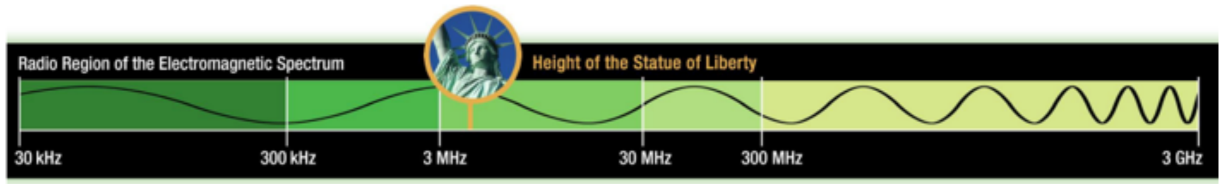
- Transverse wave with a varying electric and magnetic field
- Move with the speed of light
- Produced by moving charge particles
- Can emit and absorbed by matters
- Many applications in industry, light, laser, LED, communication, etc

Electromagnetic Spectrum



Electromagnetic Spectrum-Applications

Radio Wave Frequencies (RF)

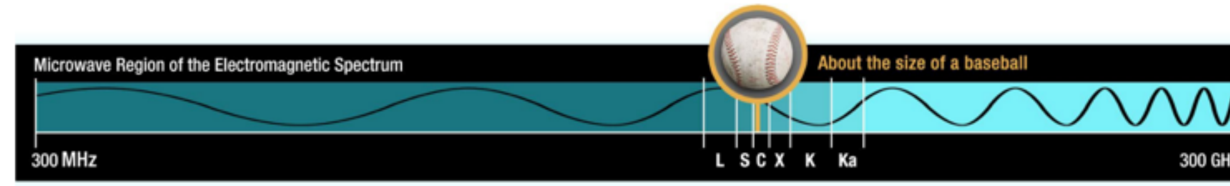


Radar
(**RA**dio **D**etection **A**nd **R**anging)

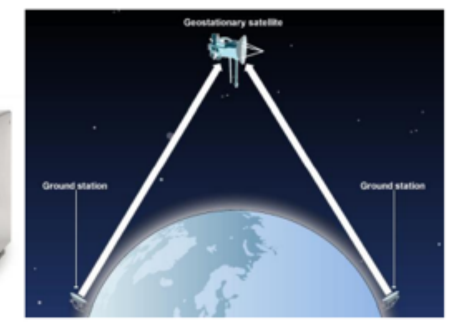
MRI
(**M**agnetic **R**esonance **I**maging)

Wireless Charging

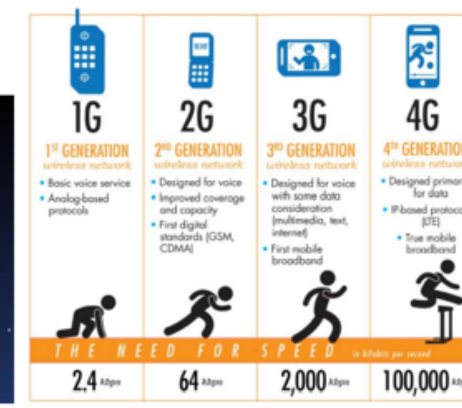
Microwaves



Microwave Oven



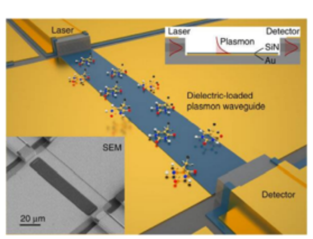
Satellite Communication



Wireless Communication

Infrareds

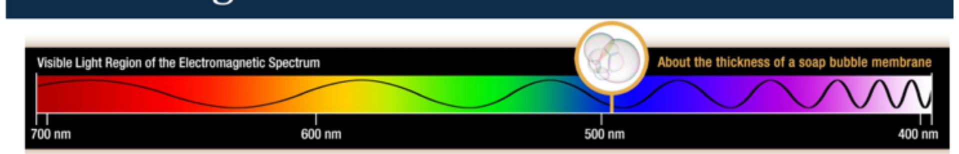
Thermal Imaging

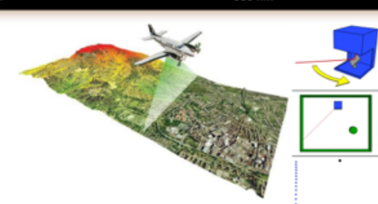


Chemical Sensing

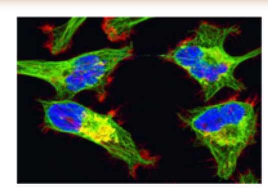
Optical Communication

Visible Light



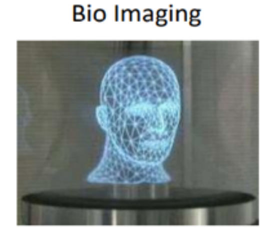


Lidar (**L**Ight **D**etection **A**nd **R**anging)



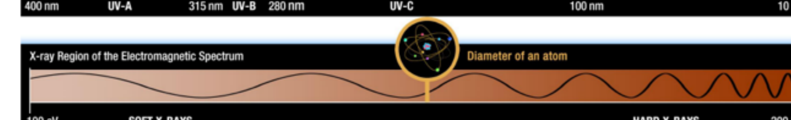
Bio Imaging

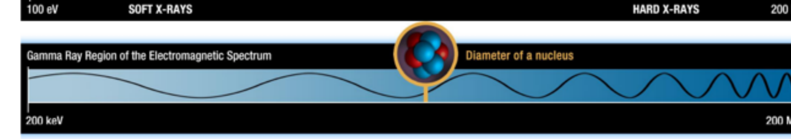
Solar Cells

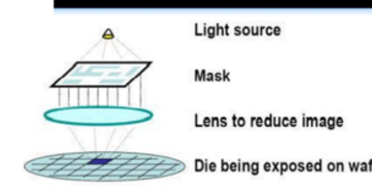


3D Display

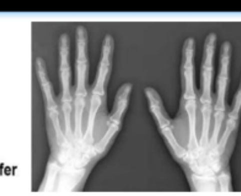
Ultraviolet, X-rays, Gamma-rays



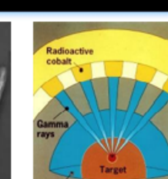




Photolithography



X-ray Imaging



Gamma Knife

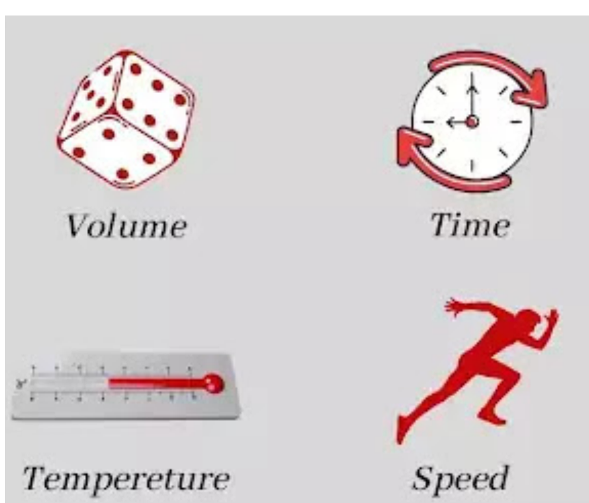
Scalar and Vector

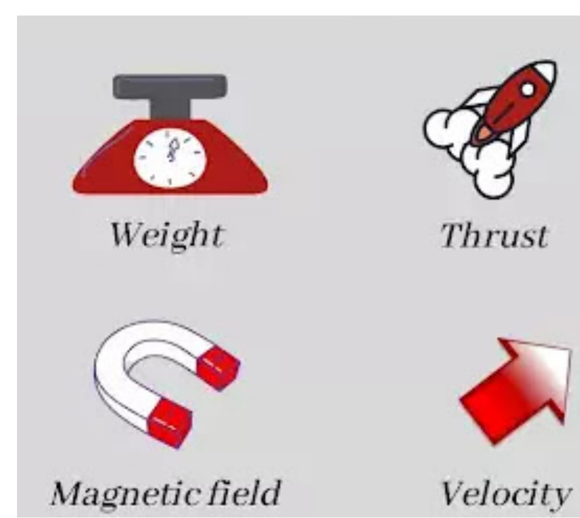
SCALAR

A scalar is a physical quantity thats us fully described by its magnitude only. It is only described by number only.

VECTOR

A vector is a physical quantity that has both magnitude and direction. Vector quantities are important in study motion, EMT etc



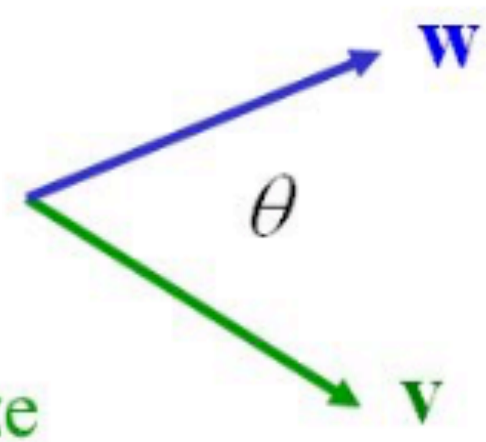


Dot (.) Products

There are two ways to multiply two vectors

- The dot product produces a scalar quantity
  - It has no direction
  - It can be pretty easily computed from geometry
  - It can be easily computed from components

$$\mathbf{v} \cdot \mathbf{w} = vw\cos\theta = v_x w_x + v_y w_y + v_z w_z$$



- The dot product of two unit vectors is easy to memorize

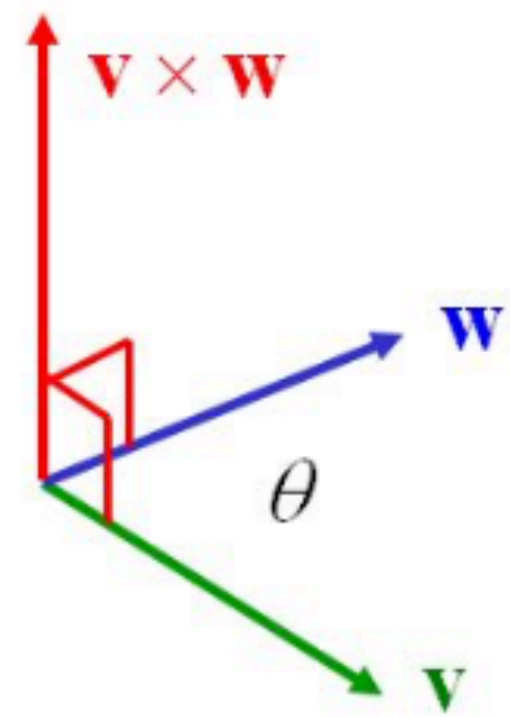
$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$
$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{i}} = 0$$
$$\hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$$
$$\hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{j}} = 0$$

- The dot product is commutative

$$\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$$

Cross (×) Product

The cross product produces a vector quantity
- It is perpendicular to both vectors
- Requires the right-hand rule
- Its magnitude can be easily computed from geometry
- It is a bit of a pain to compute from components



$$\left|\mathbf{v}\times\mathbf{w}\right| = vw\sin\theta$$

$$\mathbf{v}\times\mathbf{w} = \det\begin{pmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{pmatrix} = \left(v_y w_z - v_z w_y\right)\hat{\mathbf{i}} + \left(v_z w_x - v_x w_z\right)\hat{\mathbf{j}} + \left(v_x w_y - v_y w_x\right)\hat{\mathbf{k}}$$

Scalar field Function

Scalar field function

A **scalar field** is a physical quantity (temperature, density etc.) that is a scalar, which is also a function of multiple independent variables. It is a function which assigns a real number to every point of a part of the region of space.

T(x, y), temperature depends on x and y coordinates.

ρ **(x, y, z),** Density depends on x, y and z coordinates.

- The temperature at each point in an insulated wall
- The mass density of the atmosphere
- The water pressure at each point in an ocean

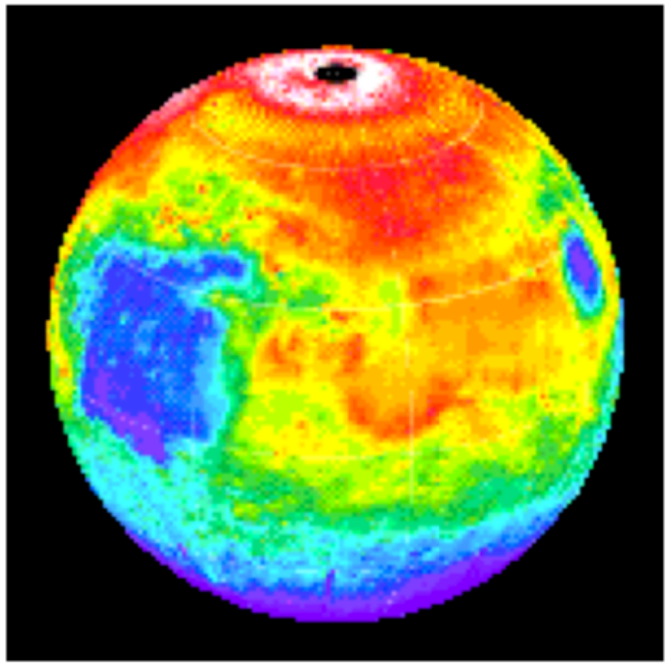


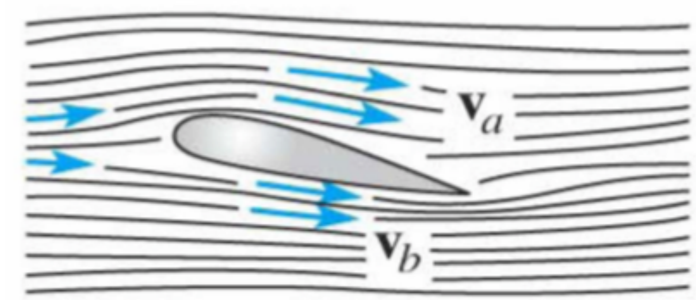
Figure 1.2.1 Nighttime temperature map for Mars

Vector field Function

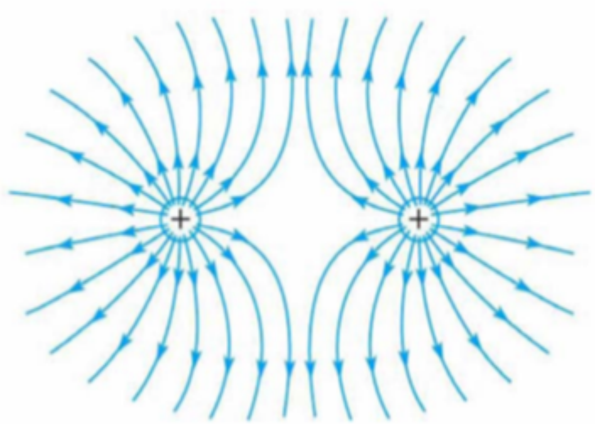
Vector field Function

A vector point function is defined as a function which assigns a vector to every point of a part of the region of space.

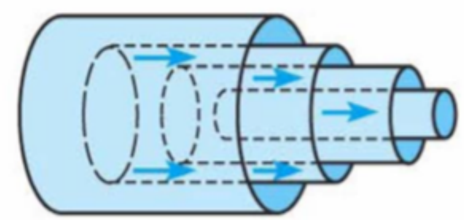
- Magnetic Field $\overrightarrow{B}(x, y, z)$
- Velocity $\overrightarrow{v}(x, y, z)$
- Gravitational Field $\overrightarrow{F}(x, y, z)$



(a) Airflow around an airplane wing



(d) Lines of force around two equal positive charges



(b) Laminar flow of blood in an artery; cylindrical layers of blood flow faster near the center of the artery

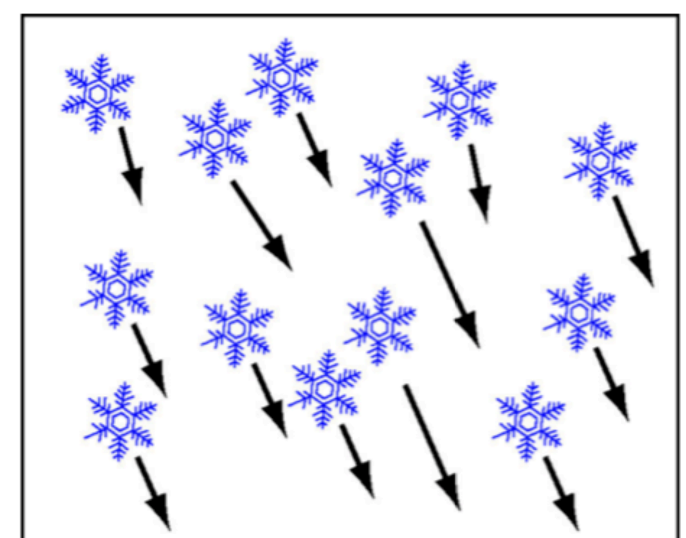
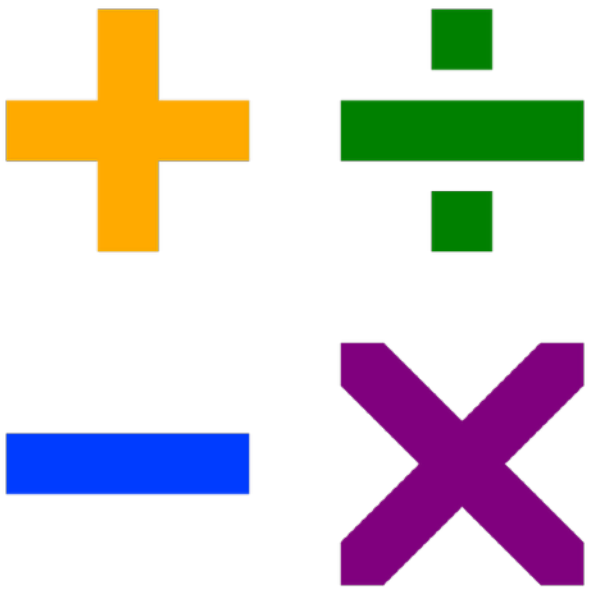


Figure 1.3.1 Falling snow.

Operator

An operator that operate over a function to give another functions



arithmetic operator

$$O = \frac{\partial}{\partial x}$$

$$function = T(x, y, z)$$

$$= x^2 + 2xz + xy^2z$$

$$\Rightarrow OT = \frac{\partial T}{\partial x}$$

$$= 2x + 2z + y^2z$$

Del ($\vec{\nabla}$) Operator

The vector differential operator ∇ , called “del” or “nabla”, is **defined** in three dimensions to be:

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

lets consider a function, $f(x, y, z) = x^2 + xy + xz$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\Rightarrow \; = \frac{\partial(x^2 + xy + xz)}{\partial x}\hat{i} + \frac{\partial(x^2 + xy + xz)}{\partial y}\hat{j} + \frac{\partial(x^2 + xy + xz)}{\partial z}\hat{k}$$

$$\Rightarrow \; = (2x + y + z)\hat{i} + x\hat{j} + x\hat{k}$$

Del ($\overrightarrow{\nabla}$) Operator

$$\overrightarrow{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Scalar function (F)

Gradient

$$\overrightarrow{\nabla} F$$

Vector function ($\overrightarrow{F}$)

Divergence

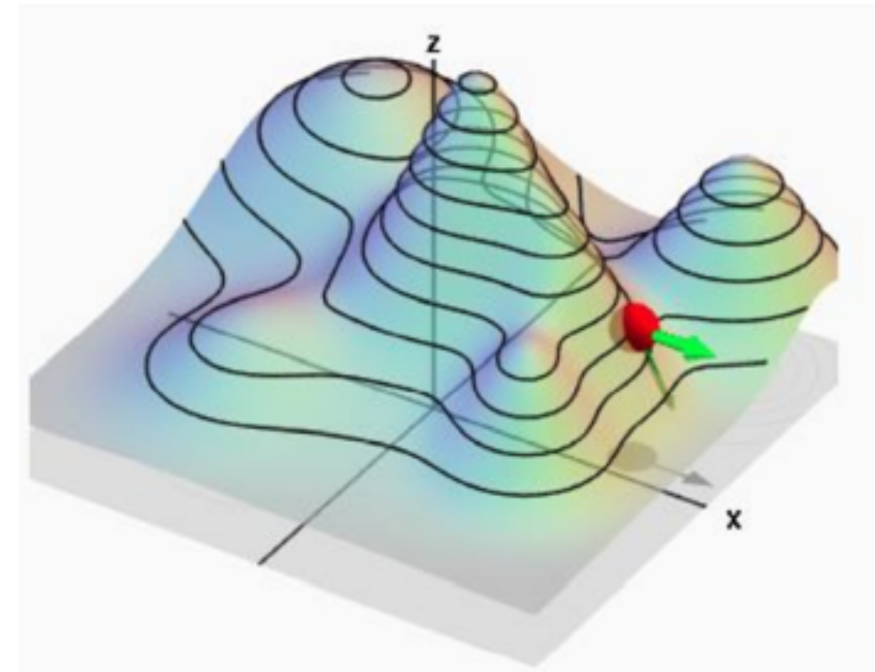
$$\overrightarrow{\nabla} \,.\, \overrightarrow{F}$$

Curl

$$\overrightarrow{\nabla} \times \overrightarrow{F}$$

Gradient of a Scalar Field ($\vec{\nabla} F$)

When the **del ($\vec{\nabla}$)** operator acts on a scalar field function $\phi(x, y, z)$, we get a vector function, called as gradient



$$\vec{\nabla}\phi = = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

A key point: F is a Scalar and the gradient of F is a vector.

Physical significance:

- $\vec{\nabla}\phi$ tells the maximum rate of change of the scaler function ϕ(x, y, z) with respect to the position in that particular direction.
- It is vector quantity

Gradient of a Scalar Field ($\overrightarrow{\nabla} F$): Example

$$\overrightarrow{\nabla}\phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

Lets consider a function, $\phi(x, y, z) = xy^2 - yz$

$$\overrightarrow{\nabla}\phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

$$\begin{aligned}
&= \frac{\partial}{\partial x}(xy^2 - yz)\boldsymbol{i} + \frac{\partial}{\partial y}(xy^2 - yz)\boldsymbol{j} \\
&\quad + \frac{\partial}{\partial z}(xy^2 - yz)\boldsymbol{k} \\
&= y^2 \times \frac{\partial}{\partial x}(x)\boldsymbol{i} + \left[x \times \frac{\partial}{\partial y}(y^2) - z \times \frac{\partial}{\partial y}(y)\right]\boldsymbol{j} \\
&\quad + (-y) \times \frac{\partial}{\partial z}(z)\boldsymbol{k} \\
&= y^2\boldsymbol{i} + (2xy - z)\boldsymbol{j} - y\boldsymbol{k}\,.
\end{aligned}$$

Divergence of a Vector Field ($\vec{\nabla}.\vec{F}$)

Let consider a vector field, $\vec{F}(x, y, z)$, then its divergence is calculated as $div\ \vec{F}(x, y, z) = \vec{\nabla}\ .\ \vec{F}$

$$\vec{\nabla}\ .\ \vec{F}(x, y, z) = (\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k})\ .\ (F_1\hat{i} + F_2\hat{j} + F_3\hat{k})$$

$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

A key point: F is a vector and the divergence of F is a scalar.

The divergence of $\boldsymbol{F}(x, y) = 3x^2\boldsymbol{i} + 2y\boldsymbol{j}$ is:

$$\boldsymbol{\nabla}\cdot\boldsymbol{F}(x, y) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y}$$

$$= \frac{\partial}{\partial x}(3x^2) + \frac{\partial}{\partial y}(2y) = 6x + 2\,.$$

Divergence of a Vector Field ($\vec{\nabla} . \vec{F}$)

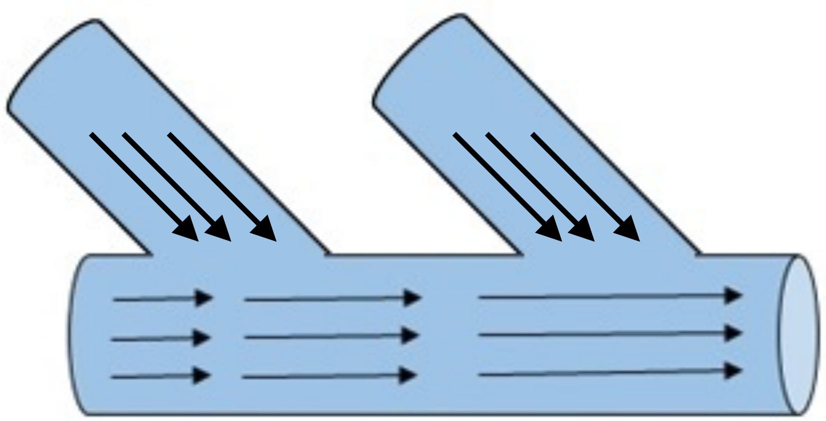
Let consider a vector field, $\vec{F}(x, y, z)$, then its divergence is calculated as $div \ \vec{F}(x, y, z) = \nabla \, . \, \vec{F}$

$$\nabla \, . \, \vec{F}(x, y, z) = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) . \left(F_1\hat{i} + F_2\hat{j} + F_3\hat{k}\right)$$

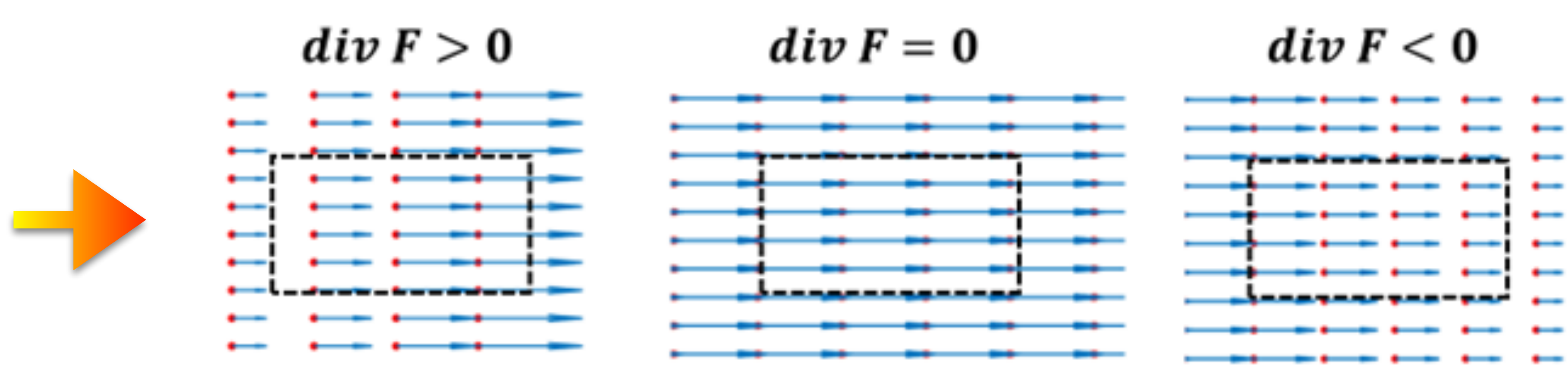
$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Physical significance:

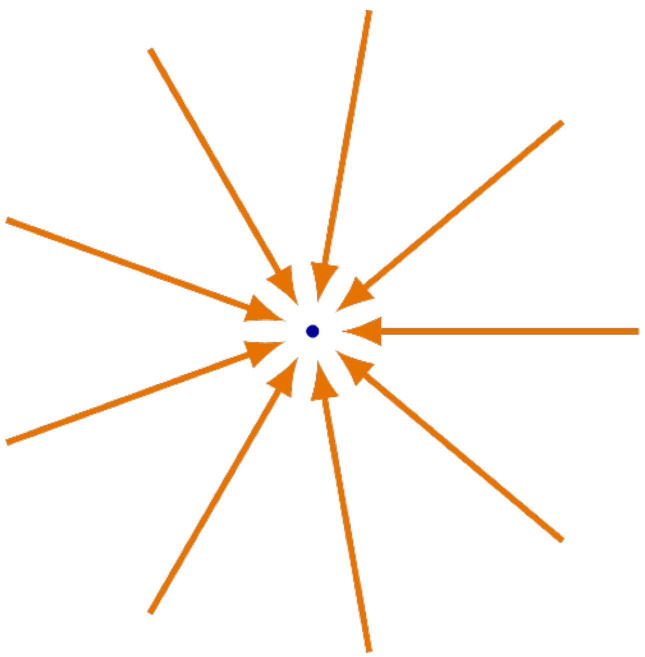
- It is a operation performed on a vector, results into a scalar
- It tells, how much flux is entering/leaving per unit volume



water flow rate

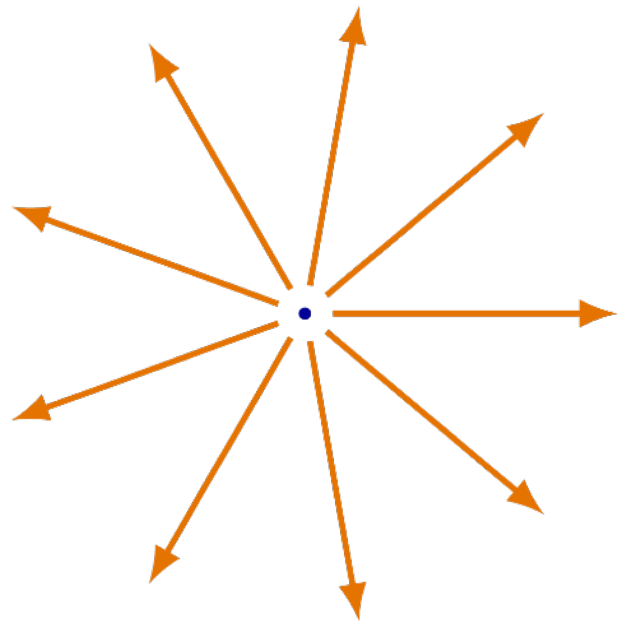


Divergence of a Vector Field ($\vec{\nabla}.\vec{F}$)



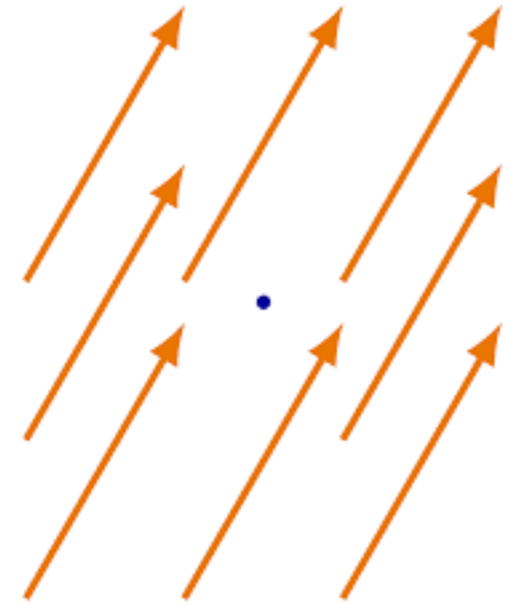
$$\nabla\,.\,\vec{V} < 0$$

Sink



$$\nabla\,.\,\vec{V} > 0$$

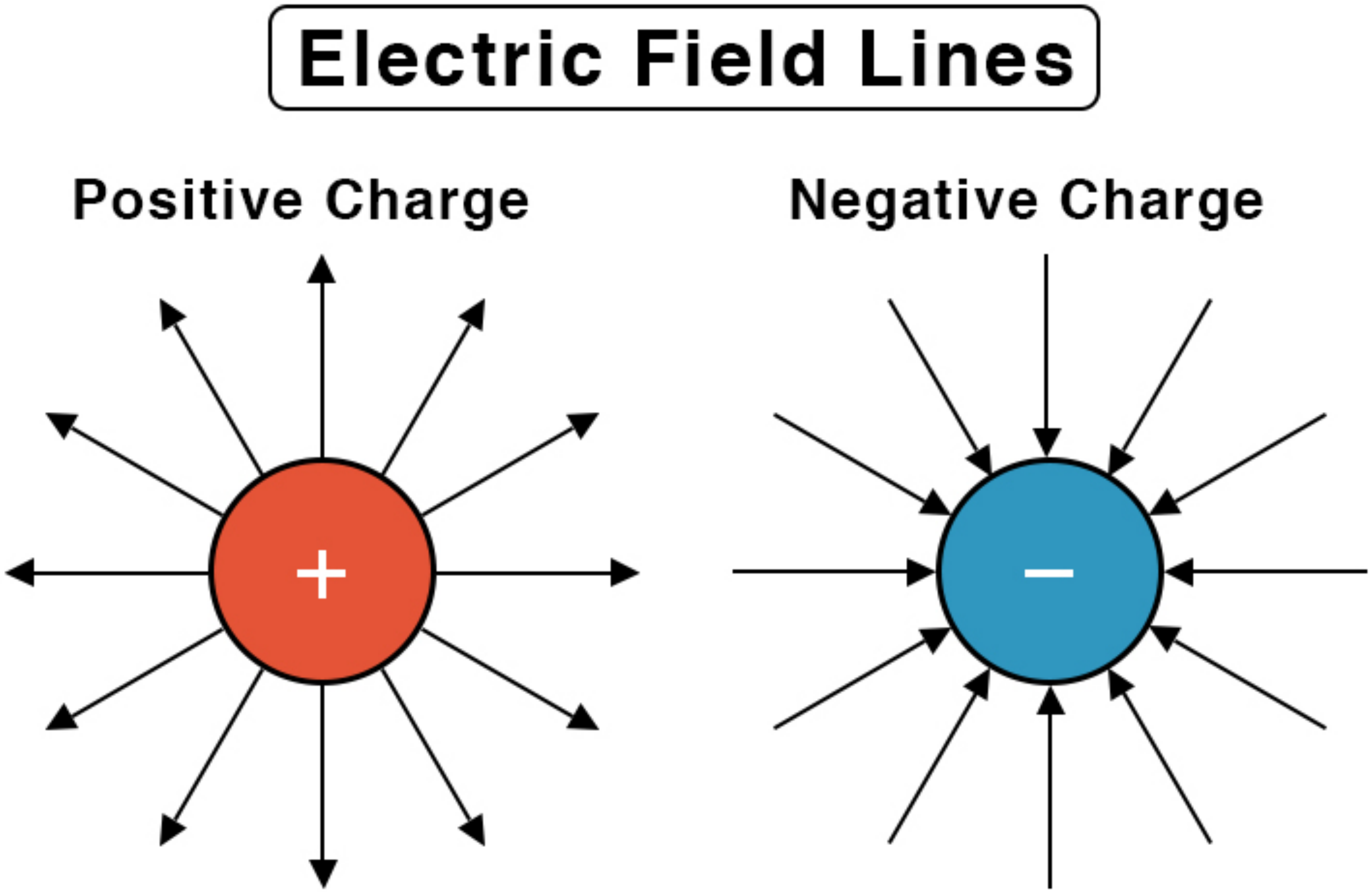
Source



$$\nabla\,.\,\vec{V} = 0$$

neither Source
nor
Sink

Divergence of a Vector Field ($\vec{\nabla}.\vec{F}$)



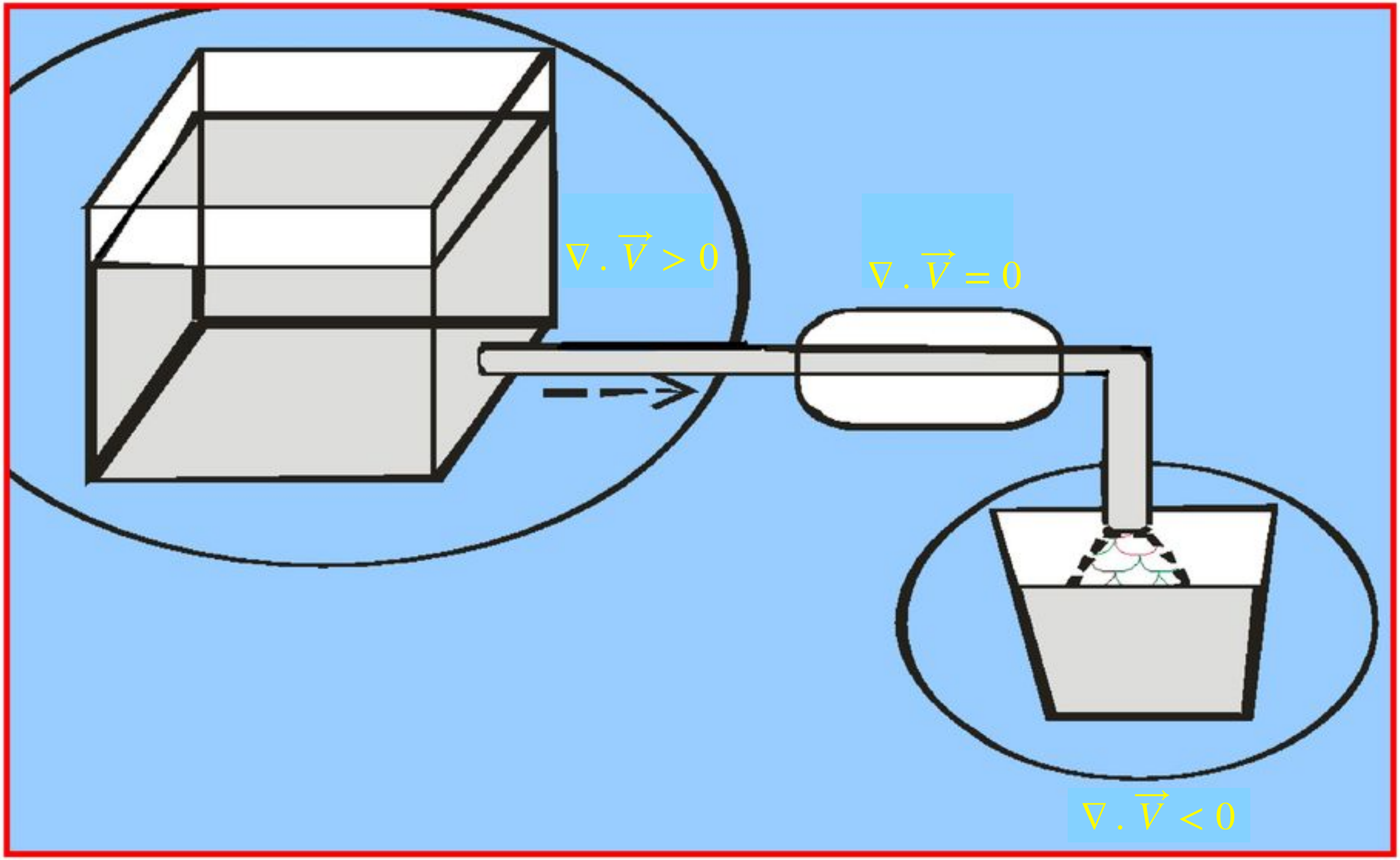
$\nabla . \vec{V} > 0$

Source

$\nabla . \vec{V} < 0$

Sink

Divergence of a Vector Field ($\vec{\nabla}.\vec{F}$)



Curl of a Vector Field ($\vec{\nabla} \times \vec{F}$)

- Consider the vector fields

$$\vec{\mathbf{F}}(x, y, z) = P(x, y, z)\hat{\mathbf{i}} + Q(x, y, z)\hat{\mathbf{j}} + R(x, y, z)\hat{\mathbf{k}}$$

- The curl of **F** is another vector field defined as:

$$\textbf{curl}\,\vec{\mathbf{F}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

- In terms of the differential operator ∇, the curl of **F**

$$\textbf{Curl}\,\vec{\mathbf{F}} = \nabla \times \vec{\mathbf{F}} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{\mathbf{i}} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\hat{\mathbf{j}} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{\mathbf{k}}$$

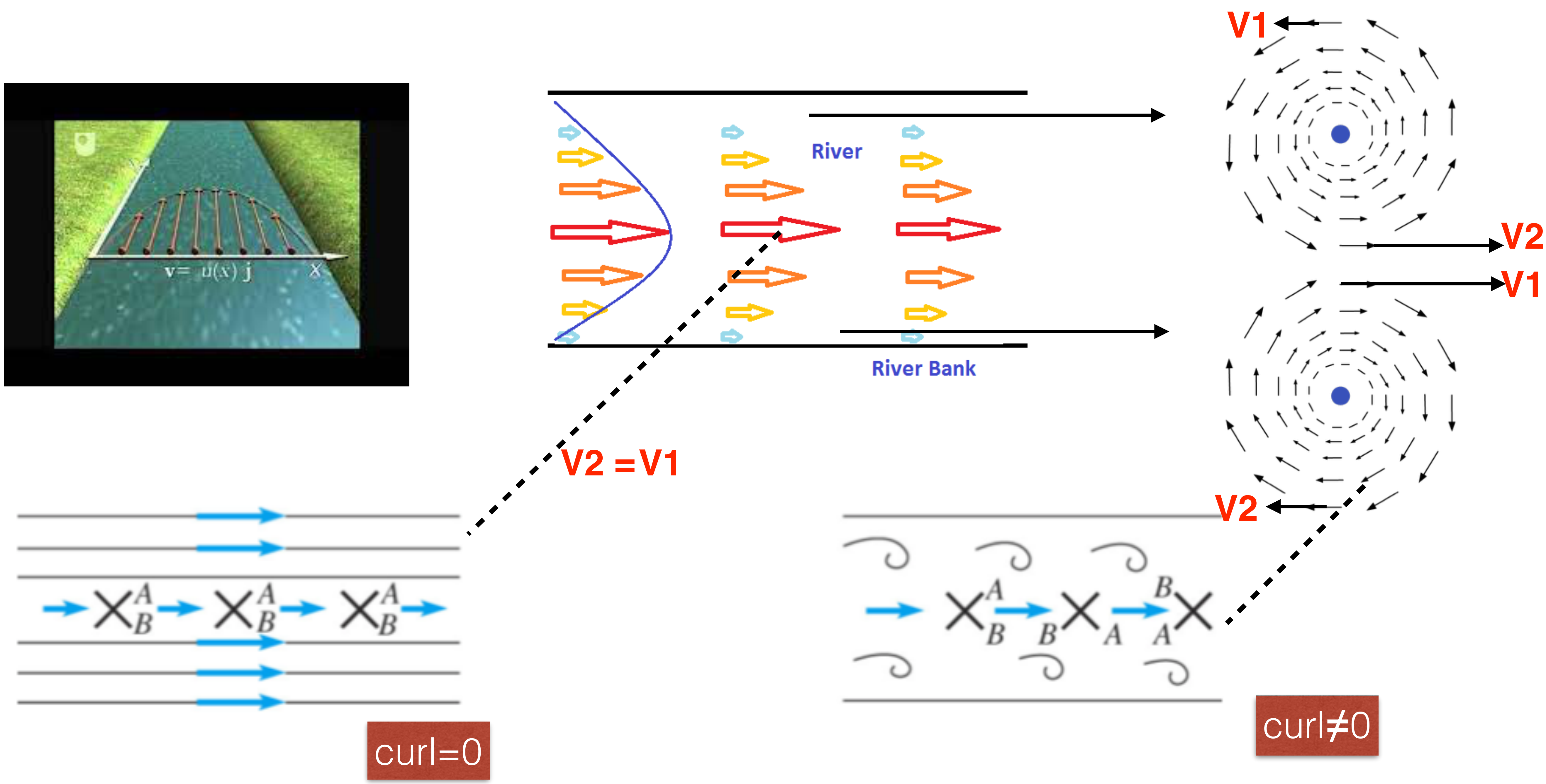
- A key point: **F** is a vector and the **curl** of **F** is a **vector**.

Direction of curl is perpendicular to the plane of circulation

Curl of a Vector Field ($\vec{\nabla} \times \vec{F}$)

Physical significance:

Curl is a measure of how much a vector field circulates or rotates about a given point



(a) Irrotational flow

(b) Rotational flow

Curl of a Vector Field ($\vec{\nabla} \times \vec{F}$)

Find a **curl** of the vector field $\vec{a}(M) = 3yz\vec{i} + 2xz\vec{j} + 5xy\vec{k}$.

$$\text{curl}\,\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3yz & 2xz & 5xy \end{vmatrix}$$

$$\Rightarrow \; = \vec{i}\cdot\left(\frac{\partial(5xy)}{\partial y} - \frac{\partial(2xz)}{\partial z}\right) - \vec{j}\cdot\left(\frac{\partial(5xy)}{\partial x} - \frac{\partial(3yz)}{\partial z}\right) + \vec{k}\cdot\left(\frac{\partial(2xz)}{\partial x} - \frac{\partial(3yz)}{\partial y}\right)$$

$$\Rightarrow \; = \vec{i}\cdot(5x - 2x) - \vec{j}\cdot(5y - 3y) + \vec{k}\cdot(2z - 3z)$$

$$\Rightarrow \; = 3x\vec{i} - 2y\vec{j} - z\vec{k}$$

To watch curl and understand it

https://www.youtube.com/watch?v=vvzTEbp9Irc&t=14s

Further Properties of $\vec{\nabla}$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{F}\right)=0$$

Divergence of Curl is Zero

$$\vec{\nabla}\times\left(\vec{\nabla}\phi\right)=0$$

Curl of Gradient is Zero

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right)=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{F}\right)-\vec{\nabla}^2\vec{F}$$

Curl of Curl is non zero

Exercise to Practice:

Exercise: Compute gradient $\vec{\nabla}\phi$ of the following scalar fields (i) $\phi(x,y,z)=xyz$, (ii) $\phi(x,y,z)=x+yz$, (iii) $\phi(x,y,z)=x^2+y^2z$

Exercise: Compute divergence $\vec{\nabla}\cdot\vec{F}$ of the following vector fields: (i) $\vec{F}(x,y,z)=x\hat{i}+y\hat{j}+z\hat{k}$, (ii) $\vec{F}(x,y,z)=x^2\hat{i}+yz\hat{j}$, (iii) $\vec{F}(x,y,z)=x^3\hat{i}+y^2\hat{j}+z\hat{k}$.

Exercise: Compute curl, $\vec{\nabla}\times\vec{F}$ of the following vector fields: (i) $\vec{F}(x,y,z)=x\hat{i}+y\hat{j}+z\hat{k}$, (ii) $\vec{F}(x,y,z)=x^2\hat{i}+yz\hat{j}$, (iii) $\vec{F}(x,y,z)=x^3\hat{i}+y^2\hat{j}+z\hat{k}$.

Exercise: Show that $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right)=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{F}\right)-\vec{\nabla}^2\vec{F}$ for the following vector fields: (i) $\vec{F}(x,y,z)=x\hat{i}+y\hat{j}+z\hat{k}$, (ii) $\vec{F}(x,y,z)=x^2\hat{i}+yz\hat{j}$.

Vector Field Function

For a Vector Field Function $\overrightarrow{F}(x, y, z)$

1.Irrotational or Conservative

$$\overrightarrow{\nabla} \times \overrightarrow{F} = 0$$

Curl of that function is Zero

2. Incompressible

$$\overrightarrow{\nabla} . \overrightarrow{F} = 0$$

Divergence of that function is Zero

Numerical to Solve at Home

1. Compute the Divergence and Curl for a vector field

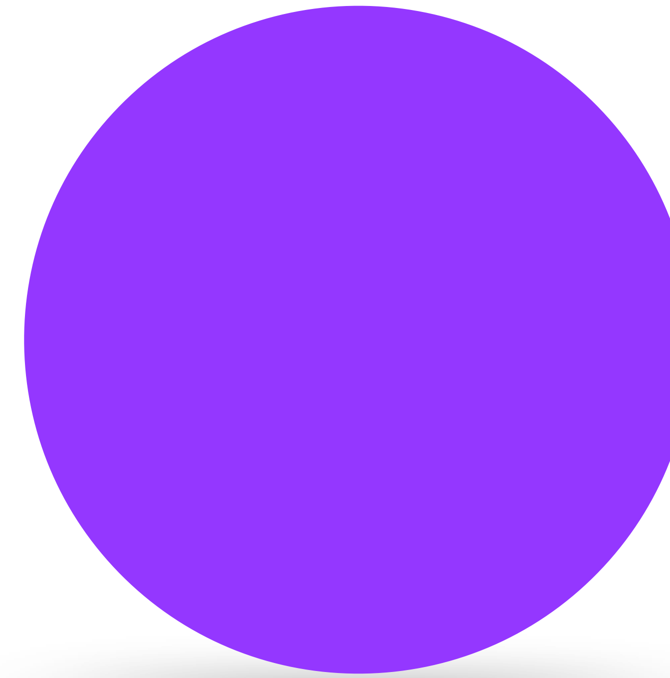
$$\overrightarrow{F} = x^2 y\hat{i} - (z^3 - 3x)\hat{j} + 4y^2\hat{k}$$

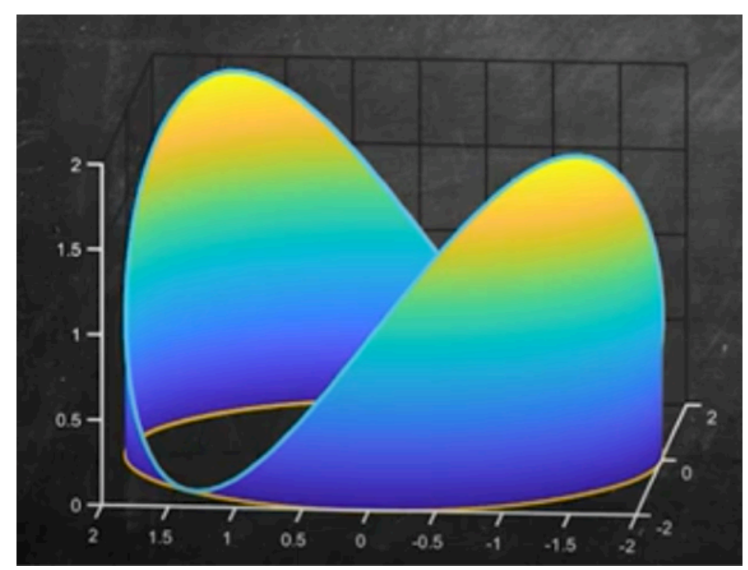
2. For the above vector field, $\overrightarrow{F}$, the curl of the gradient is zero, i.e $\nabla \times (\nabla \overrightarrow{F}) = 0$

3. Prove that the following vector field $\overrightarrow{F}$ is conservative:

$$\overrightarrow{F} = \left(4y^2 + \frac{3x^2y}{z^2}\right)\hat{i} + \left(8xy + \frac{x^3}{z^2}\right)\hat{j} + \left(11 - \frac{2x^3y}{z^3}\right)\hat{k}$$

Area of simple shape



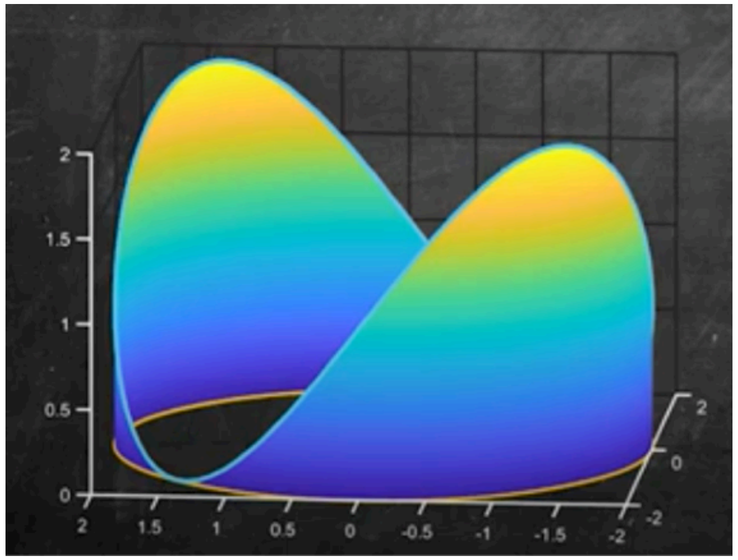


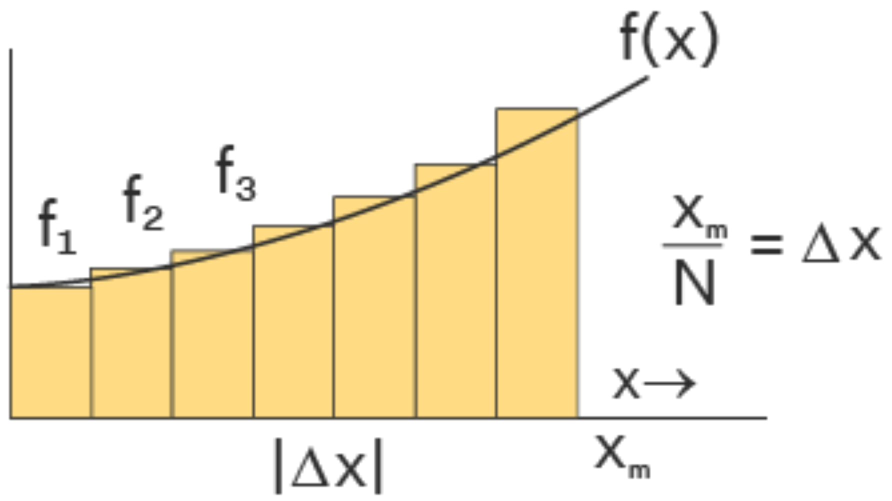
?

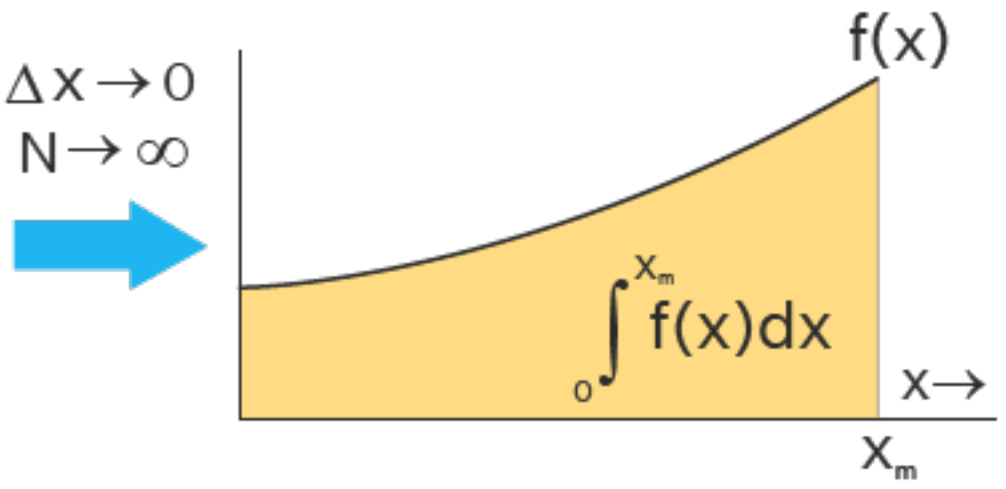
Integration

For a function, f(x), the integration can be defined for a point a—to—b is

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$







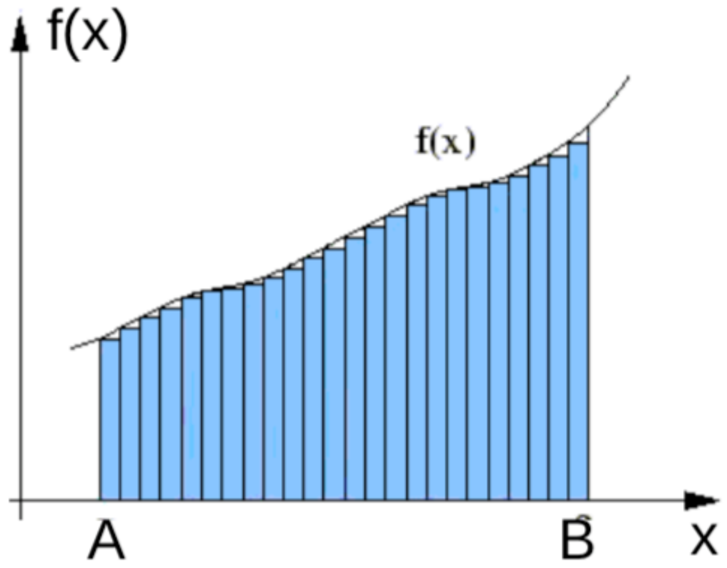
$$\text{Area} = \int_0^{X_m} f(x)dx = \lim_{\Delta x\to 0}\sum_{i=1}^{N} f_i(x)\Delta x$$

The integration denotes the summation of discrete data

Line, Surface, and Volume Integral

Function of single variable

$$\int_{x=A}^{x=B} f(x)\,dx = \lim_{\delta x \to \infty} \sum_{x=A}^{x=B} f(x)\,\delta x$$



For function of multiple independent variables (scalar and vector fields)

1. **Line integral**
2. **Surface integral**
3. **Volume integral**

Line Integral

A line integral is integral in which the function to be integrated is determined along a curve in the coordinate system. The function which is to be integrated may be either a scalar field or a vector field. We can integrate a scalar-valued function or vector-valued function along a curve. The value of the line integral can be evaluated by adding all the values of points on the vector field.

Consider a scalar function $\phi(x, y)$. In this case, the line-integral is defined as

$$\int_A^B \phi(x, y) dr$$

where dr is the infinitesimal line segment along a specific path between A and B

If the path is closed, in which case $A = B$, we write the integral as

$$\oint_C \phi(x, y) dr$$

Line Integral

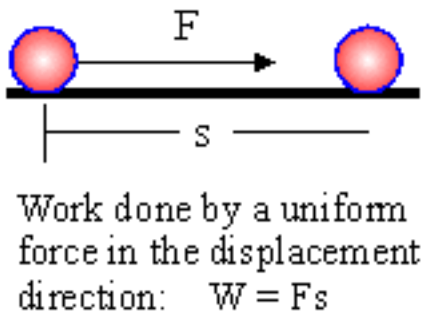
If the function is a vector the line-integral then defined as

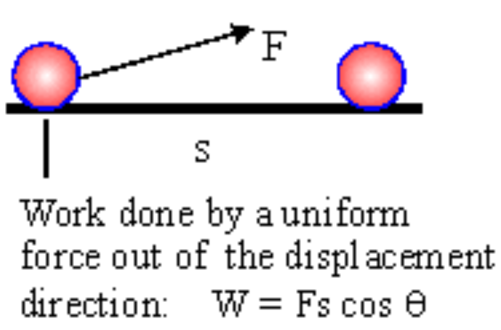
$$\int_A^B \vec{F} \cdot d\vec{r}$$

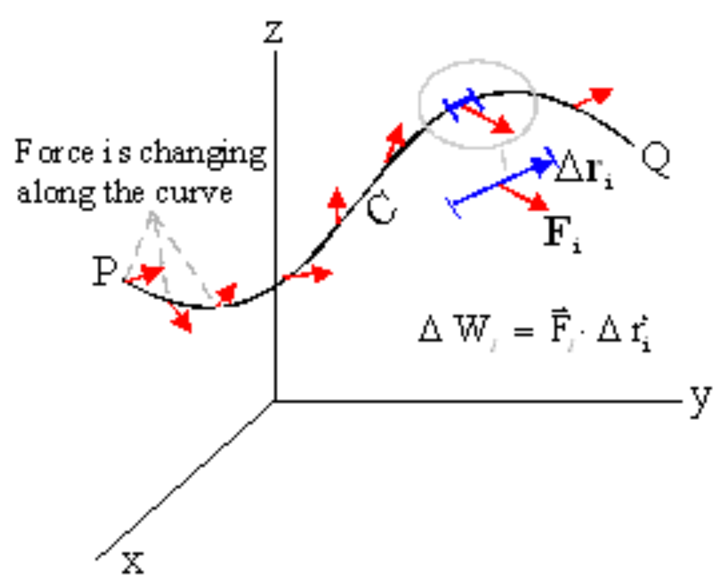
Note that this time angle between the line segment dr and the vector filed F becomes relevant. If the path is closed, like before, A = B, we write the integral is written as:

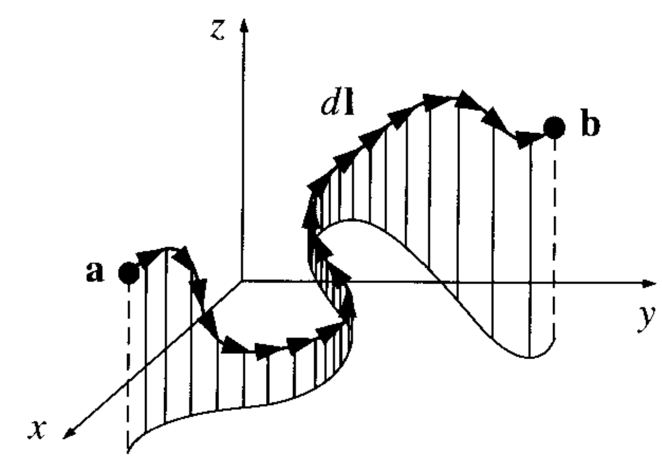
$$\oint_C \vec{F} \cdot d\vec{r}$$

Suppose F = F (x, y, z) is the force acting on a particle which moves along the curve C given by r = [x (t), y (t), z (t)], and with its initial point at P and terminal point Q: What is the work done in moving the particle form P to Q



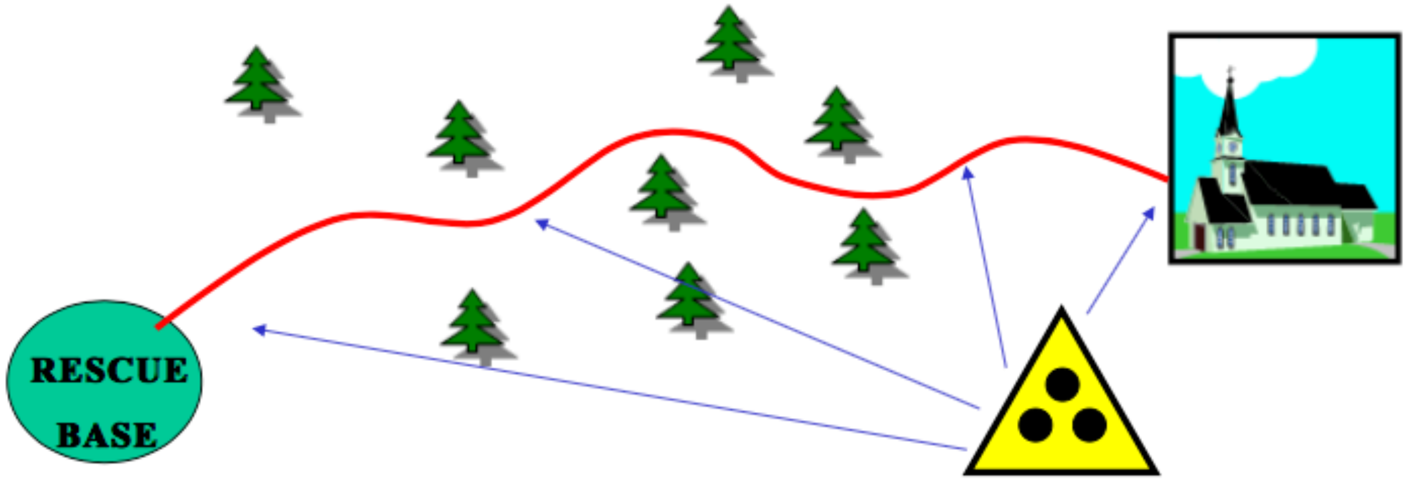


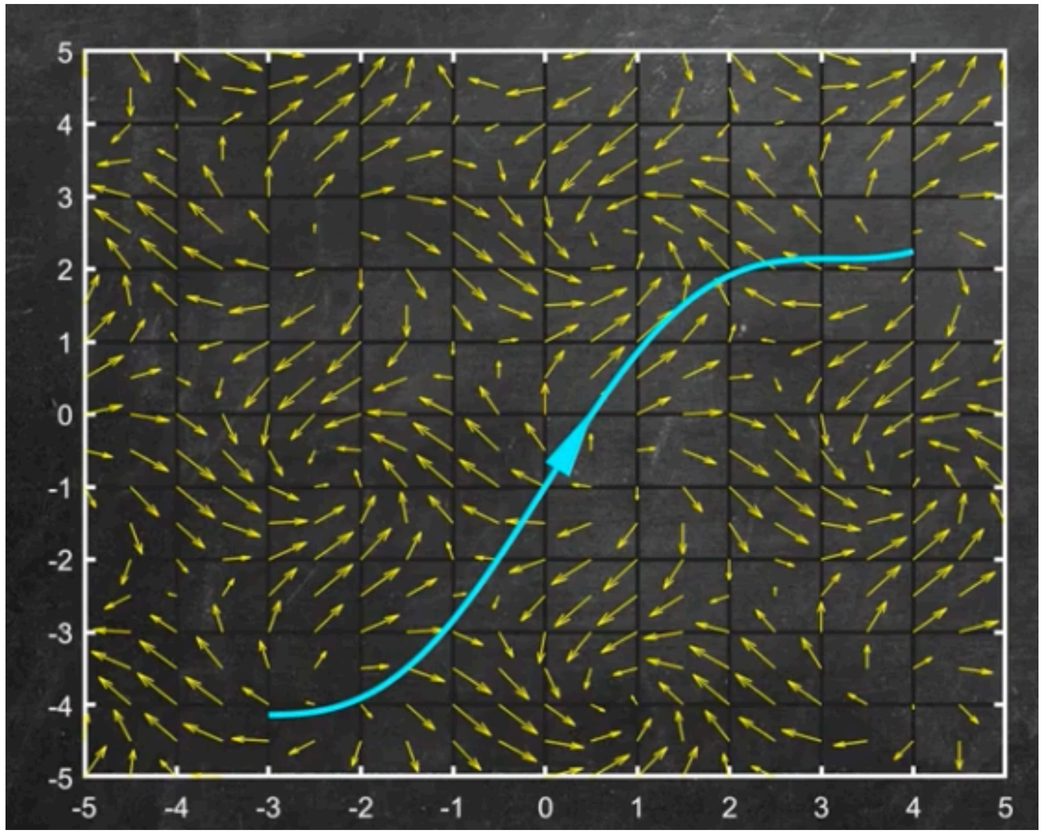




Line Integral

A rescue team follows a path in a danger area where for each position the degree of radiation is defined. Compute the total amount of radiation gathered by the rescue team along the path.





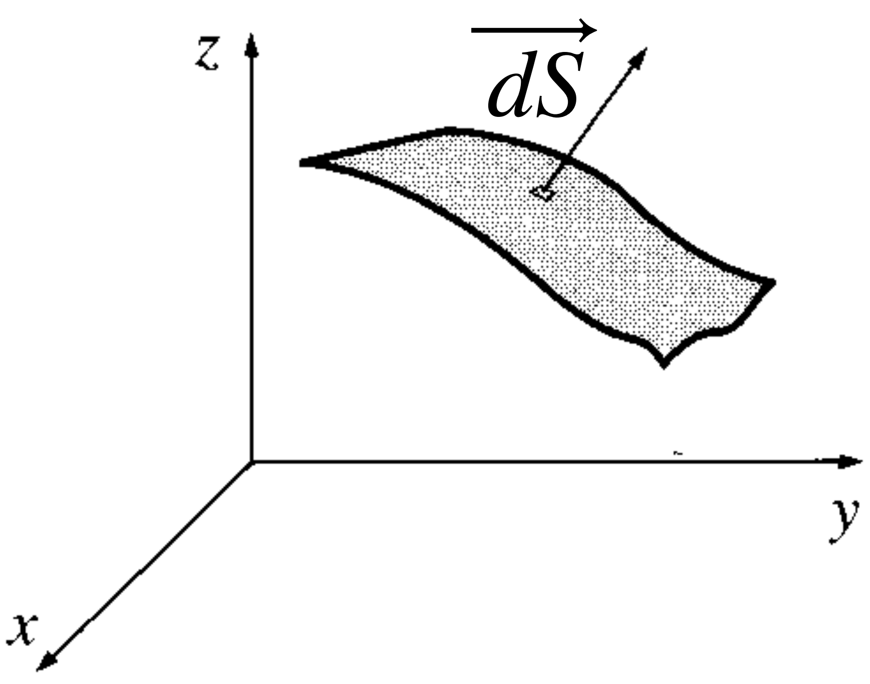
charge particle in a electric filed

Surface Integral

Surface integral of a vector field $\vec{F}$ (x, y, z) along the surface S is defined as

$$\iint_S \vec{F}(x,y,z)\cdot\vec{dS} = \iint_S \left(\vec{F}\cdot\hat{n}\right) dS$$

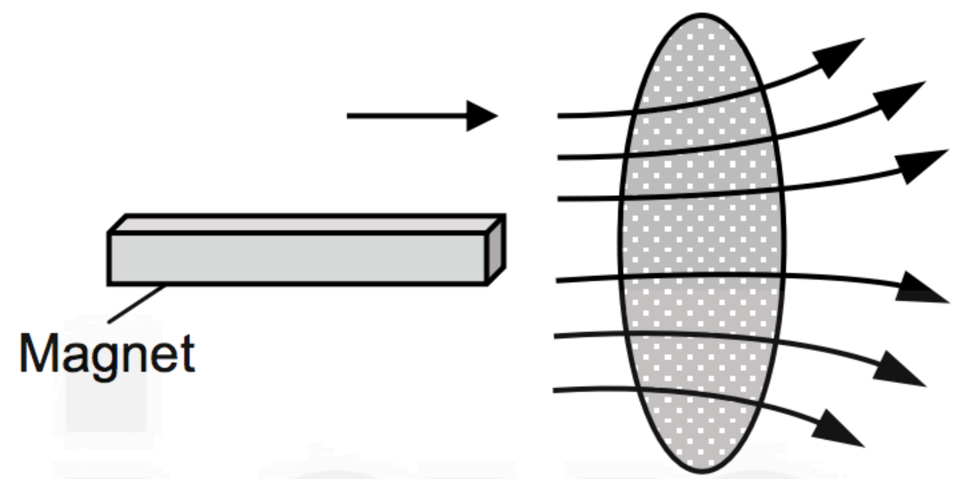
where $\overrightarrow{dS}$ is an infinitesimal vector perpendicular to the surface element and n̂ is a unit vector in the direction of $\overrightarrow{dS}$.



Surface Integral

Surface integral of a vector field $\vec{F}$ (x, y, z) along the surface S, and the **surface is closed**, then it is defined as

$$\oiint \vec{F}(x,y) \cdot d\vec{S}$$

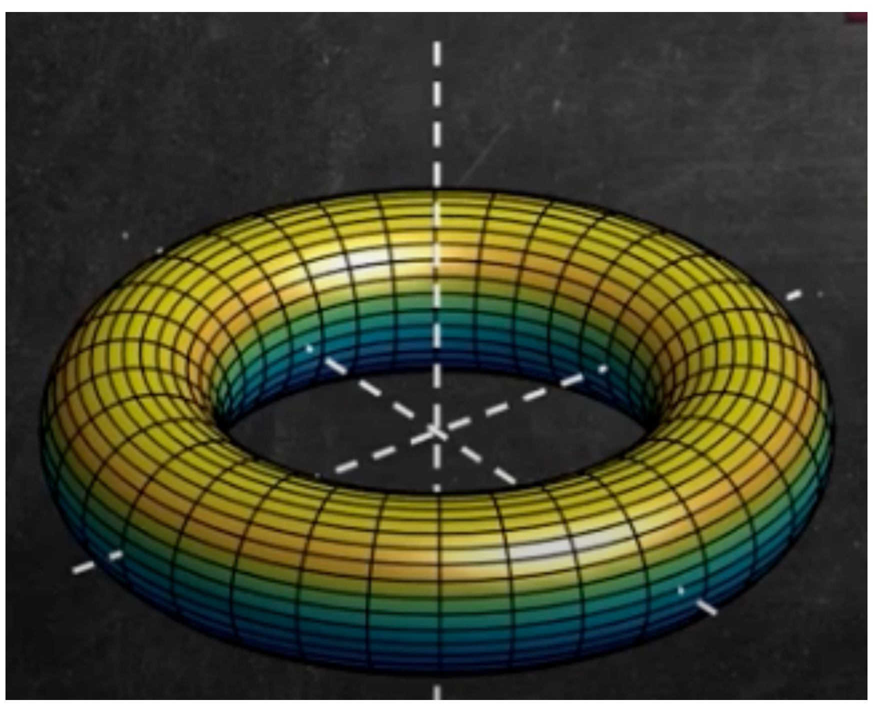


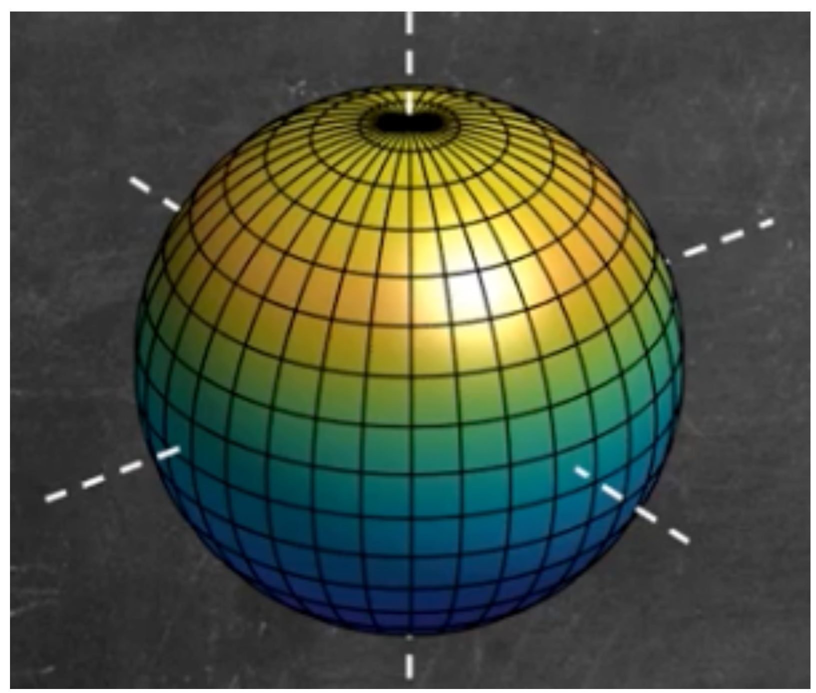
To determine the magnetic flux, we have to integrate the magnetic field vector, $\overrightarrow{B}$ **over the area enclosed by the coil.** $\phi_B = \int \overrightarrow{B} \cdot \overrightarrow{dS}$ **represents the magnetic flux passing through the enclosed surface.**

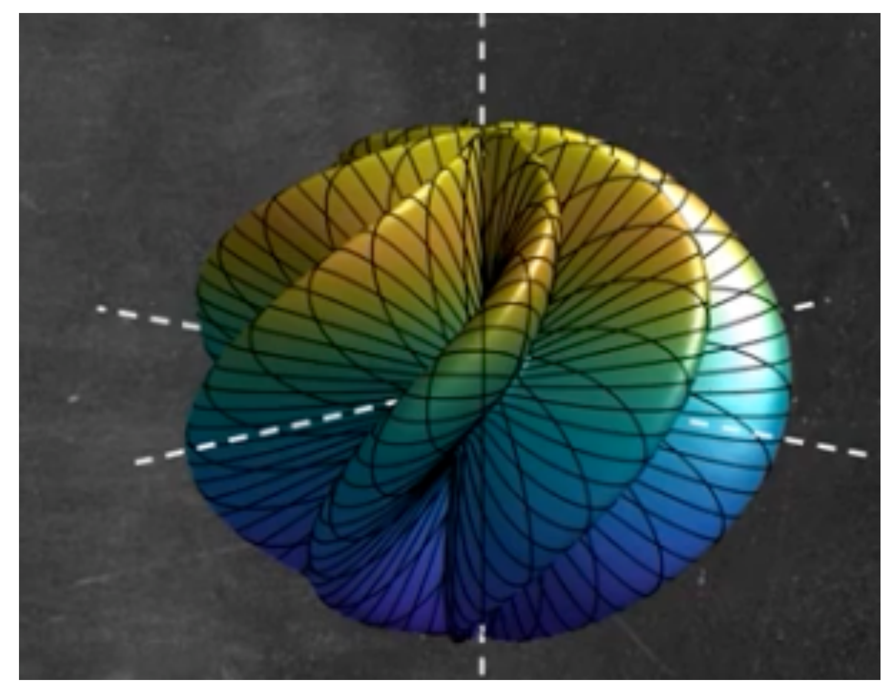
Surface Integral

Surface integral of a vector field $\vec{F}(x, y, z)$ along the surface S, and the **surface is closed**, then it is defined as

$$\oiint \vec{F}(x, y) \cdot d\vec{S}$$



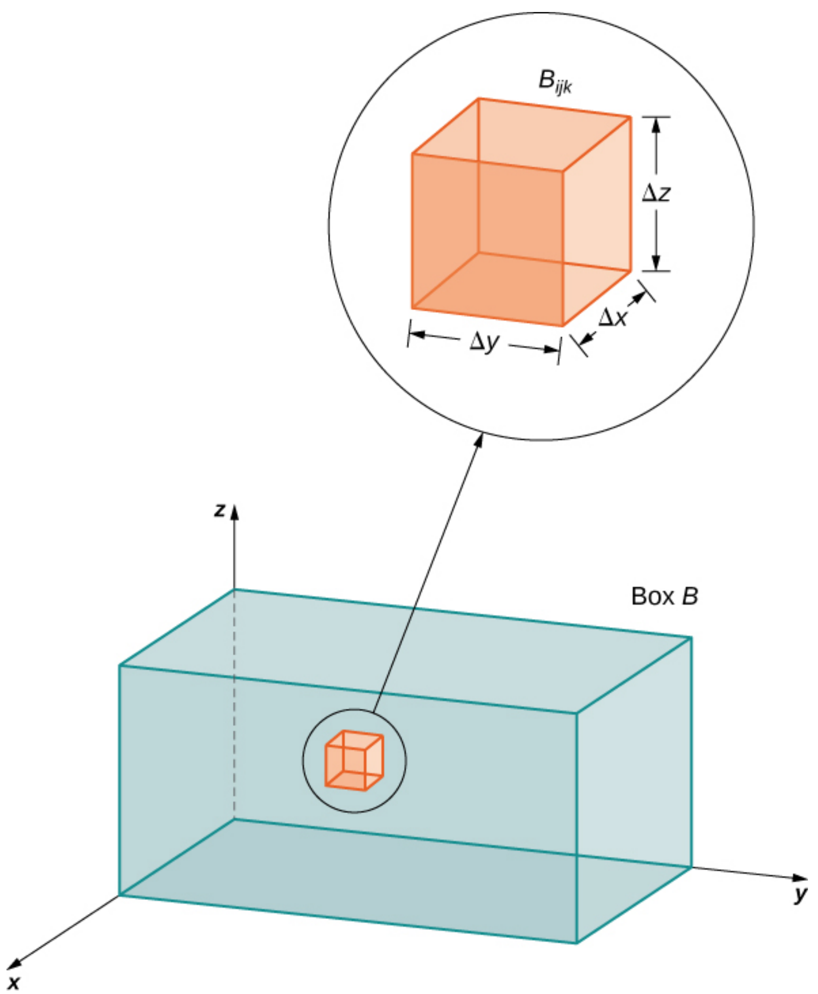


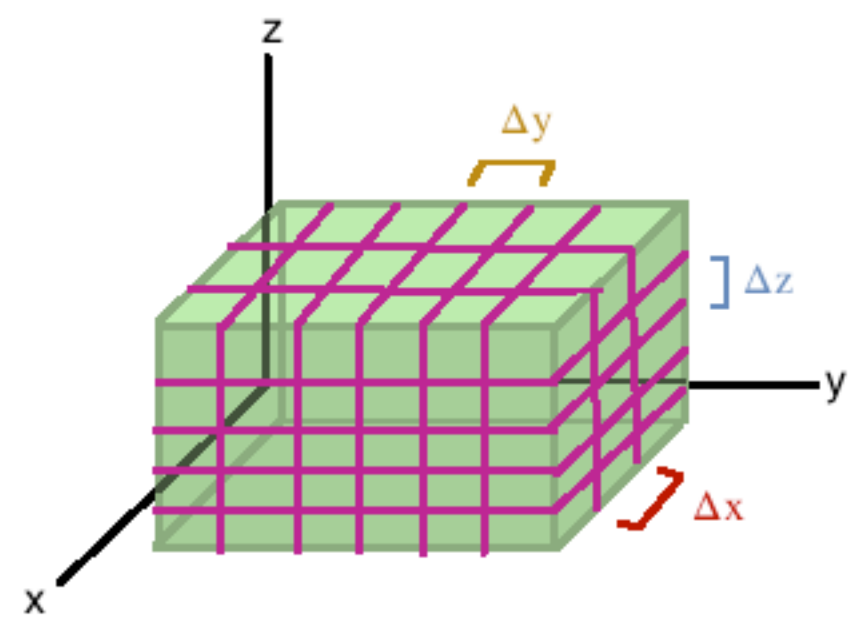


Volume Integral

Volume integral of a scalar field f(x,y) within a region V is written as

$$\iiint_{\mathcal{V}} f(x,y,z)dV = \iiint_{\mathcal{V}} f(x,y,z)dxdydz$$





Important Integrals in E-M Theory

1. Line Integral

$$\varepsilon = \oint_L \overrightarrow{\mathrm{E}} \cdot \overrightarrow{dl}$$

2. Surface Integral

$$I = \oint_S \vec{\mathrm{J}} \cdot \overrightarrow{dS}$$

E= Electric Field
J= Current Density
ρ= Charge Density
I= Current
Ɛ= Potential
Q_{in}= Total charge enclosed

3. Volume Integral

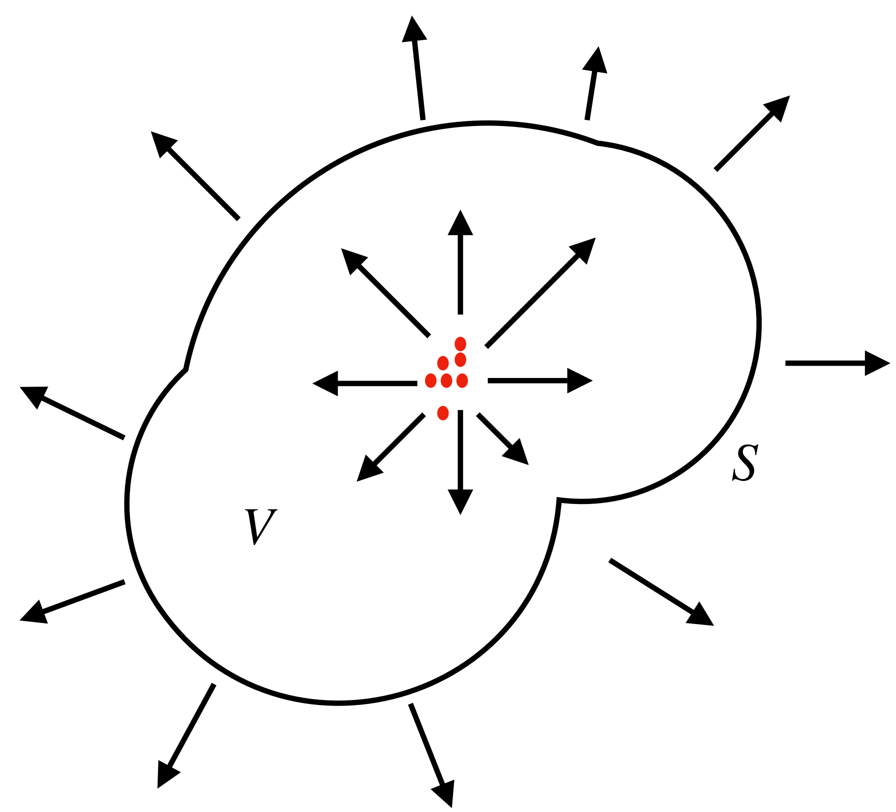
$$Q_{in} = \int_V \rho \cdot dV$$

Gauss-Divergence Theorem

The normal surface integral of a vector function $\overrightarrow{F}(x, y, z)$ over the boundary of a closed region is equal to the volume integral of the divergence of the vector function, $\overrightarrow{\nabla} . \overrightarrow{F}(x, y, z)$, taken throughout that region

$$\oint_S \overrightarrow{\mathrm{F}} \cdot \overrightarrow{dS} = \int_{\mathrm{V}} (\overrightarrow{\nabla} \cdot \overrightarrow{\mathrm{F}}) dV$$

Total outward flux of $\overrightarrow{\mathrm{F}}$ through a closed surface S is equal to the volume integral of the divergence of $\overrightarrow{\mathrm{F}}$ over volume V enclosed by S.



Gauss-Divergence Theorem

$$\oint_S \vec{F} \cdot \vec{dS}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{dS} = dS_x\hat{i} + dS_y\hat{j} + dS_z\hat{k}$$

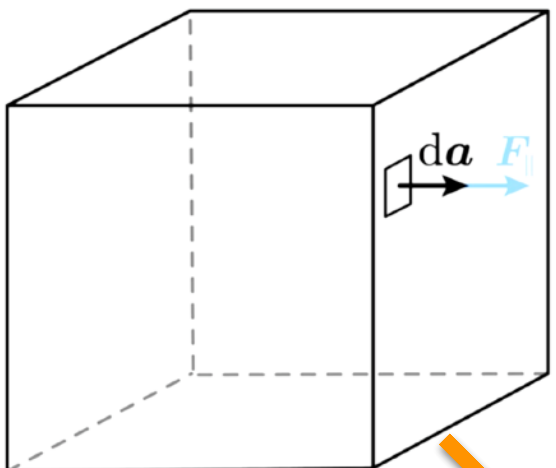
$$\vec{F} \cdot \vec{dS} = F_x dS_x + F_y dS_y + F_z dS_z$$
$$= Flux = \Phi$$

$$\int_V (\vec{\nabla} \cdot \vec{F}) dV$$

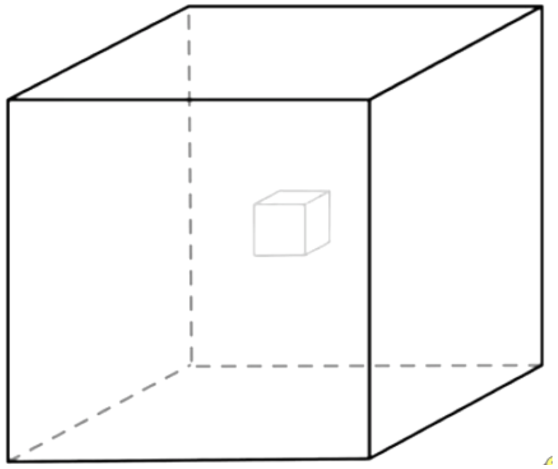
$$\vec{\nabla} \equiv \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$$

$$dV = dxdydz$$

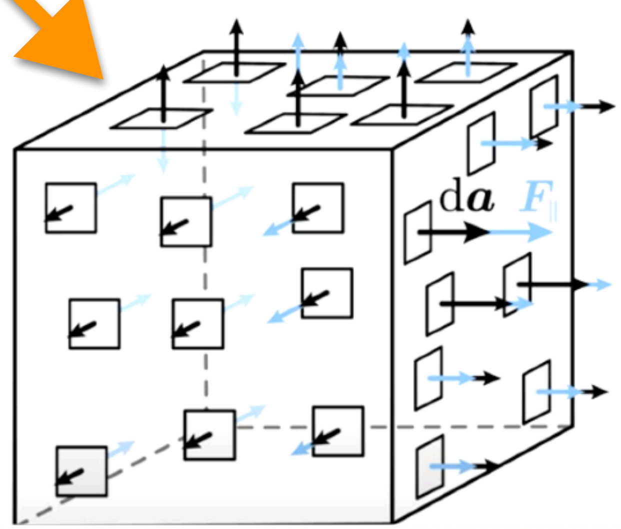
$$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$



The surface integral of a vector field over a closed surface is equal to the volume integral of the divergence over the region inside the surface.



$$\oint_S \vec{F} \cdot \vec{dS} =$$



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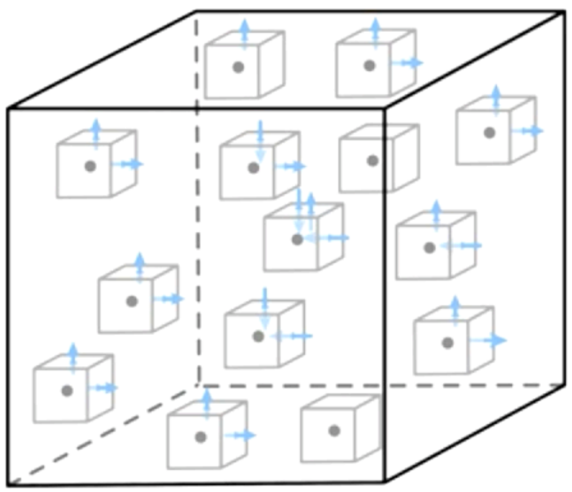


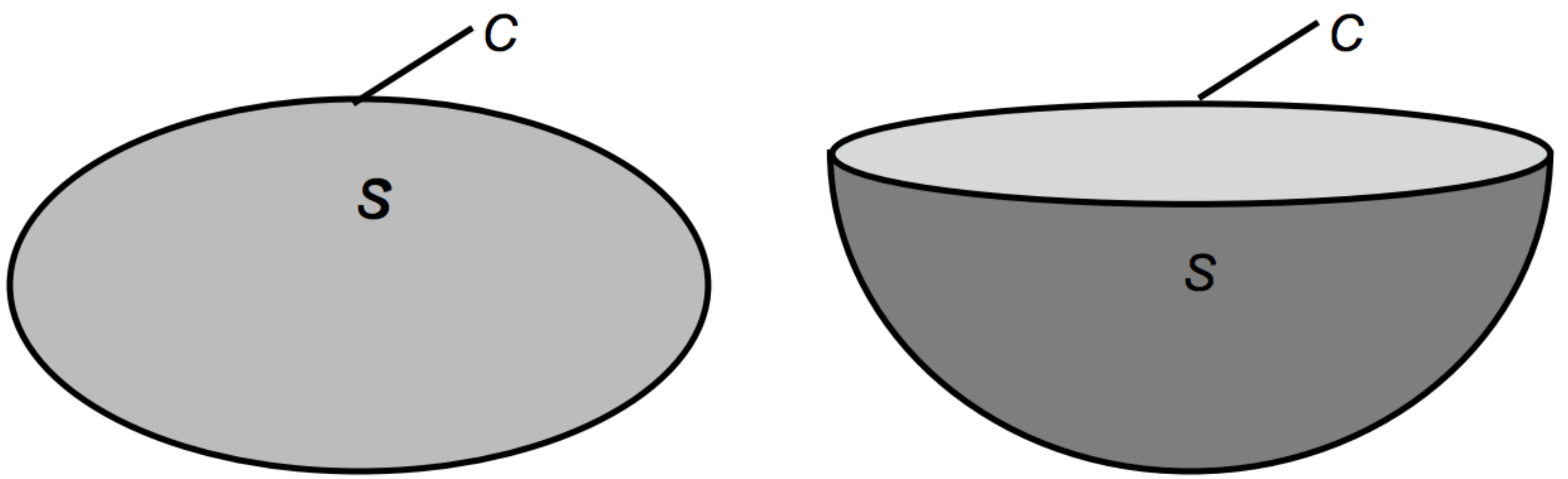
$$= \int_V (\vec{\nabla} \cdot \vec{F}) dV$$

$$\int \text{(faucets within the volume)} = \iint \text{(flow out through the surface)}$$

Stokes Theorem

The line integral of a vector field, $\overrightarrow{\mathrm{F}}$ over a loop (loop means closed path) is equal to the flux of its curl through the enclosed surface

$$\oint_L \overrightarrow{\mathrm{F}} \cdot \overrightarrow{dl} = \int_S (\overrightarrow{\nabla} \times \overrightarrow{\mathrm{F}}) \cdot \overrightarrow{dS}$$

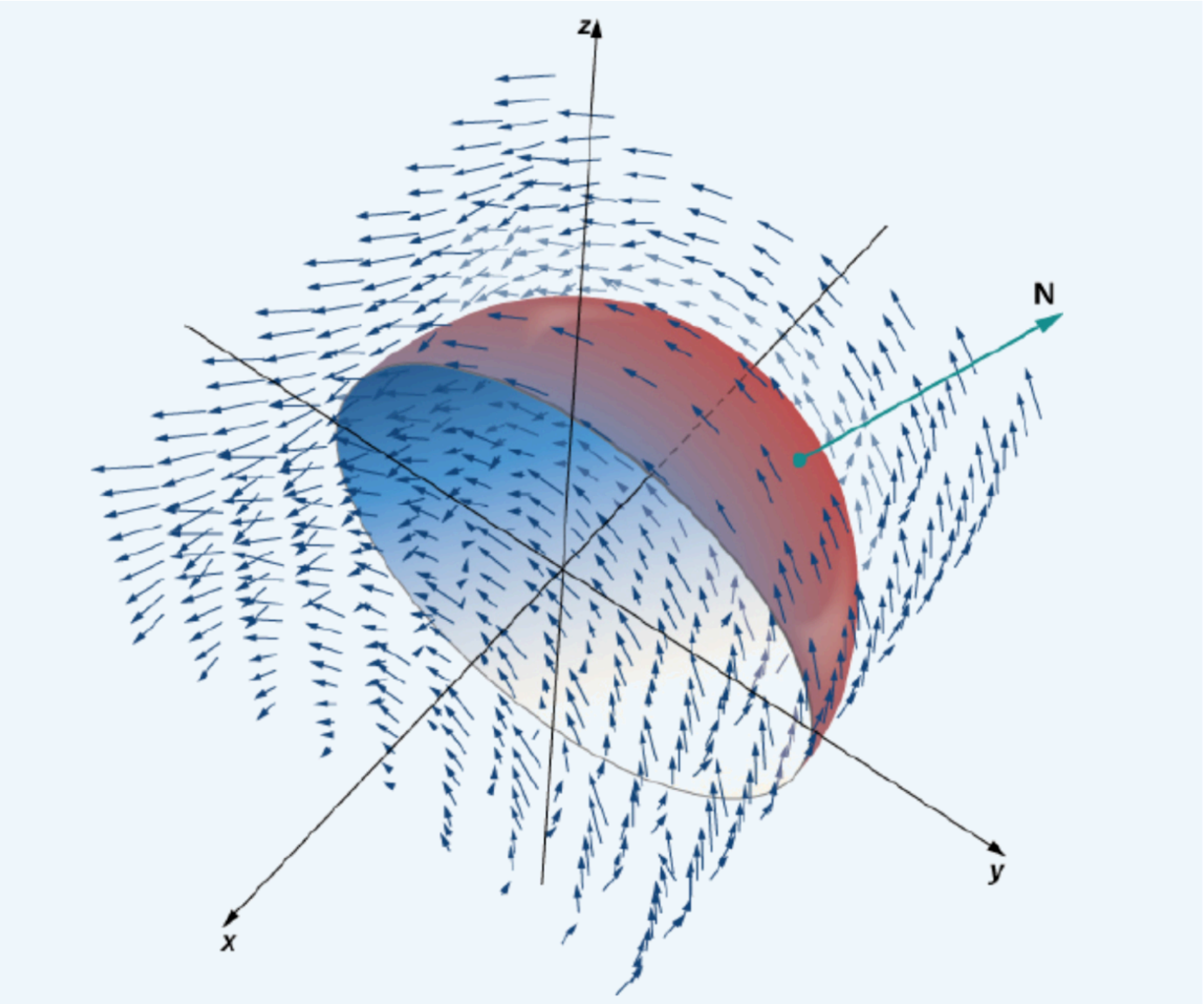


Closed path line (C) is associated with a surface S

Stokes Theorem

The line integral of a vector field, $\overrightarrow{\mathrm{F}}$ over a loop (loop means closed path) is equal to the flux of its curl through the enclosed surface

$$\oint_L \overrightarrow{\mathrm{F}} \cdot \overrightarrow{dl} = \int_S (\overrightarrow{\nabla} \times \overrightarrow{\mathrm{F}}) \cdot \overrightarrow{dS}$$



Stokes Theorem

The line integral of a vector field, $\overrightarrow{\mathrm{F}}$ over a loop (loop means closed path) is equal to the flux of its curl through the enclosed surface

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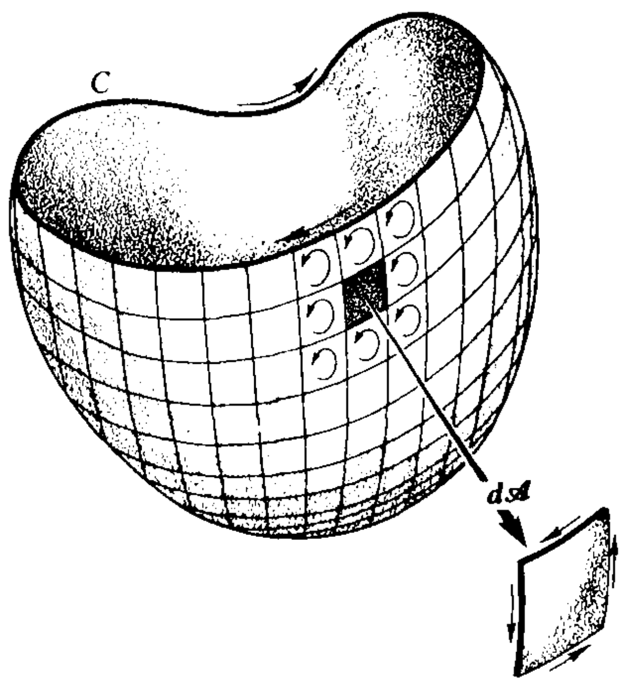
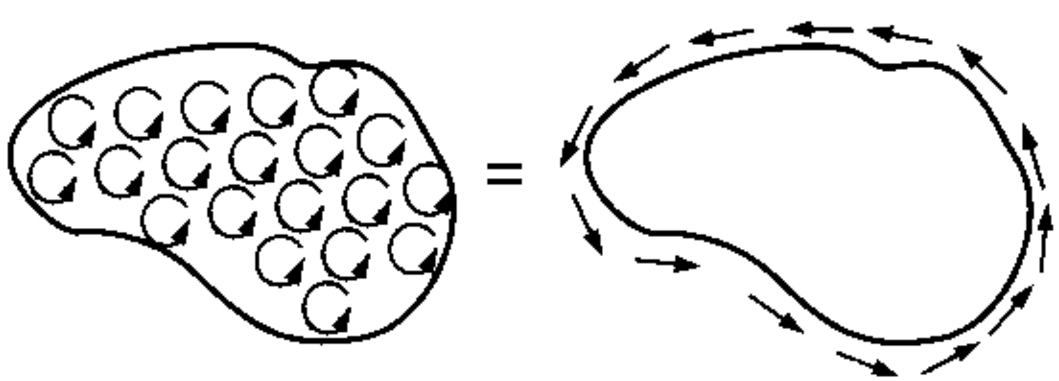




It relates the microscopic circulation of a vector field with the macroscopic circulations

Four Laws of Electricity and Magnetism

1. Gauss's law: electrostatics

2. Gauss law: magnatostatics

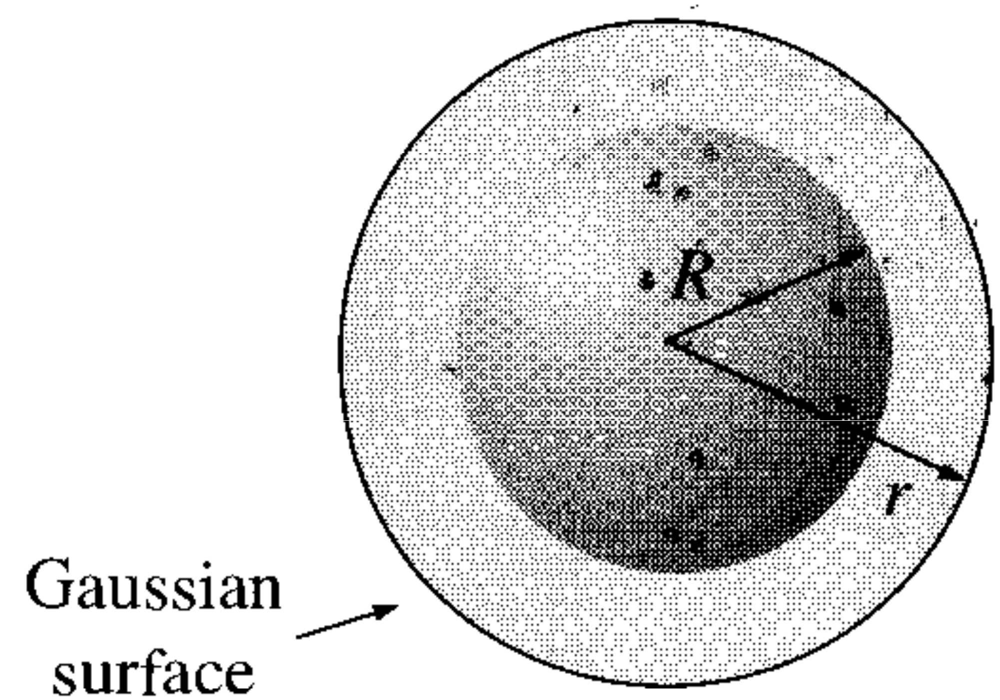
3. Faraday's law:

4. Ampere's law:

Gauss's Law

The net electric flux through a closed surface (3D) is $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface

$$\Phi_E = \oint \vec{E} \cdot \vec{da} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

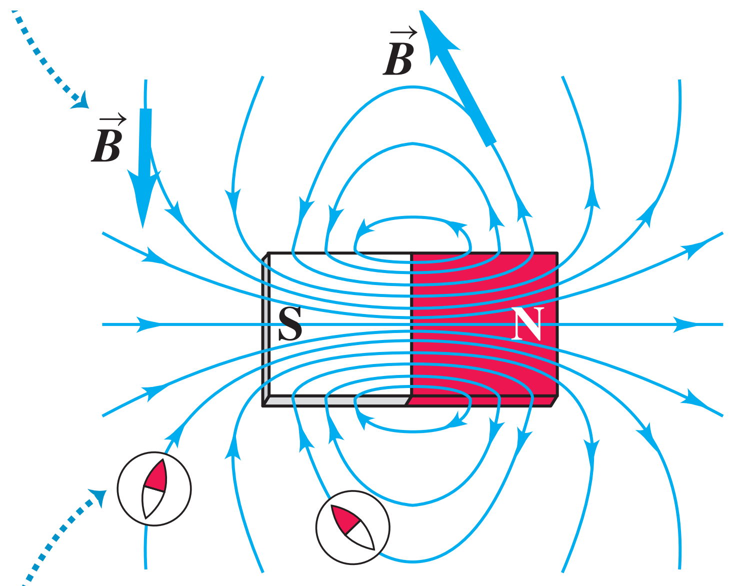


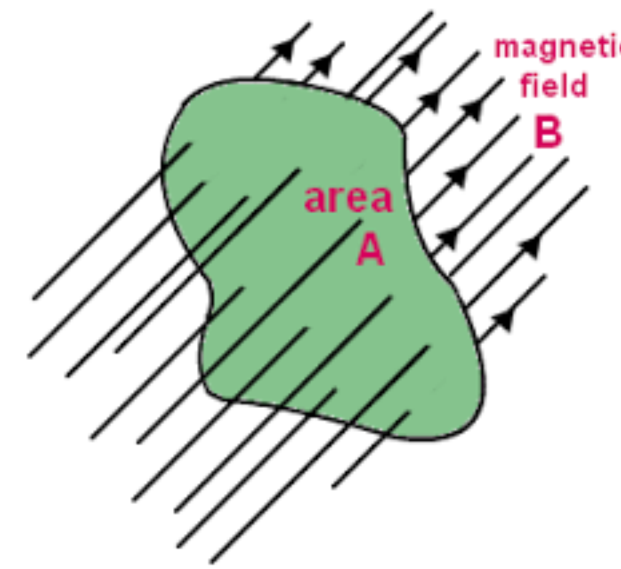
Electric flux through any closed surface is measurement of charge enclosed

Gauss's Law of magnetism

The net magnetic flux through any closed surface is Zero

$$\Phi_B = \oint \overrightarrow{B} \cdot \overrightarrow{da} = 0$$





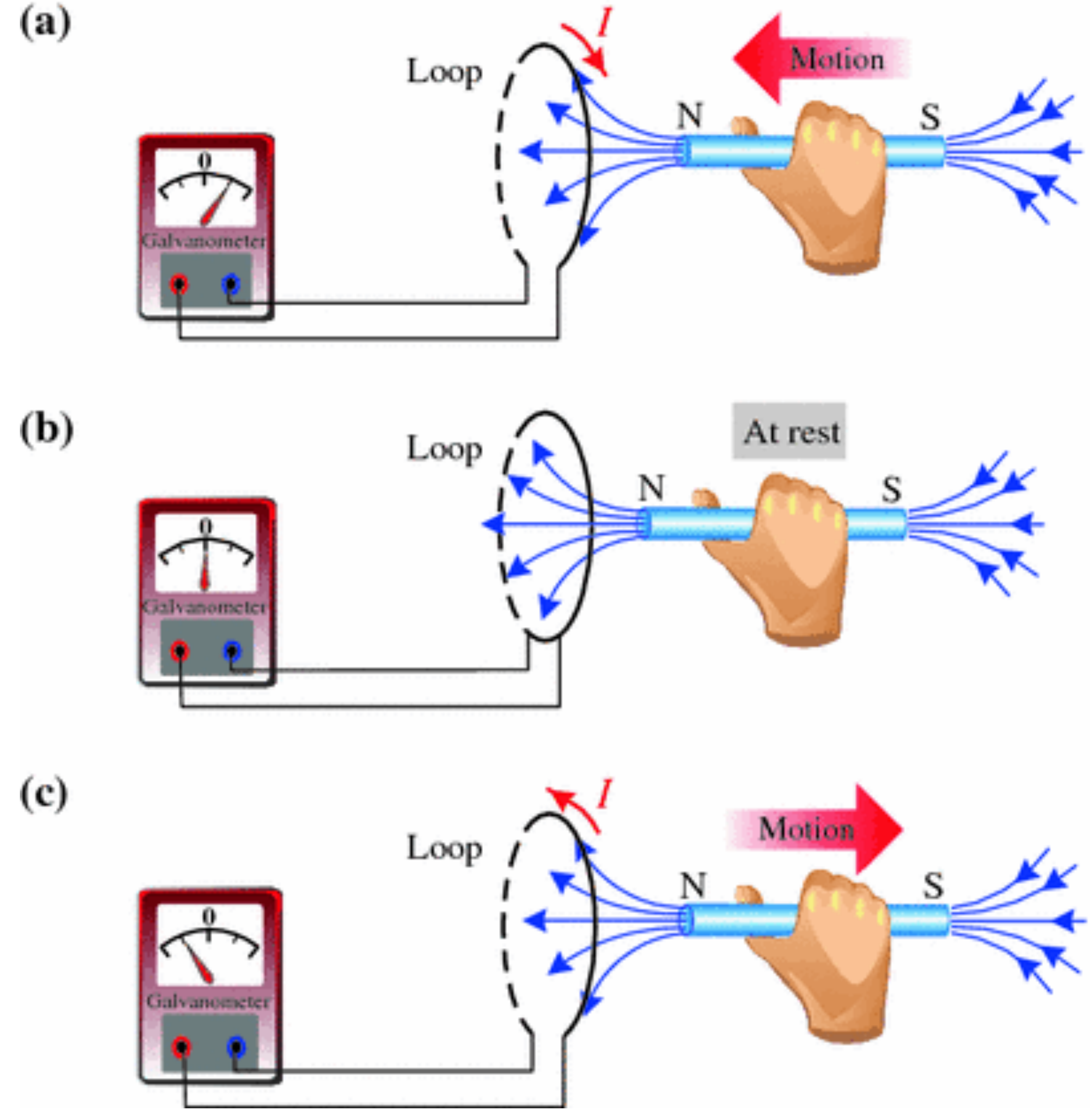
The number of magnetic field lines that enter a closed volume must equal the number that leave that volume. This implies that magnetic field lines cannot begin or end at any point. If they did, it would mean that isolated magnetic monopoles existed at those points: **Magnetic monopoles does not exist**

Faraday's Law of Induction

Whenever a conductor is placed in a varying magnetic field, an electromotive force is induced. If the conductor circuit is closed, a current is induced, which is called induced current." the induced emf in a coil is equal to the rate of change of flux.

$$\varepsilon = -\frac{\partial \Phi_B}{\partial t}$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \Phi_B}{\partial t}$$

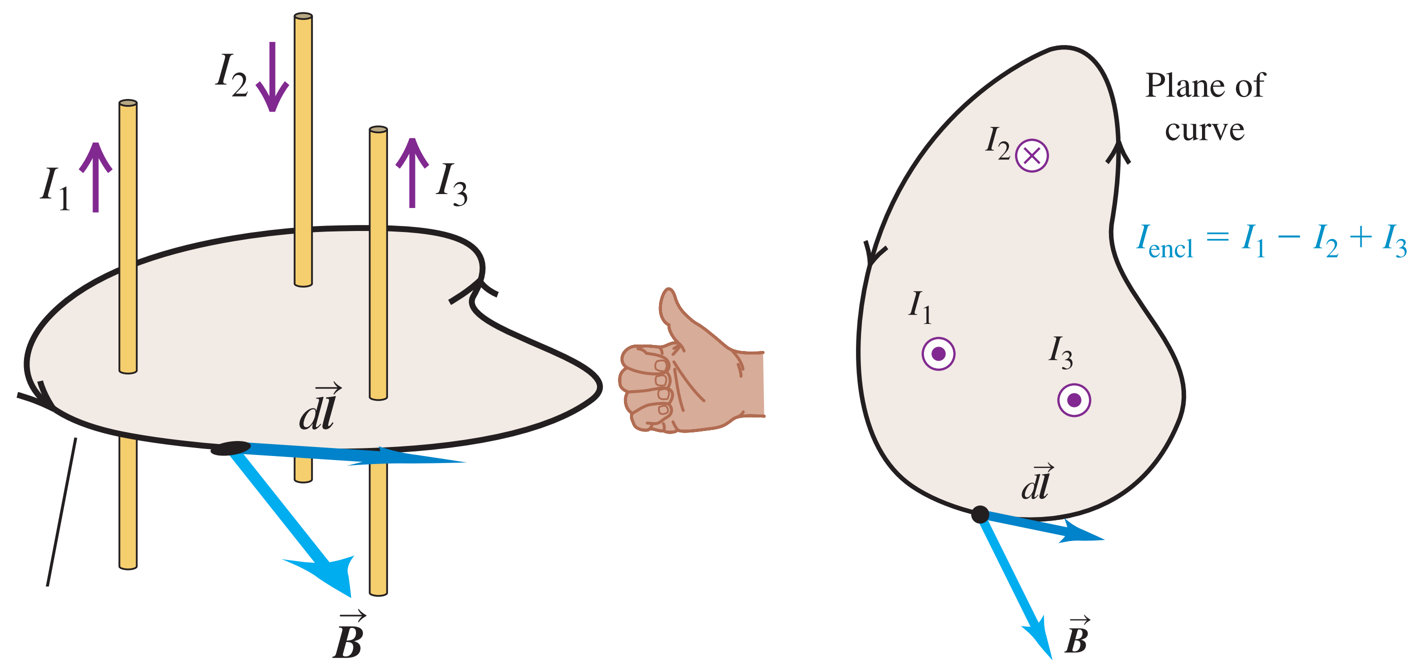


Change in Magnetic flux create an Electric field

Ampere's Law

The magnetic field in space around an electric current is proportional to the electric current which serves as its source

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$$



It relates the creation of magnetic field due to the electric current

Integral and differential forms Maxwell Equations

Law	Integral form	Theorem	Differential form
Gauss's Law	$\oint \vec{E} \cdot \vec{da} = \frac{Q_{encl}}{\epsilon_0}$	Gauss divergences theorem	$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
Gauss's Law for magnetism	$\oint \vec{B} \cdot \vec{da} = 0$	Gauss divergences theorem	$\vec{\nabla} \cdot \vec{B} = 0$
Faraday's Law	$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \Phi_B}{\partial t}$	Stoke's theorem	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampere's Law	$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$	Stoke's theorem	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

Maxwell Equations: Integral ⇒ Differentiation form

Integral form

Gauss's Law $\oint \overrightarrow{\mathrm{E}} \cdot \overrightarrow{da} = \frac{\mathrm{Q_{encl}}}{\epsilon_0}$

Gauss divergences theorem

$$\Rightarrow \oint_S \overrightarrow{\mathrm{E}} \cdot \overrightarrow{da} = \int_{\mathrm{V}} (\overrightarrow{\nabla} \cdot \overrightarrow{\mathrm{E}}) dV \Rightarrow \int_{\mathrm{V}} (\overrightarrow{\nabla} \cdot \overrightarrow{\mathrm{E}}) dV = \frac{\mathrm{Q_{encl}}}{\epsilon_0}$$

$$\Rightarrow \int_{\mathrm{V}} (\overrightarrow{\nabla} \cdot \overrightarrow{\mathrm{E}}) dV = \frac{1}{\epsilon_0} \int_{\mathrm{V}} \rho dV$$

$$\Rightarrow \overrightarrow{\nabla} \cdot \overrightarrow{E} = \frac{\rho}{\epsilon_0}$$

Differential form

Maxwell Equations: Integral $\Rightarrow$ Differentiation form

Integral form

Gauss's Law magnetism

$$\oint \overrightarrow{\mathrm{B}} \cdot \overrightarrow{da} = 0$$

Gauss divergences theorem

$$\Rightarrow \oint_S \overrightarrow{\mathrm{B}} \cdot \overrightarrow{da} = \int_{\mathrm{V}} (\overrightarrow{\nabla} \cdot \overrightarrow{\mathrm{B}}) dV \quad \Rightarrow \quad \int_{\mathrm{V}} (\overrightarrow{\nabla} \cdot \overrightarrow{\mathrm{B}}) dV = 0$$

$$\Rightarrow \overrightarrow{\nabla} \cdot \overrightarrow{B} = 0$$

Differential form

Maxwell Equations: Integral $\Rightarrow$ Differentiation form

Integral form

Faraday's Law

$$\varepsilon = -\frac{\partial \Phi_B}{\partial t}$$

$$-\frac{\partial \Phi_B}{\partial t} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot\vec{dS}$$

$$\Rightarrow \oint \vec{E}\cdot\vec{dl} = -\frac{\partial \Phi_B}{\partial t}$$

$$= -\int_S \frac{\partial \vec{B}}{\partial t}\cdot\vec{dS}$$

Apply Stoke's theorem

$$\Rightarrow \oint_L \vec{E}\cdot\vec{dl} = \int_S (\vec{\nabla}\times\vec{E})\cdot\vec{dS}$$

$$\Rightarrow \quad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Differential form

Integral form

Ampere's Law $$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$$

$$I = \int_S \vec{J} \cdot \vec{dS}$$

Apply Stoke's theorem

$$\Rightarrow \oint_L \vec{B} \cdot \vec{dl} = \int_S (\vec{\nabla} \times \vec{B}) \cdot \vec{dS}$$

$$\Rightarrow \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Differential form

Integral and differential forms Maxwell Equations

Law	Integral form	Theorem	Differential form
Gauss's Law	$\oint \vec{E}\cdot\vec{da} = \frac{Q_{encl}}{\epsilon_0}$	Gauss divergences theorem	$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$
Gauss's Law for magnetism	$\oint \vec{B}\cdot\vec{da} = 0$	Gauss divergences theorem	$\vec{\nabla}\cdot\vec{B} = 0$
Faraday's Law	$\oint \vec{E}\cdot\vec{dl} = -\frac{\partial\Phi_B}{\partial t}$	Stoke's theorem	$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
Ampere's Law	$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{encl}$	Stoke's theorem	$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

Maxwell Equations

Physical Significance

		Physical Significance
Gauss's Law	$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$	Net electric flux through a closed surface (3D) is times the net charge enclosed by the surface
Gauss's Law for magnetism	$\vec{\nabla} \cdot \vec{B} = 0$	magnetic monopoles cannot exist
Faraday's Law	$\vec{\nabla} \times \vec{\mathrm{E}} = -\frac{\partial \vec{\mathrm{B}}}{\partial \mathrm{t}}$	Time-varying magnetic flux produce electric field
Ampere's Law	$\vec{\nabla} \times \vec{\mathrm{B}} = \mu_0 \vec{\mathrm{J}}$	A magnetic field is produced due to conduction current density

Maxwell Equations

Gauss's Law $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

Gauss's Law for magnetism $\vec{\nabla} \cdot \vec{B} = 0$

Faraday's Law $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

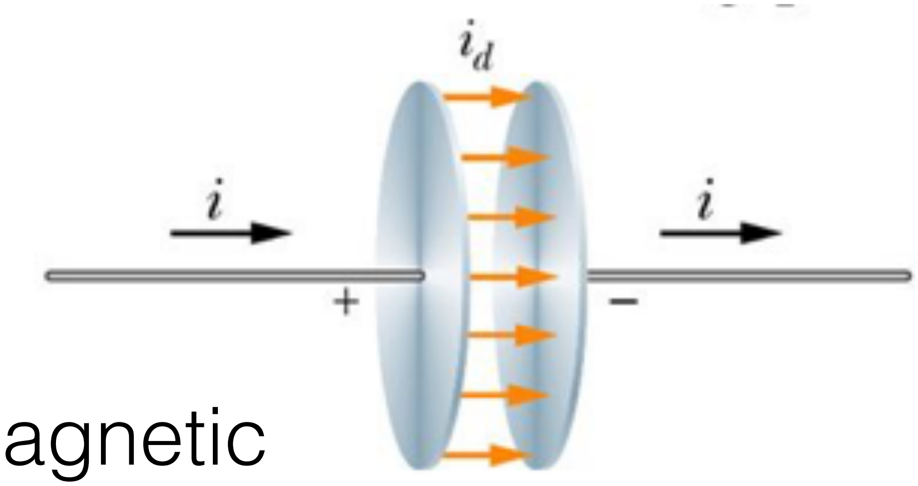
Ampere's Law $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

Anomalies with this equation

Displacement Current

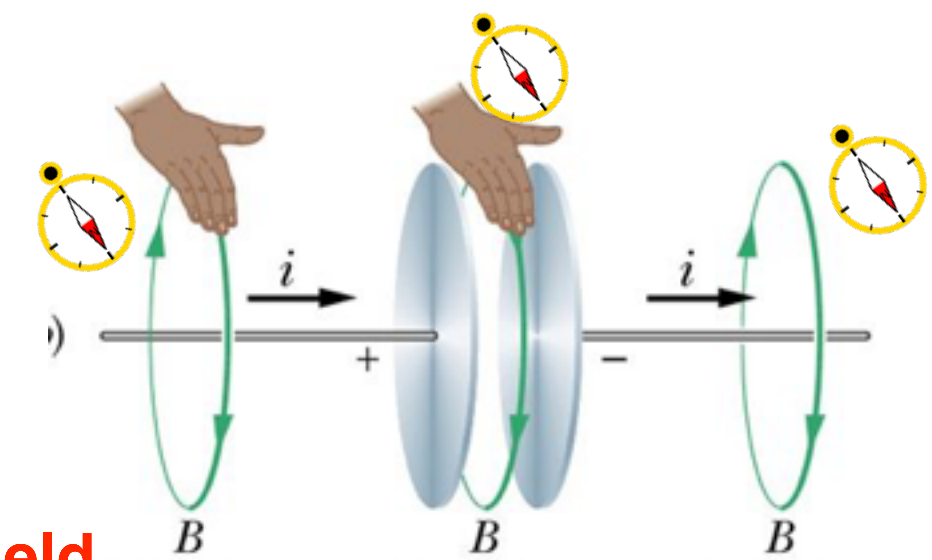
Assume a capacitor of radius, **r**, is connected to a circuit with a flowing current of **I**

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$



According to appears law, there must be no magnetic field inside the capacitor, as I=0

However, Maxwell found the same magnetic field with same deflection inside the capacitor, so he predict that:



The change in Electric Field $\Rightarrow$ **Creates the Magnetic Field**

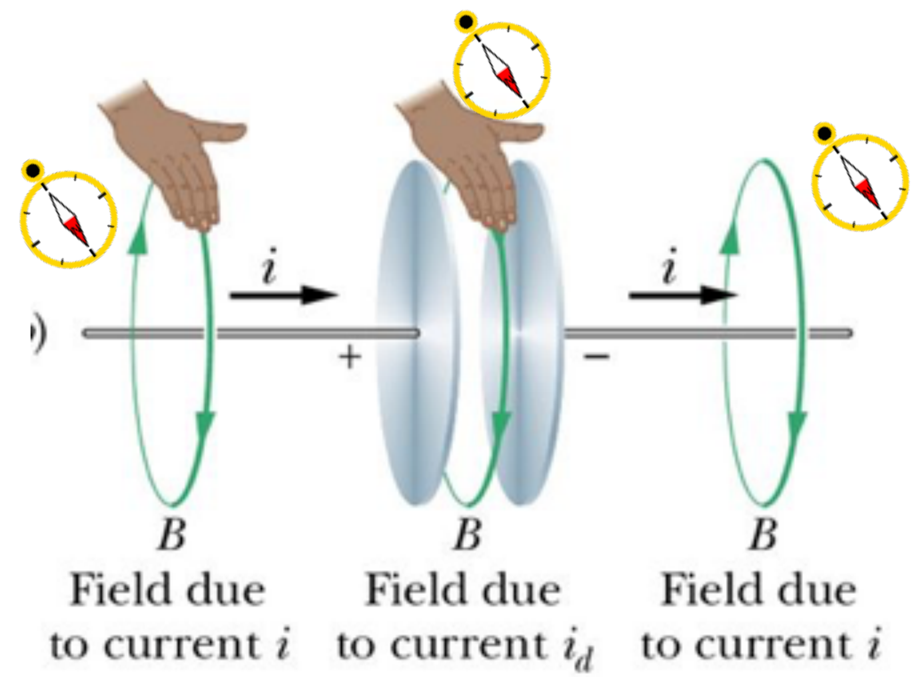
$$\oint \vec{B} \cdot \vec{dS} \propto \frac{\partial E}{\partial t} \Rightarrow \oint \vec{B} \cdot \vec{dS} = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \Rightarrow \mu_0 \epsilon_0 \frac{\partial E}{\partial t} = i_d$$

displacement current

Displacement Current: Ampere-Maxwell Law

Maxwell said that not only current produces a magnetic field but a **changing electric field** in vacuum/free space also produces a magnetic field

Ampere's Law $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$



Corrected

$$\vec{\nabla} \times \vec{B} = \mu_0 \left[\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right]$$

Ampere's-Maxwell Law

Conduction current

Displacement current

Modified Maxwell Equations

Physical Significance

Gauss's Law

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Net electric flux through a closed surface (3D) is times the net charge enclosed by the surface

Gauss's Law for magnetism

$$\vec{\nabla} \cdot \vec{B} = 0$$

magnetic monopoles cannot exist

Faradav's Law

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Time-varying magnetic flux produce electric field

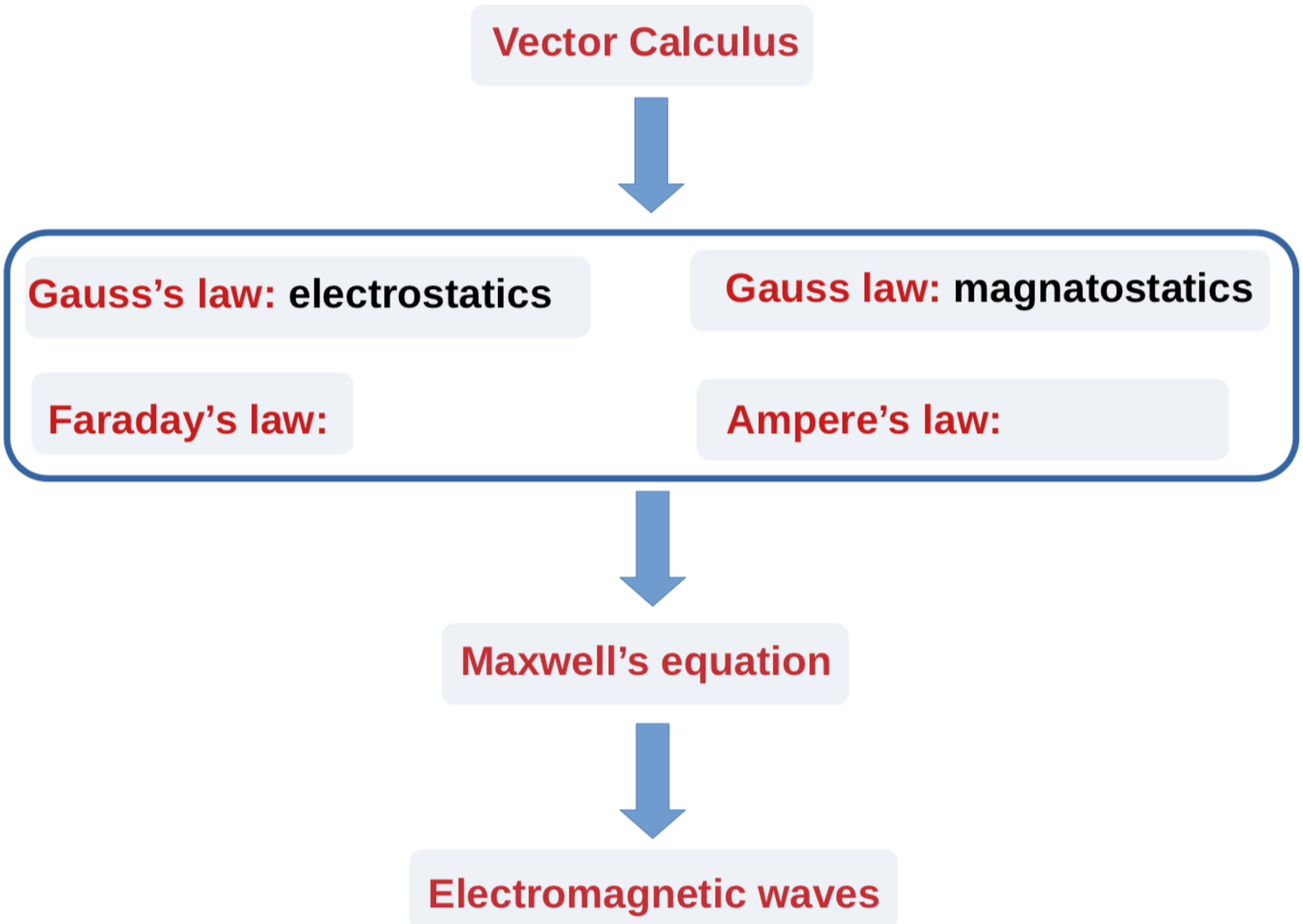
Ampere's-Maxwell Law

$$\vec{\nabla} \times \vec{B} = \mu_0 \left[\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right]$$

modified

A magnetic field is produced due to conduction current density and varying electric field

Maxwell Equations



Modified Maxwell Equations

Gauss's Law

$$\overrightarrow{\nabla} \cdot \overrightarrow{E} = \frac{\rho}{\epsilon_0}$$

Gauss's Law for magnetism

$$\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0$$

Faradav's Law

$$\overrightarrow{\nabla} \times \overrightarrow{\mathrm{E}} = -\frac{\partial \overrightarrow{\mathrm{B}}}{\partial \mathrm{t}}$$

Ampere's-Maxwell Law

$$\overrightarrow{\nabla} \times \overrightarrow{\mathrm{B}} = \mu_0 \left[\overrightarrow{\mathrm{J}} + \epsilon_0 \frac{\partial \overrightarrow{E}}{\partial t}\right]$$

Maxwell Equations in free space

A free space means there is no source of current or Charge;

$$\rho = 0, \quad J = 0$$

General form

Gauss's Law $\vec{\nabla} \cdot \vec{E} = \dfrac{\rho}{\epsilon_0}$

Gauss's Law for magnetism $\vec{\nabla} \cdot \vec{B} = 0$

Faradav's Law $\vec{\nabla} \times \vec{\mathrm{E}} = -\dfrac{\partial \vec{\mathrm{B}}}{\partial \mathrm{t}}$

Ampere's-Maxwell Law $\vec{\nabla} \times \vec{\mathrm{B}} = \mu_0 \left[\vec{\mathrm{J}} + \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}\right]$

Free Space

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{\mathrm{E}} = -\frac{\partial \vec{\mathrm{B}}}{\partial \mathrm{t}}$$

$$\vec{\nabla} \times \vec{\mathrm{B}} = \mu_0\, \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Electromagnetic wave equation in free space (Derivation)

A free space means there is no source of current or Charge; $\rho = 0, \quad J = 0$, considering the Maxwell equation: $\vec{\nabla} \cdot \vec{E} = 0 \ldots\ldots(1)$ $\quad \vec{\nabla} \cdot \vec{B} = 0 \ldots\ldots(2)$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \ldots\ldots(3) \qquad \vec{\nabla} \times \vec{B} = \mu_0 \, \epsilon_0 \frac{\partial \vec{E}}{\partial t} \ldots\ldots(4)$$

Lets operate Curl on eqn-3

$$\Rightarrow \quad \vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$\Rightarrow \quad \vec{\nabla}(\vec{\nabla} \,.\, \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

using Maxwell equation-1 & 4

$$\Rightarrow \quad \vec{\nabla}(0) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}(\mu_0 \, \epsilon_0 \frac{\partial \vec{E}}{\partial t})$$

$$\Rightarrow \quad \nabla^2 \vec{E} = \mu_0 \, \epsilon_0 \frac{\partial}{\partial t}(\frac{\partial \vec{E}}{\partial t})$$

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$$

$$\Rightarrow \nabla^2 \vec{E} = \mu_0 \, \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \Rightarrow \quad \frac{\partial^2 \vec{E}}{\partial t^2} = v^2 \, \nabla^2 \vec{E}$$

Lets operate Curl on eqn-4

$$\Rightarrow \quad \vec{\nabla} \times \vec{\nabla} \times \vec{B} = \vec{\nabla} \times (\mu_0 \, \epsilon_0 \frac{\partial \vec{E}}{\partial t})$$

$$\Rightarrow \quad \vec{\nabla}(\vec{\nabla} \,.\, \vec{B}) - \nabla^2 \vec{B} = \mu_0 \, \epsilon_0 \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E})$$

using Maxwell equation-2 & 3

$$\Rightarrow \quad \vec{\nabla}(0) - \nabla^2 \vec{B} = \mu_0 \, \epsilon_0 \frac{\partial}{\partial t}(\frac{-\partial \vec{B}}{\partial t})$$

$$\Rightarrow \quad -\nabla^2 \vec{B} = -\mu_0 \, \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$$

$$\Rightarrow \quad \frac{\partial^2 \vec{B}}{\partial t^2} = v^2 \, \nabla^2 \vec{B}$$

Electromagnetic wave equation in free space

$$\Rightarrow \quad \frac{\partial^2 \overrightarrow{E}}{\partial t^2} = v^2 \; \nabla^2 \overrightarrow{E} \qquad \Rightarrow \quad \frac{\partial^2 \overrightarrow{B}}{\partial t^2} = v^2 \; \nabla^2 \overrightarrow{B}$$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

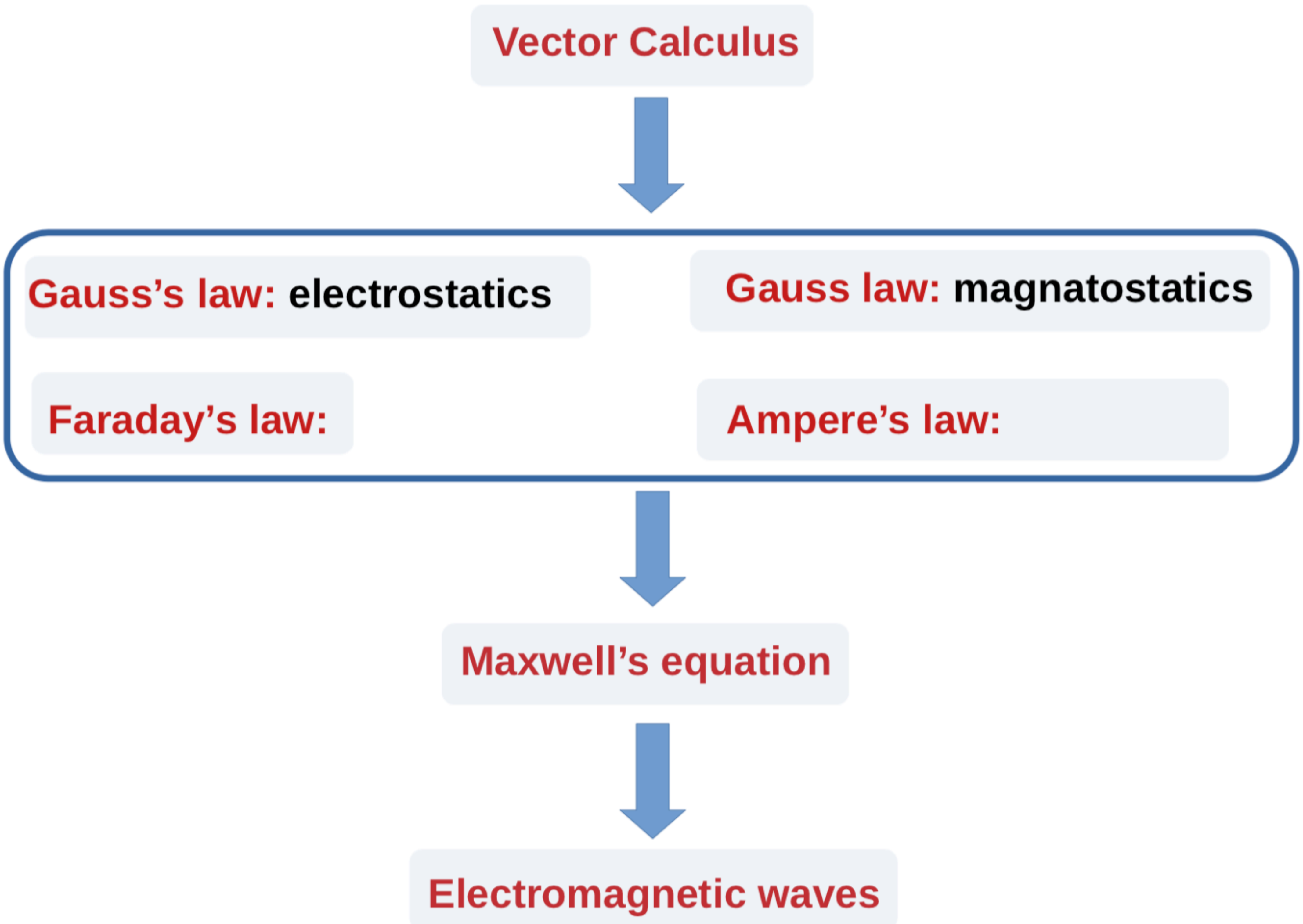
c, is the speed of EM wave. If we substitute the value of

$$\epsilon_0 = 8.854 \times 10^-12 \; Fm^{-1}, \; and \; \mu_0 = 4\pi \times 10^{-7}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99776 \; \times \; 10^8 \; m/s = c \; (Speed \; of \; Light)$$

This conclude that the EM wave travels in speed of light in free space

EM Wave Equation Solution



Objective of Class

Wave equation of Electric field and Magnetic field derived from Maxwell's Equations

$$\frac{\partial^2 \overrightarrow{E}}{\partial t^2} = c^2 \; \nabla^2 \overrightarrow{E} \qquad \Rightarrow \qquad \frac{\partial^2 \overrightarrow{B}}{\partial t^2} = c^2 \; \nabla^2 \overrightarrow{B}$$

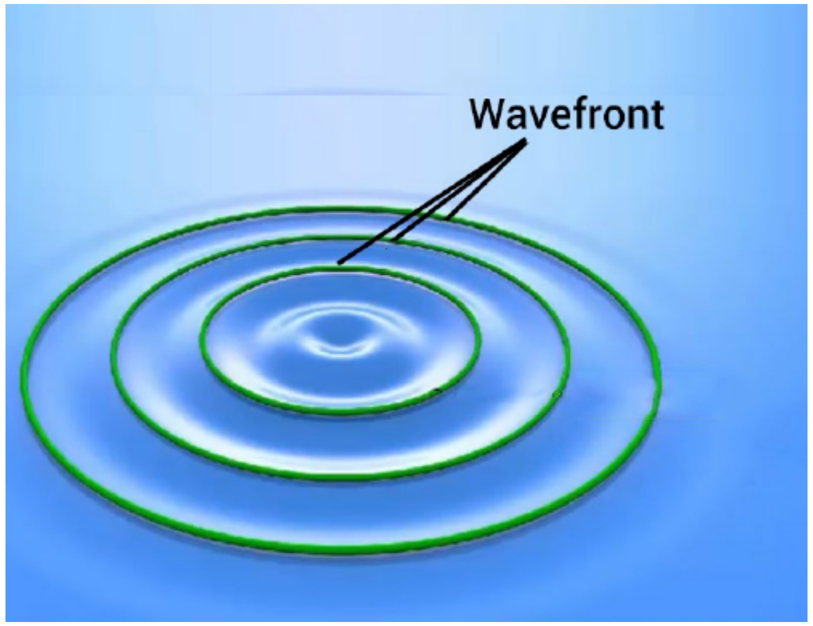
Next goal is to discuss the plane-wave solution

→ **Find the solution to the wave equation**

→ **Show that electromagntic wave is a transverse wave**

→ **Both $\vec{E}$ and $\vec{B}$ are perpendicular to each other and both are perpendicular to the direction of propagation.**

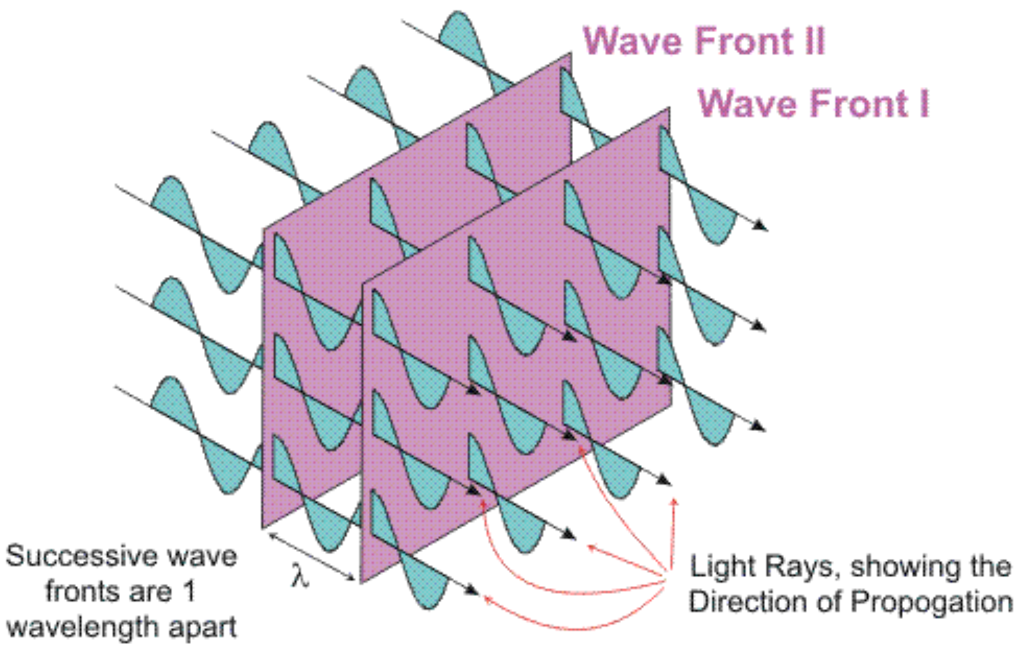
Wavefronts

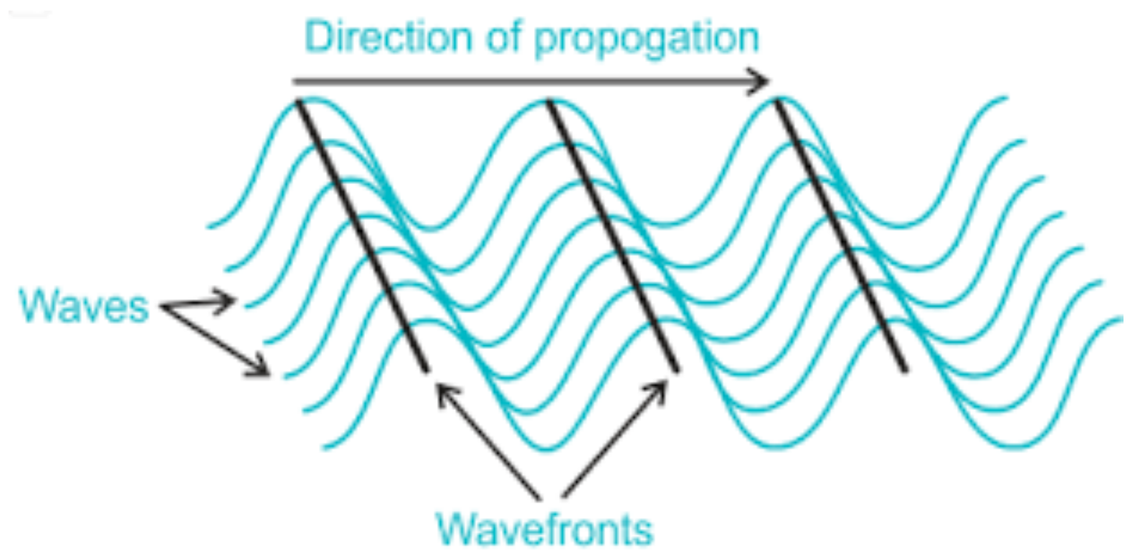


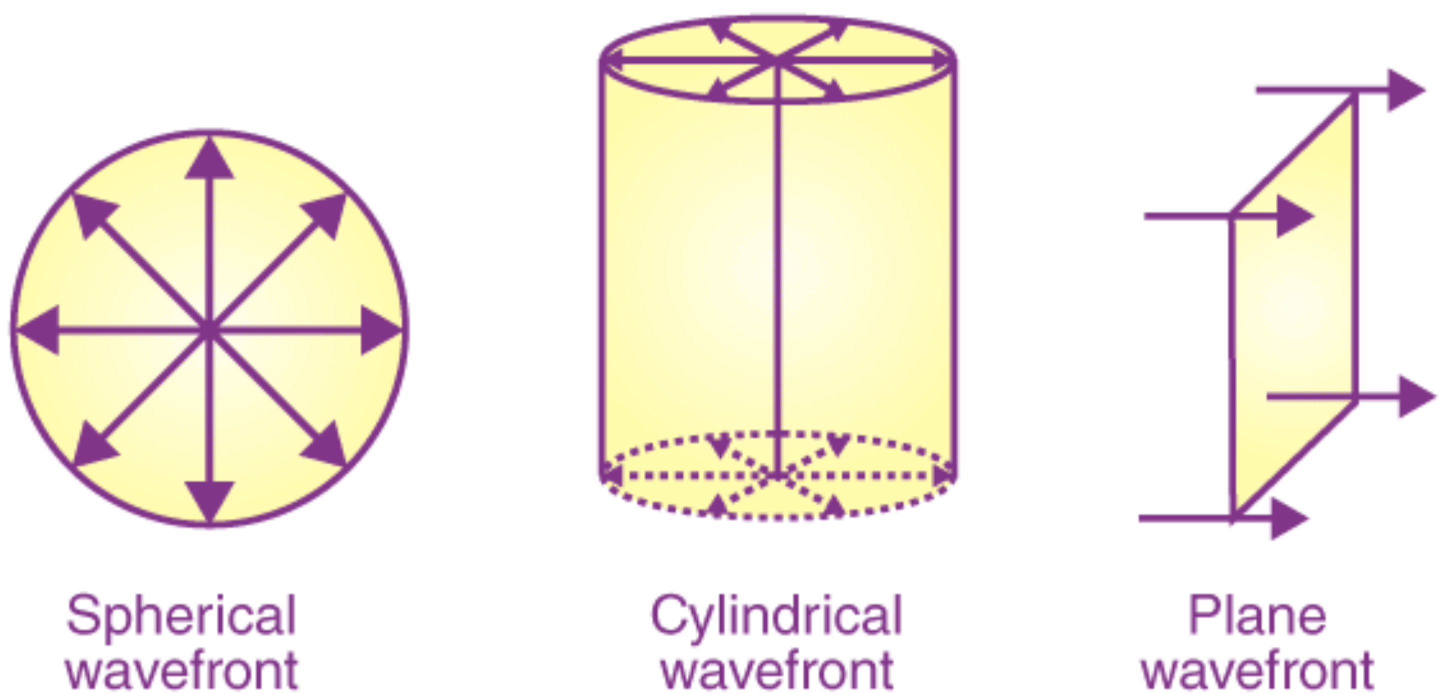
Wave Fronts are the parallel surfaces connecting equivalent points on adjacent waves or the locus of all the particles of the medium which are in the same state of vibrations

Wavefronts and its Types

Wave Fronts **are the parallel surfaces connecting equivalent points on adjacent waves**

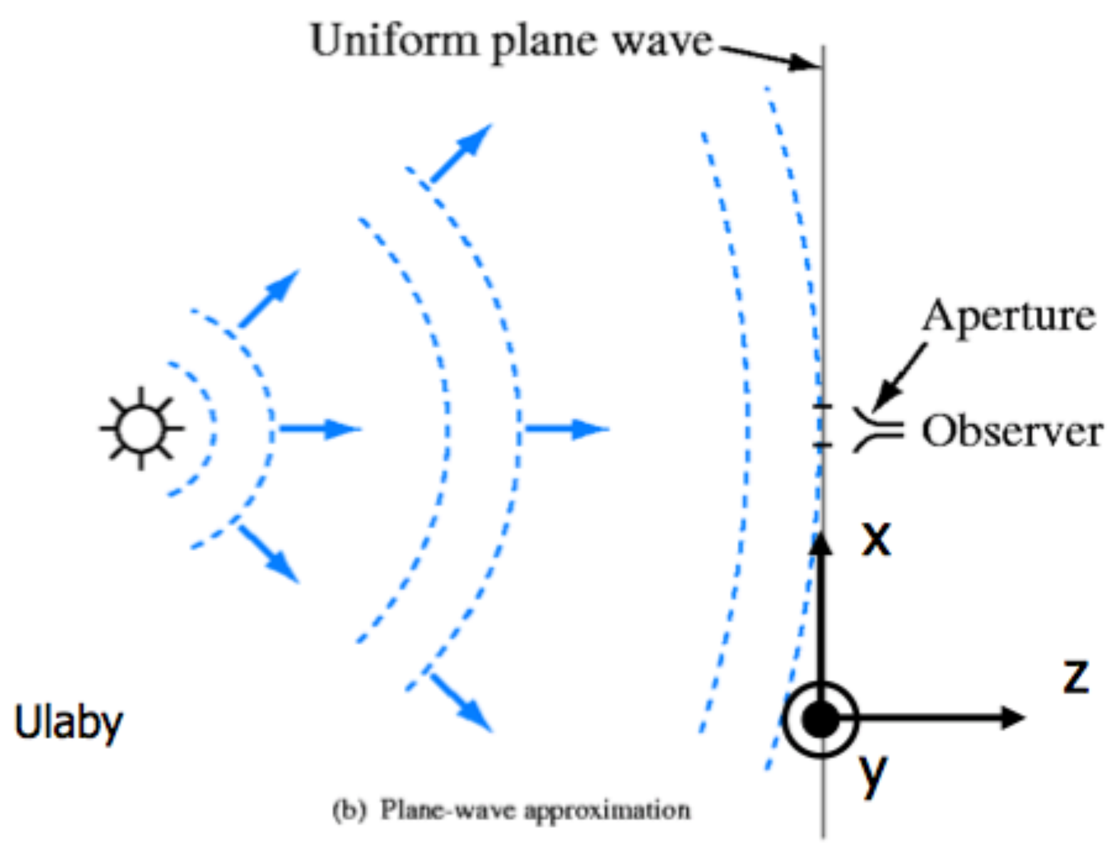


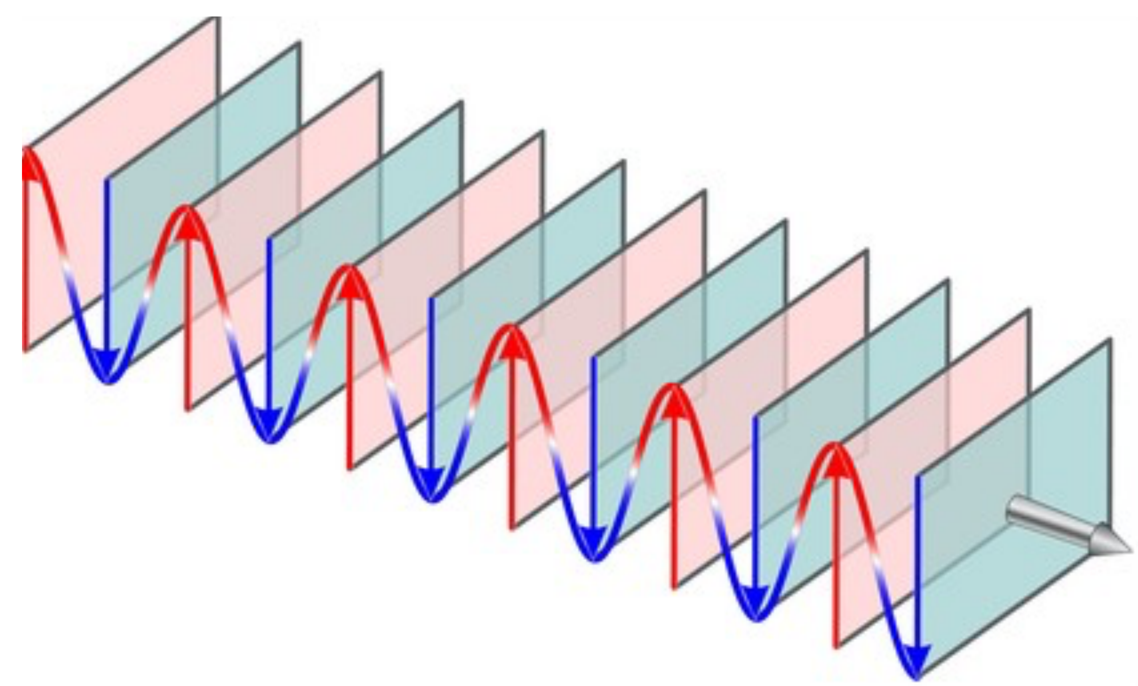




Plane Waves

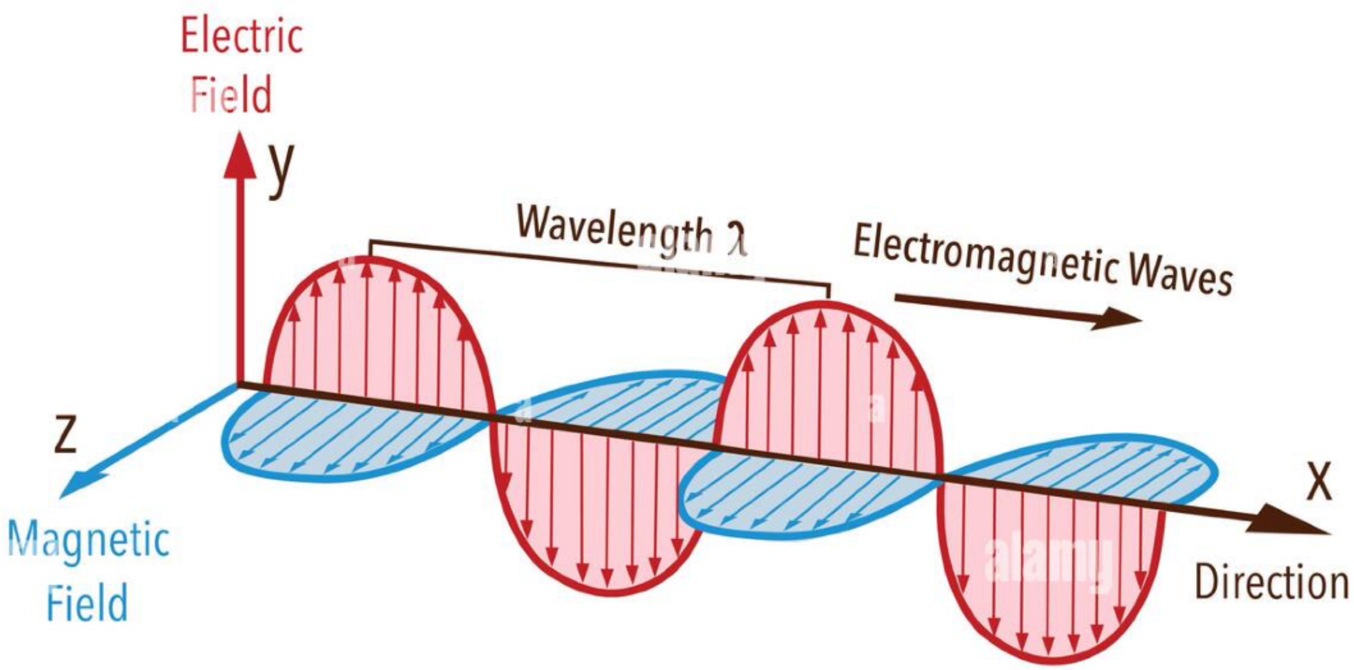
A plane wave is defined as a wave whose value remains constant throughout a plane (constant phase on surface) and is transverse to the direction of the propagation of the wave

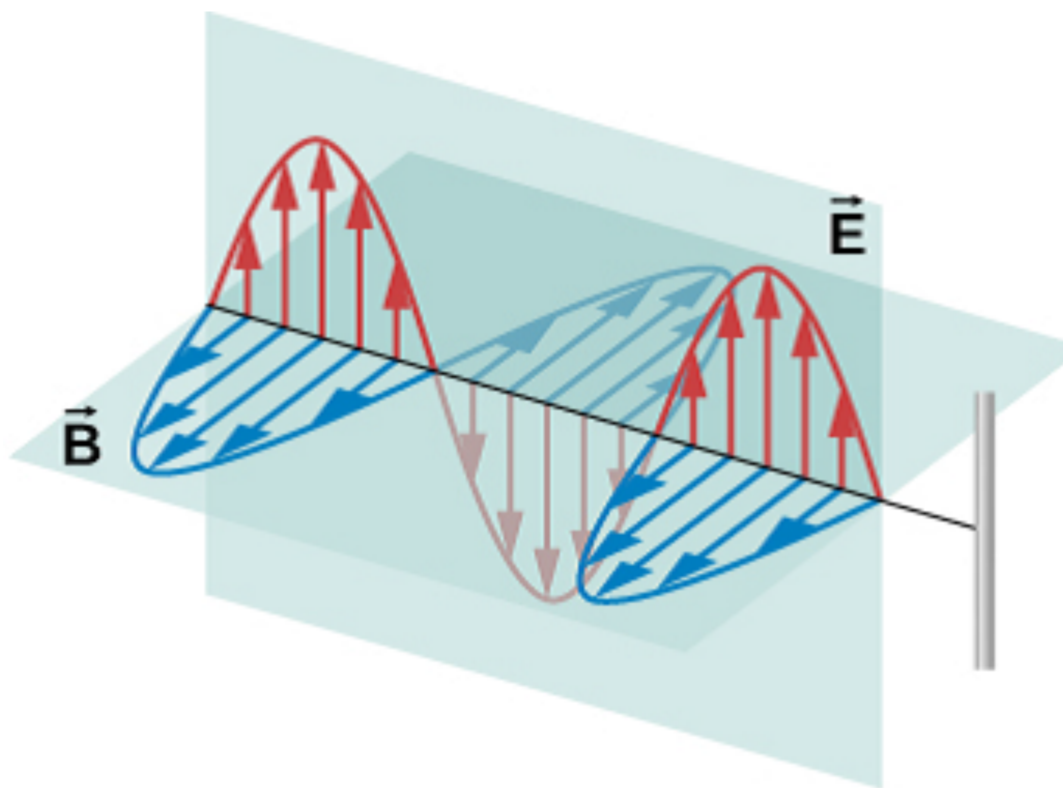




It is a wave in which the amplitude is constant over all the points of that plane which is perpendicular to the direction of the propagation. i. e. for a given time, the disturbance has a constant phase on the surface.

Electromagnetic Waves





- Transverse wave with a varying electric and magnetic field
- Move with the speed of light
- E-is in Y-X plane
- B is in Z-X plane

Objective of Class

Wave equation of Electric field and Magnetic field derived from Maxwell's Equations

$$\frac{\partial^2 \vec{E}}{\partial t^2} = c^2 \nabla^2 \vec{E} \quad \Rightarrow \quad \frac{\partial^2 \vec{B}}{\partial t^2} = c^2 \nabla^2 \vec{B}$$

Next goal is to discuss the plane-wave solution

→ **Find the solution to the wave equation**

→ **Show that electromagntic wave is a transverse wave**

→ **Both $\vec{E}$ and $\vec{B}$ are perpendicular to each other and both are perpendicular to the direction of propagation.**

Transverse nature of EM Wave

$$\frac{\partial^2 \vec{E}}{\partial t^2} = c^2 \nabla^2 \vec{E} \quad \Rightarrow \quad \frac{\partial^2 \vec{B}}{\partial t^2} = c^2 \nabla^2 \vec{B}$$

The most familiar form of solution to Maxwell's equation is of the form:

$$\vec{E}(x, y, z, t) = \vec{E}_0 \, Sin(\vec{k} \, . \vec{r} - \omega t + \phi)$$

$$\vec{B}(x, y, z, t) = \vec{B}_0 \, Sin(\vec{k} \, . \vec{r} - \omega t + \phi)$$

Where, $\vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{E}_0 = E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k} \qquad \vec{B}_0 = B_{0x}\hat{i} + B_{0y}\hat{j} + B_{0z}\hat{k}$$

using Maxwell equation in free space

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0 \quad \Rightarrow \quad \begin{matrix} \vec{E} \perp \vec{k} \\ \vec{B} \perp \vec{k} \end{matrix} \quad \Rightarrow \quad \vec{E} \perp \vec{B}$$

will prove it

Transverse nature of EM Wave

$$\frac{\partial^2 \vec{E}}{\partial t^2} = c^2 \nabla^2 \vec{E} \quad \Rightarrow \quad \frac{\partial^2 \vec{B}}{\partial t^2} = c^2 \nabla^2 \vec{B}$$

The most familiar form of solution to Maxwell's equation is of the form:

$$\vec{E}(x, y, z, t) = \vec{E}_0 \, Sin(\vec{k} . \vec{r} - \omega t + \phi)$$

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Where, $\vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{B}_0 = B_{0x}\hat{i} + B_{0y}\hat{j} + B_{0z}\hat{k}$$

Transverse nature of EM Wave

using Maxwell equation (Faraday's law) in free space $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

lets assume, the Electric field in y-direction

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{B}_0 = -\hat{k}\left(\frac{k}{\omega}\right)\vec{E}_y$$

$$\Rightarrow \vec{B}_0 = -\hat{k}\left(\frac{1}{c}\right)\vec{E}_y$$

If $\vec{E}$ oscillates in the y-direction, $\vec{B}$ will oscillates on the z-direction and rise versa….i.e mutually orthogonal to each other

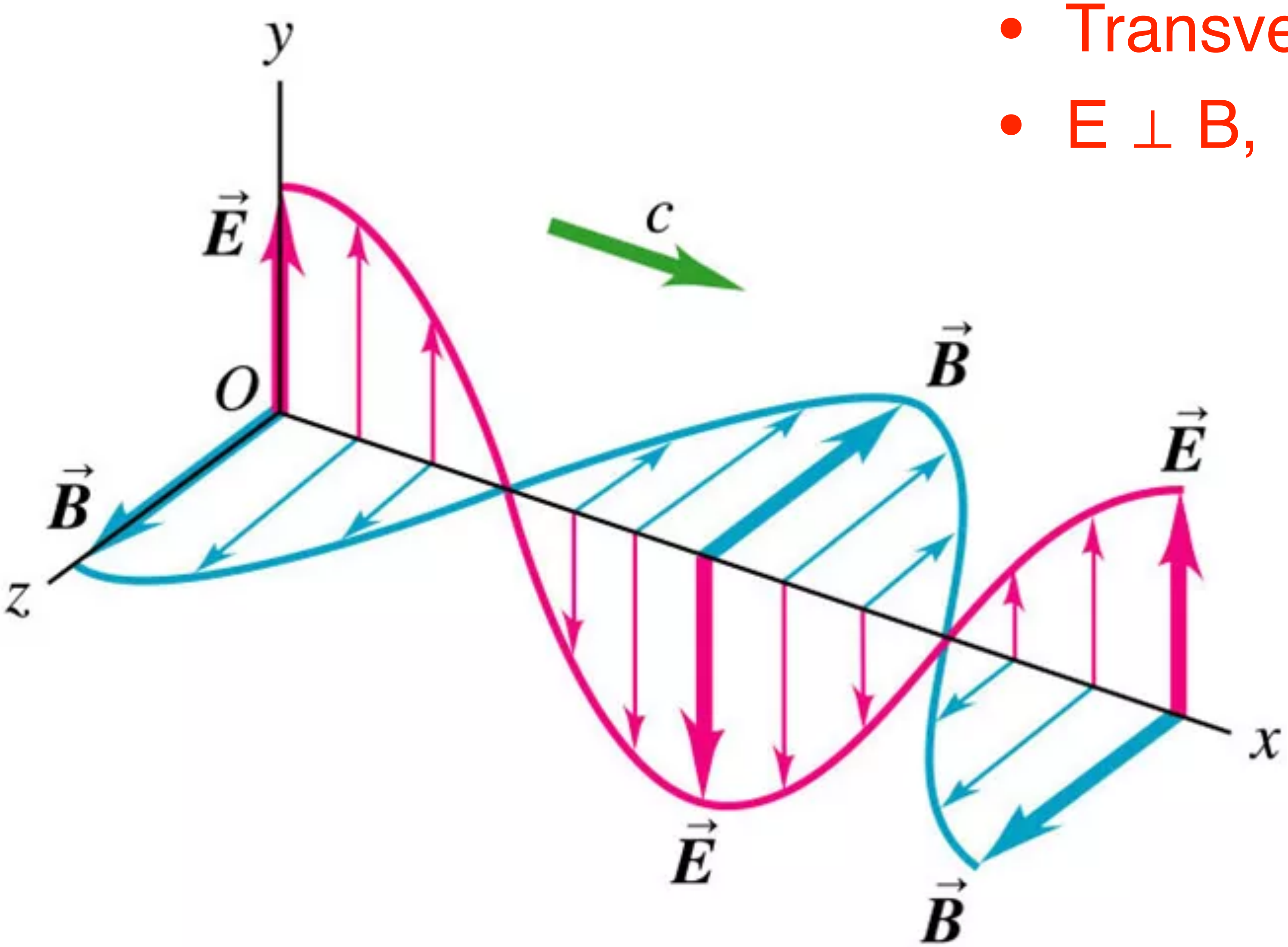
similarly, If the Electric field in z-direction

$$\Rightarrow \vec{B}_0 = -\hat{j}\left(\frac{1}{c}\right)\vec{E}_z$$

Plane Electromagnetic Waves

A plane EM wave is travelling in the x-direction, then the E and B fields are mutually orthogonal to each other

- Transverse in nature
- E ⊥ B, & E, B ⊥direction of propagation



$$\overrightarrow{E} = \hat{y}\ E_{max}\ Cos(kx - \omega t)$$

$$\overrightarrow{B} = \hat{z}\ E_{max}\ Cos(kx - \omega t)$$

$$B_{max} = \frac{E_{max}}{c}\ ;\ c = \frac{\omega}{k}$$

Concept: Accelerating Charge Particle

Lets consider a charge particle:

Rest, v=0

Electric Field

$E \neq 0, \quad B = 0$

Charge in motion ($v \neq 0$)

V= Constant

$(E \neq 0, \quad B \neq 0)$

E, B are constant

Accelerating

$(E \neq 0, \quad B \neq 0)$

E, B are time varying

EM wave

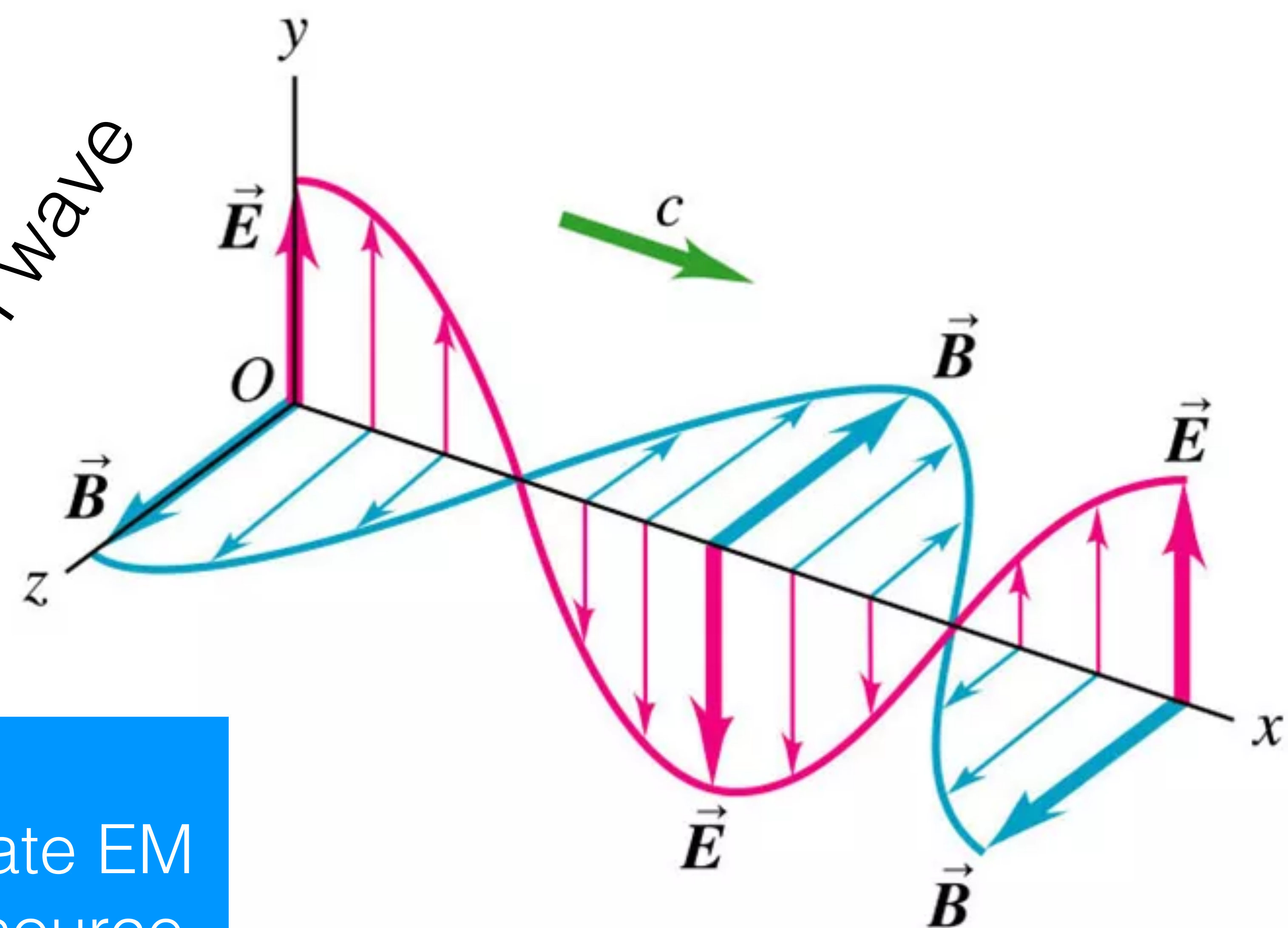
Concept: Accelerating Charge Particle

Accelerating

$(E \neq 0, \;\; B \neq 0)$

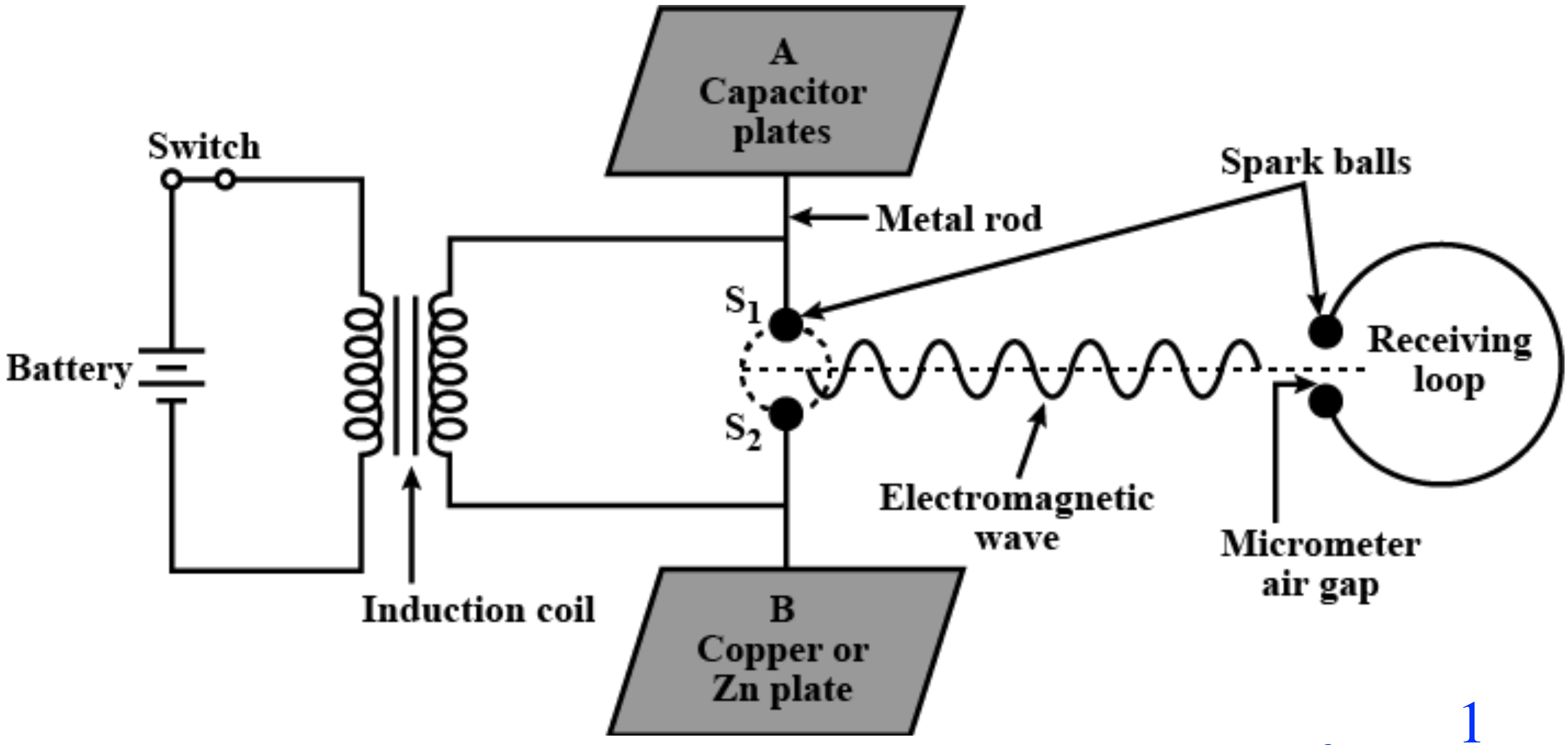
E, B are time varying

EM wave



Accelerating charge particles/ oscillating charge particles create EM waves. So the LC circuit is the source of the EM wave

Hertz Experiment



A-B plates= 60 cm
S1-S2 ball= 2 cm

$$f = \frac{1}{\sqrt{LC}}$$

$$f = 5 \times 10^7 Hz$$

Importance of Hertz Experiment

- Heinrich Hertz between 1885 and 1889 had an exceptional influence on the subsequent development of science and technology
- In 1888, hertz was the first to experimentally generate and detect electromagnetic waves and proved the theory predicted by Maxwell in 1865
- He used an oscillator made of polished brass knobs, connected to an induction coil and separated by a tiny gap over which sparks could leap.
- If Maxwell's predictions were correct, electromagnetic waves would be transmitted during each series of sparks.

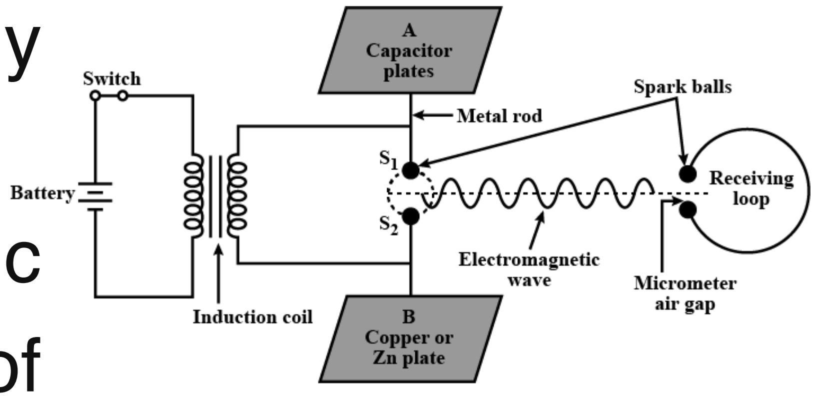
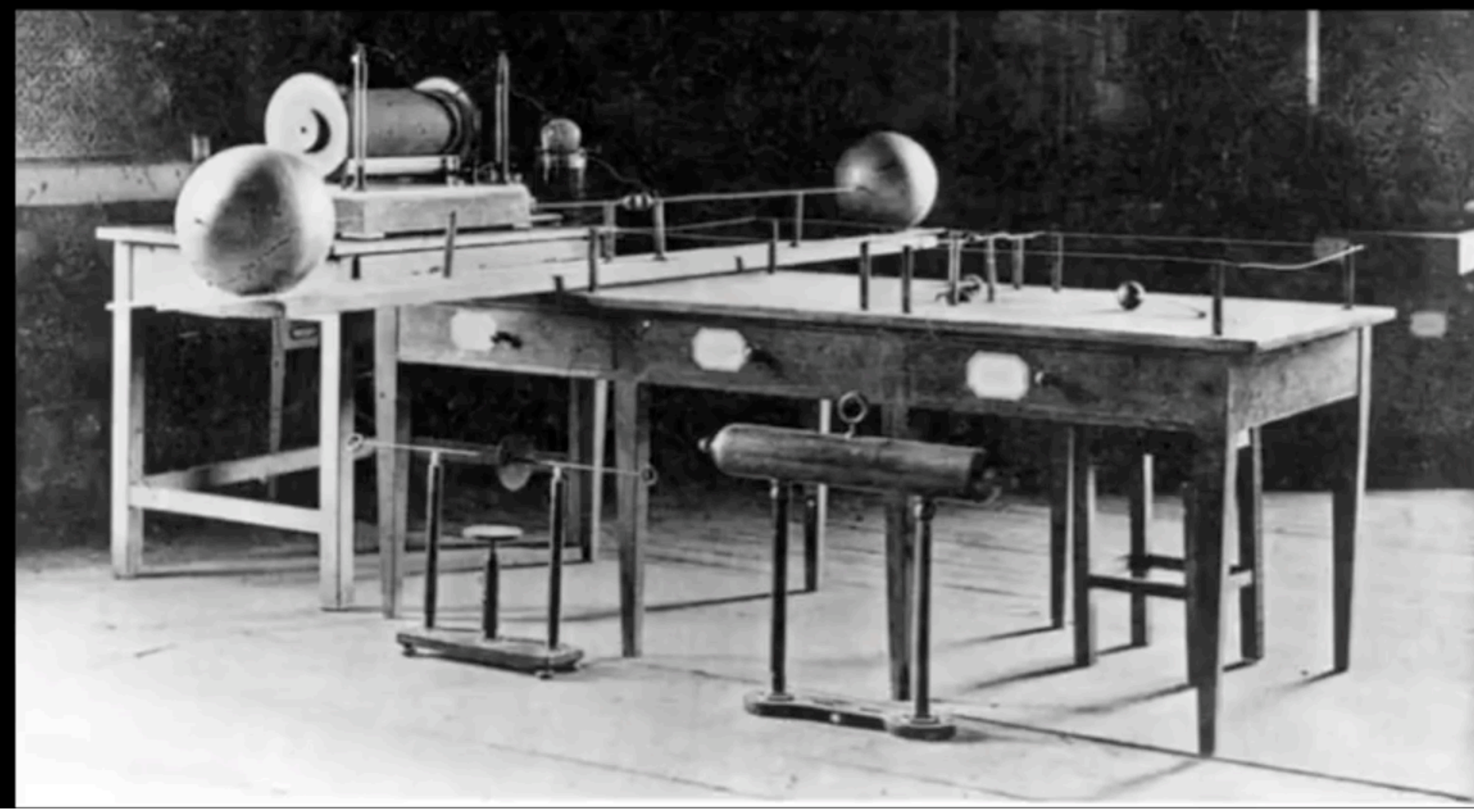


Fig : Sketch of the apparatus used by Hertz for producingand detecting radiowaves

Hertz Experiment



Important Question on Mod-2

1. **Numerical Problems on Gradient, Divergence and Curl for any vector Field**
2. **Explain Gauss Divergence Theorem and Stokes theorem with diagram and their physical significance**
3. **Write Maxwell's Equations in integral/differential form and explain their physical significance**
4. **Convert Maxwell's equation from integral form to differential form using Gauss divergence theorem and stokes theorem**
5. **Derive wave equation for electric field and magnetic field component of the EM wave and prove that it travels at speed of light**
6. **Derive plane EM wave equation and show it is transverse in nature**
7. **With stable schematic digram explain the Hertz Experiment**

CAT-1 QP Pattern

Part – A (5 x 10 = 50)

Answer ALL Questions

Sl. No	**Questions**	**Max Marks**
1	Module 1 – Descriptive Type	**10**
2	Module 1 – Descriptive Type + Numericals	**5+5**
3	(a) Module 1 – Descriptive Type (b) Module 1 – Numericals	**5** **5**
4	Module 2 – Descriptive Type	**10**
5	(a) Module 2 – Descriptive Type (b) Module 2 – Descriptive Type	**5** **5**

Thank You