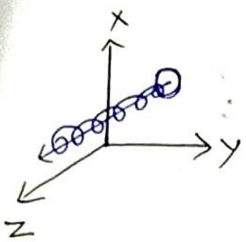
I. module - 2

Rotation

~~Module - 3~~

① ℬ



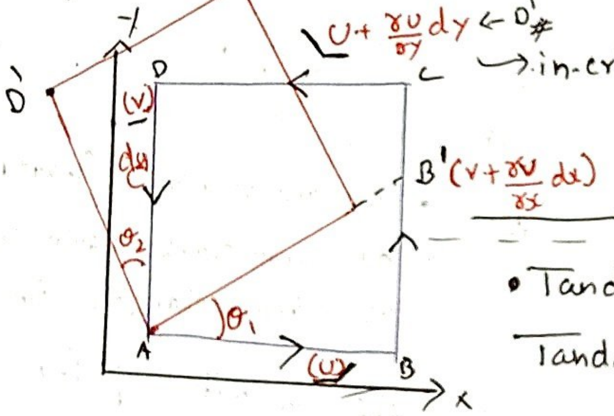
$\Rightarrow x = 0$ ∴ non - rotation

$\Rightarrow x \neq 0$ ∴ rotation.

To Prove the equation

$$\boxed{\omega\vartheta = \omega_x i + \omega_y j + \omega_z k} \rightarrow ①$$

The fluid element in x & y plane.



→ in-criment.

To "+" Gone changes.

→ in-crimant.

$\Rightarrow D = v \times T$

- $\tan d\alpha = \frac{BB'}{dx}$

$$\tan d\alpha = \frac{\frac{\partial v}{\partial x} \cancel{dx} \times dt}{\cancel{dx}}$$

$$\boxed{\frac{d\alpha}{dt} = \frac{dv}{dx}}$$

$$\boxed{\omega_{AB} = \frac{\partial v}{\partial x}}$$

$$\boxed{\omega_{AD} = \frac{\partial u}{\partial y}}$$

- $\tan d\alpha = \frac{OD'}{dy}$

$$\tan d\alpha = \frac{\frac{\partial u}{dy} dy \times dt}{dy}$$

$$\frac{d\alpha}{dt} = \frac{du}{dy}$$

$$\boxed{\omega_{AD} = \frac{\partial u}{\partial y}}$$

①

$$\vec{\omega} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

②

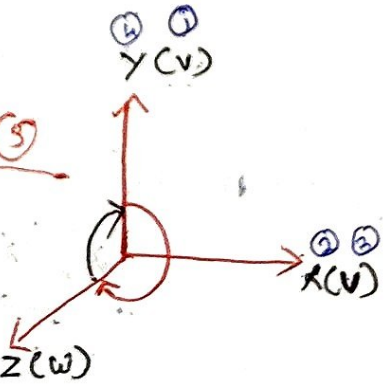
- $\omega_x = \frac{\omega_{AB} + \omega_{AD}}{2}$

→ anti-clock wise "(-)"

- $\omega_x = \frac{1}{2} \left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right] \rightarrow ②$
- $\omega_y = \frac{1}{2} \left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right] \rightarrow ③$
- $\omega_z = \frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] \rightarrow ④$ ex:- ①②③④

- $\omega_y = \frac{\quad}{2}$
- $\omega_z = \frac{\omega_{AB} +}{2}$

⑤



bub ②, ③, ④ on ①

eq ① :- $\omega = \omega_x i + \omega_y j + \omega_z k.$

- $\vec{\omega} = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]\hat{i} + \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]\hat{j} + \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]\hat{k}$ #

Vorticity and Circulation

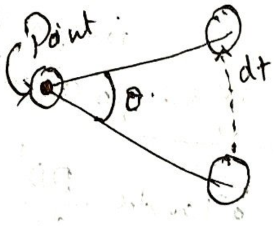
Vorticity (Ω)

⇒ It endicate wheather a Partical (→ air (or) water) is rotating or not.

⇒ Angular velocity in general use and Vorticity in fluid mechanics.

- **Angular velocity** :-

⇒ It tells how much fast the angle changes when the object moves along a circular path.



$$\boxed{\omega = \frac{d\theta}{dt}}$$

- ω = angular velocity (in rad per sec
- $d\theta$ = small change angular displacement
- dt = small change in time.

⇒ mathamatically Vorticity is twice the angle velocity of the Partical about its own axis.

⇒ 2ω #

$\Omega = 2\vec{\omega} \rightarrow$ ①

$$\Rightarrow \vec{\omega} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix} = \frac{1}{2}[\nabla \times \vec{V}]$$

$\vec{\omega} = \frac{1}{2}[\nabla \times \vec{V}] \rightarrow$ ②

$\vec{\Omega}$ ⇒ Exist ⇒ Rotational

$\vec{\Omega}$ ⇒ Not Exist ⇒ $0\hat{i} + 0\hat{j} + 0\hat{z}$ ⇒ Ir-Rotational

Put ① in eq ②

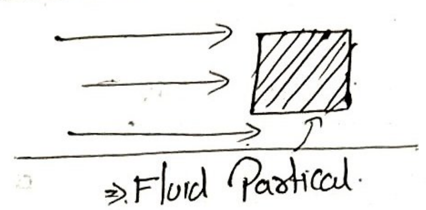
$\cancel{2}[\frac{1}{\cancel{2}}(\nabla \times \vec{V})]$

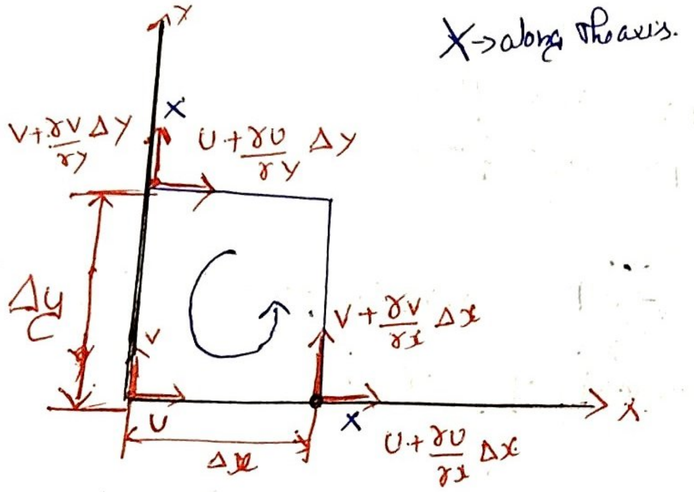
$$\boxed{\vec{\Omega} = \nabla \times \vec{V}}$$ ⇒ Curl of velocity vector (→ how much the field rotates around The points.)

I_n Circulation :- (Γ

⇒ Line integral of Tangential velocity Component.

⇒ Circulation around a closed contour is sum of vorticity enclosed with in it.





$$\Gamma = \oint_c \vec{V}\cdot\vec{ds}$$

$$d\Gamma = (u\times\Delta x) - \left[V+\frac{\partial v}{\partial x}\Delta x\right]\Delta y - \left[u+\frac{\partial u}{\partial y}\Delta y\right]\Delta x - V\Delta y$$

$$= \cancel{u\Delta x}^{0} + \cancel{V\Delta y}^{0} + \frac{\partial v}{\partial x}\Delta x\Delta y - \cancel{u\Delta x} - \frac{\partial u}{\partial y}\Delta y\Delta x - \cancel{V\Delta y}^{0}$$

$$= \underbrace{\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]}\Delta x\Delta y$$

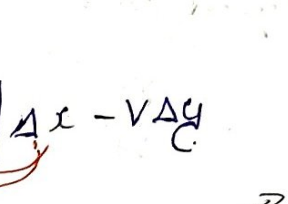
$$\boxed{2\omega_z = \Omega}$$

$$\Rightarrow 2[\omega_z]\,\Delta x\Delta y$$

$\boxed{d\Gamma = 2\omega_z\, dA}$ → differential circulation.

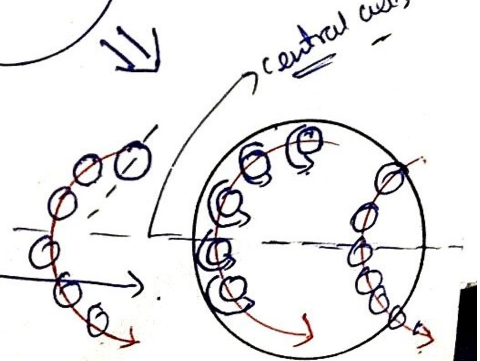
$$\boxed{\Gamma = \oint_c \vec{V}\cdot\vec{ds} = 2\int \omega_z\, dA}$$

$$\underset{\text{circulation}}{\Gamma} = \int \underset{\text{vorticity}}{\Omega}\, dA.$$



⇒ All fluid Partical. will flow at the central axis of pipe.

⇓



⇒ circulation is sum of vorticity in the area.

Stream function and Potential function

Stream function :- (ψ) → Scalar → psi

⇒ A function to describe an incompressible flow

⇒ Introduce by lagrange.

⇒ for 2D. flow only.

⇒ Stream function ≠ compressible flow.
= incompressible flow.

ψ → Partial Derivative; P.D →

$$\Rightarrow \; U = \frac{\partial \psi}{\partial y} \; ; \; V = -\frac{\partial \psi}{\partial x}$$

$\frac{\partial \psi}{\partial x}$ ⇒ giving in the velocity of y-direction

$\frac{\partial \psi}{\partial y}$ ⇒ givenig in the velocity of x-direction.

Continuity equation :-

$$\cancel{\frac{\partial \rho}{\partial t}}^{\to 0} + \nabla(\rho\vec{V}) = 0 \Rightarrow \cancel{\rho}^{0}_{\text{const}}(\nabla \vec{V}) = 0$$

$$\nabla \vec{V} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}U + \frac{\partial}{\partial y}V = 0$$

$$\Rightarrow \frac{\partial}{\partial x}\left[\frac{\partial \psi}{\partial y}\right] + \frac{\partial}{\partial y}\left[-\frac{\partial \psi}{\partial x}\right] = 0$$

$$\Rightarrow \cancel{\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y}}^{\to 0} = 0$$

$0 = 0$

LHS = RHS.

∴ Continuity is Satisfy

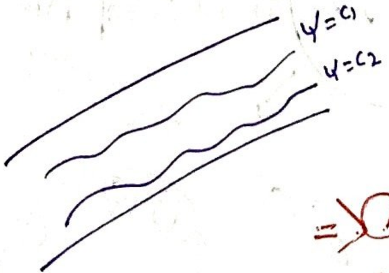
So flow its exist

Uses of ψ :-

⇒ U and V are replaced by single varible. (ψ).

$$\psi \begin{cases} \to U = \frac{\partial \psi}{\partial y} \\ \to V = -\frac{\partial \psi}{\partial x} \end{cases}$$

⇒ Stream lines. Can be easily drawn.



eqn of stream line = $\frac{dx}{u} \times \frac{dy}{v}$

$$v\,dx - u\,dy = 0$$

$$-\frac{\partial \psi}{\partial x}dx - \frac{\partial \psi}{\partial y}dy = 0$$

$$\underbrace{\frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy}_{} = 0$$

$$\partial \psi = 0$$

$$\int d\psi = \int 0 \Rightarrow \boxed{\psi = C}$$

⇒ Stream function is equal to a Constant along the Stream line.

Important point :-

- $\psi \Rightarrow$ Continuity equation should be satisifid.

  $\Downarrow$

  The flow is exist #

- Laplace eq^n is satisify $\Leftrightarrow$ flow is irrotational of ψ

  of ψ

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \Rightarrow \text{Laplass } eq^n \text{ of } \psi$$

ϕ ✓ → Ir-rot

ϕ × ⇒ rotat

Potential function (or) velocity potential function (ϕ)

← Scaler.

⇒ A function to describe 3D irrotational flow.

⇒ Applicable for 2D as well as 3D flow.

Vector identity ⇒ Curl of divergence of a scaler $f^n = 0$

$$\left.\begin{aligned} &\nabla \times \nabla\phi = 0 \\ \text{Irrot} \Rightarrow\ &\nabla \times \vec{V} = 0 \end{aligned}\right]$$

→ vorticity = 0 ⇒ no rotation.

$\vec{V} = \nabla\phi \Rightarrow$ velocity potential function

$$U = \frac{\partial\phi}{\partial x} ; V = \frac{\partial\phi}{\partial y} , w = \frac{\partial\phi}{\partial z}.$$

Lines of $\phi = c$ ⇒ equivipotential lines.

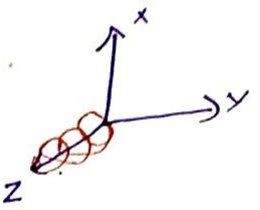
Important point :-

- ϕ exist ⇒ flow is irrotational.
- Laplace eq^n is satisfy ⇒ flow is possible.

Problem

①

A Stream function is given by $\psi = 3x^2 - y^3$, Determine the magnitude of velocity components at the point (3, 1)



Sol:-

$\Rightarrow \psi = 3x^2 - y^2.$

$\Rightarrow \underset{x,\,y}{(3,1)}$

∵ They gave x, y so we want to find z-direction rotation.

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \rightarrow ①$$

$\frac{\partial \psi}{\partial y} = u$ $\qquad$ $-\frac{\partial \psi}{\partial x} = v$

$\frac{(3x^2-y^3)}{\partial y} = u$ $\qquad$ $\frac{\partial(3x^2-y^3)}{\partial x} = v$

$-3y^2 = u$ $\qquad$ $-6x = v$

$\boxed{u = -3y^2} \rightarrow ②$ $\qquad$ $\boxed{v = -6x} \rightarrow ③$

$\boxed{V = -3y^2 i - 6x j}$

velocity field.

but eq ② & ③ on ①

$$\omega_z = \frac{1}{2}\left[-6 - (-6)\right] = 0$$

$\boxed{\omega = 0}$

∵ So, it is NO-rotational.

For magnitude $(\overset{x}{3}, \overset{y}{1})$

$V = -3(1)i - 6(3)j$

$V = 3i - 18j$

$V = \sqrt{u^2 + v^2}$

$= \sqrt{3^2 + 18^2}$

$= \sqrt{9 + 324}$

$= \sqrt{333} \approx 3\sqrt{37}$

$= 18.24$ #

Problem①

In a two-dimentional, incomplete flow the fluid velocity components are given by U=X-4Y ; V=-Y-4X show that the flow satisfies the continuity equation and obtain the expression of the stream function. If the flow is potential (Irrotation) obtain also the expression for the velocity potential.

Sol:-

$$U = X - 4Y \; ; \; V = -Y - 4X$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}$$

$$-1 + 1 = 0 \; \#$$

- $$\frac{\partial \psi}{\partial y} = U \Rightarrow \int \partial \psi \Rightarrow \int \overset{\curvearrowright (x-4y)}{U} dy$$

$$\int d\psi = \int (x-4y) dy$$

$$\int d\psi = xy - \frac{\overset{2}{\cancel{4}} y^2}{\cancel{2}} + f(x) + C_1$$

$$\boxed{\psi = xy - 2y^2 + f(x) + C_1} \rightarrow ①$$

- $$\frac{-\partial \psi}{\partial x} = V \qquad \int d\psi = \int (y + 4x) dx$$

$$\boxed{\psi = xy + 2x^2 + f(y) + C_2} \rightarrow ②$$

Compare eq ① & ②

- $$\psi = xy + 2x^2 - 2y^2 + C \; \#$$

ii] $$\omega_z = \frac{1}{2}\left[\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right] = 0$$

$$\omega_z = \frac{1}{2}\left[\cancel{x}\overset{0}{\cancel{-4y}}\overset{0}{\cancel{+y}}\overset{0}{\cancel{+4x}}\right]$$

$$\boxed{w_z = 0} \; \#$$

- $$\frac{\partial U}{\partial x}$$

$$\frac{\partial (x - 4y)}{\partial x}$$

$$\frac{\partial U}{\partial x} = 1$$

- $$\frac{\partial V}{\partial x}$$

$$\frac{\partial (-y - 4x)}{\partial y}$$

$$\frac{\partial U}{\partial y} = -1$$

$$\frac{\partial \psi}{\partial y} = U$$

$$\frac{\partial (xy + 2x^2 - 2y^2)}{\partial y}$$

$$\boxed{x - 4y = U}$$

$$\frac{\partial \psi}{\partial x} = V$$

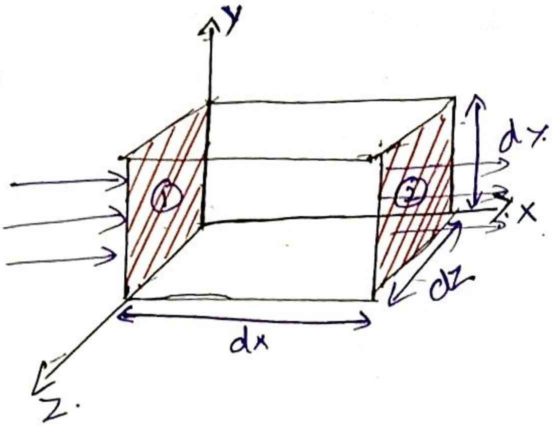
$$\frac{\partial (xy + 2x^2 + 2y^2)}{\partial x}$$

$$\boxed{y + 4x = V}$$

Module-3

Differential and kinatic of Continuity Equation

Formula:- $\overset{\text{In}}{\dot{M}_1} - \overset{\text{out}}{\dot{M}_2} = \overset{\text{Net}}{\frac{dm}{dt}}$



x-direction:-

$$\overset{\text{In}}{\rho\, dydz\, u} - \left[\overset{\text{out}}{\rho u + \frac{\partial \rho u}{\partial x} dx}\right] = \overset{\text{Net}}{-\frac{\partial \rho u}{\partial x} dx\, dy dz} \quad \#$$

(or) $\rho u \cdot dydz$

Y-direction:-

$$-\frac{\partial \rho v}{\partial y} dxdydz \quad \#$$

Z-direction:-

$$-\frac{\partial \rho w}{\partial z} dxdydz \quad \#$$

$$M_1 - M_2 = \frac{dv}{dz}$$

$$\left[-\frac{\partial \rho u}{\partial x} - \frac{\partial \rho v}{\partial y} - \frac{\partial \rho w}{\partial z}\right] \cancel{dxdydz} = \frac{d}{dt}\left(\rho \cancel{dxdydz}\right)$$

$$\frac{\partial \rho}{\partial T} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z}$$

Total net flow in unit time = change of mass flow in control volume in unit.

$$\boxed{\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = -\frac{\partial \rho}{\partial T}}$$ ⇒ un-Study Case #

In-Compressible:-

⇒ density will ~~Const~~ Cancel

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Study flow:-

⇒ time will cancel.

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

Bernolies

⇒ it comes from applying the law of conservation of energy to a moving fluid.

⇒ The general form of bernoulli's equation (along a streamline, for an encompressible, non-viscous, steady flow, neglecting heat/work interaction.

$$\underset{\swarrow \atop P.E}{\frac{P}{\rho g}} + \underset{\swarrow \atop K.E}{\frac{V^2}{2g}} + \underset{\swarrow \atop \text{Space}}{Z} = \text{constant}$$

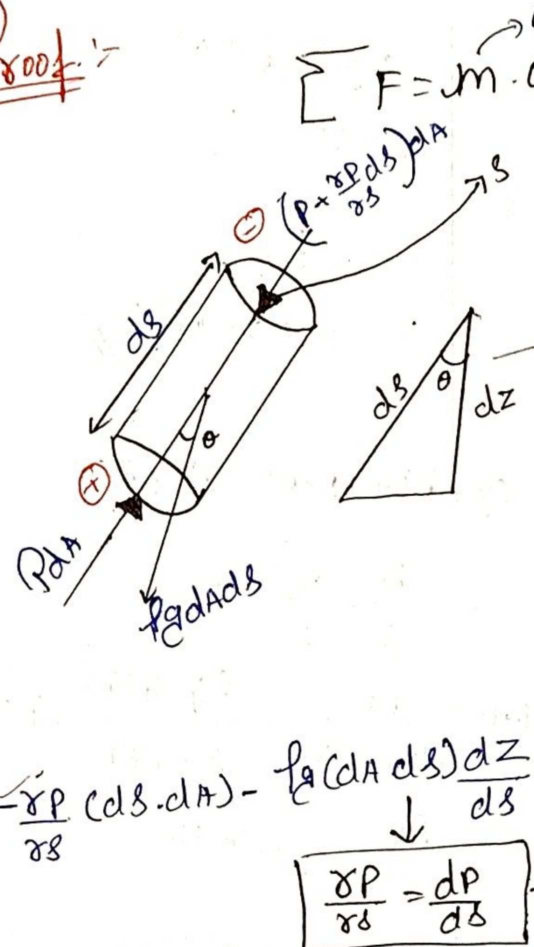
Internal friction. ×
Pressure force. ✓
Shear force. ×
Gravitation Force. ✓

Proof:-

$$\sum F = \overset{\nearrow \text{mass}}{m} \cdot a \rightarrow \text{acceleration.}$$

Pressure force is zero.

$$\sum F = +\cancel{P}^{\,0} \cdot dA - \left[\cancel{P}^{\,0} + \frac{\partial P}{\partial s} ds\right] dA - \rho g\, dA\, ds \cos\theta$$



$$\cos\theta = \frac{dz}{ds}$$

$$\sum F = -\frac{\partial P}{\partial s} (ds\, dA) - \rho g (dA\, ds) \frac{dz}{ds} = m \cdot a$$

RHS:-

$$\Rightarrow m \cdot a$$

$$\Rightarrow \rho (dA \cdot ds) \frac{dv}{dt} \times \frac{ds}{ds}$$

→ (v) → to remove time. because it is steady flow.

$$\Rightarrow \rho (dA \cdot ds)\, v \cdot \frac{dv}{ds} \;\#$$

$$\Rightarrow -\frac{\partial P}{\partial s} (ds \cdot dA) - \rho g (dA\, ds) \frac{dz}{ds} = \rho (dA \cdot ds)\, v \cdot \frac{dv}{ds}$$

$$\boxed{\frac{\partial P}{\partial s} = \frac{dP}{ds}}$$ → change partial to finate differential.

$$\Rightarrow -\frac{dP}{\cancel{ds}} \cancel{(dA \cdot ds)} - \rho g \cancel{(dA \cdot ds)} \frac{dz}{\cancel{ds}} = \rho \cancel{(dA\, ds)}\, v \frac{dv}{\cancel{ds}}$$

$$\boxed{\frac{dP}{\rho} + g\, dz + v\, dv = 0}$$

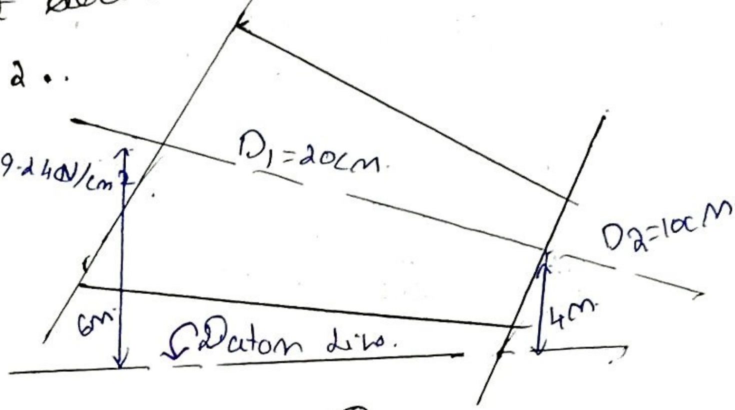
It is called eqn of euler's eqn of the motion:-

⇒ by using integration by parts.

$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z = C.$$

⑮

① The water is flowing through pipe having diameter 20cm and 10cm at section 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6m above datum and section 2 is 4 above datum. If the pressure at section 1 is 39.24 N/cm², find the Pressure at section 1 is 39.24 N/cm², find the intensity of Pressure at section 2..



Given:-

Section ①

$D_1 = 20cm \Rightarrow 0.2m$

$A_1 = \pi/4 \, (0.2) \Rightarrow 0.0314 m^2$

$P_1 = 39.24 N/cm^2$

$= 39.24 \times 10^4 N/m^2$

$Z_1 = 6m$

Section ②

$D_2 = 0.10m$

$A_2 = \pi/4 \, (0.1)^2$

$= 0.0078 m^2$

$Z_2 = 4m$

$P_2 = ?$

Rate flow:-

$Q = 35 \, lit/s$

$= \frac{35}{1000}$

$= 0.035 m^3/s$ #

Now; $\underbrace{Q = A_1 v_1 = A_2 v_2}$

Contitic equation.

$_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114 m/s$

$_2 = \frac{Q}{A_2} = \frac{0.035}{0.00785} = 4.456 m/s$

Applying bernoulli's equation section ① & ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2.$$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

$$40 + 0.063 + 6.0 = \frac{P_2}{9810} + 1.012 + 4.0$$

$$46.063 = \frac{P_2}{9810} + 5.012.$$

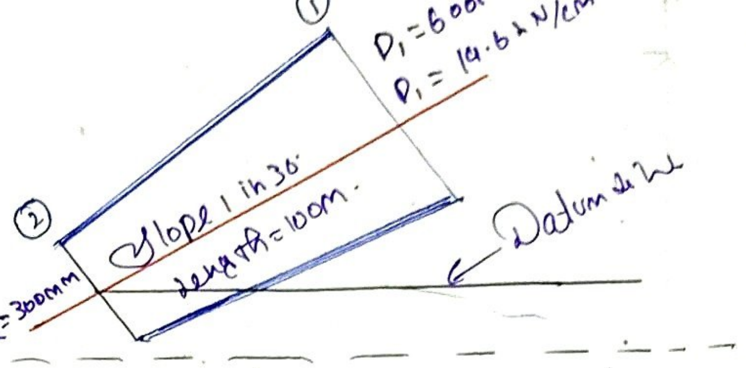
$$\frac{P_2}{9810} = 46.063 - 5.012 \Rightarrow 41.051$$

$$P_2 = 41.051 \times 9810 N/m^2.$$

$$P_2 = \frac{41.051 \times 9810}{10^4} N/cm^2$$

$$= 40.27 N/cm^2$$ #

② The water is flowing through a taper pipe of length 100m having diameters 600mm at the upper end and 300mm at a lower end, at the rate of 50 liters/s. The pipe has a slope of 1 is 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm².



Sol:-

length of pipe;

$\Rightarrow L = 100M$.

- Dia. upper end, $D_1 = 600mm = 0.6m$.

$\rightsquigarrow$ Area, $A_1 = \frac{\pi}{4} D_1^2 \Rightarrow \frac{\pi}{4}(0.6)^2$

$\Rightarrow 0.2827m^2$

P_1 at upper end

$\Rightarrow 19.62\ N/cm^2$.

$\Rightarrow 19.62 \times 10^4 N/m^2$

- Dia. lower end, $D_2 = 300mm = 0.3m$.

$\rightsquigarrow$ Area, $A_2 = \frac{\pi}{4} D_2^2 \Rightarrow \frac{\pi}{4}(0.3)^2$

$\Rightarrow 0.07068m^2$

- Q = rate of flow $\Rightarrow$ 50 liters/s $\Rightarrow \frac{50}{1000} = 0.05 m^3/s$

Let the datum line passes through the centre of the lower end. $\boxed{Z_2 = 0}$

Also slope is 1 to 30 means $Z_1 = \frac{1}{30} \times 100 = \frac{10}{3} m$

Also we know $\boxed{Q = A_1 V_1 = A_2 V_2}$

$V_1 = Q/A \Rightarrow \frac{0.05}{0.2827} \Rightarrow 0.1768\ m/sec \Rightarrow 0.177 m/s$

$V_2 = Q/A = \frac{0.05}{0.07068} = 0.7074\ m/sec \Rightarrow 0.707 m/s$

by Applying bernoulli's equation.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{\rho g} + \frac{v_2^2}{2g} + Z_2.$$

$$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{0.177^2}{2 \times 9.81} + \frac{10}{3} = \frac{P_2}{\rho g} + \frac{0.707^2}{2 \times 2.81} + 0$$

$$20 + 0.001596 + 3.334 = \frac{P_2}{\rho g} + 0.0254.$$

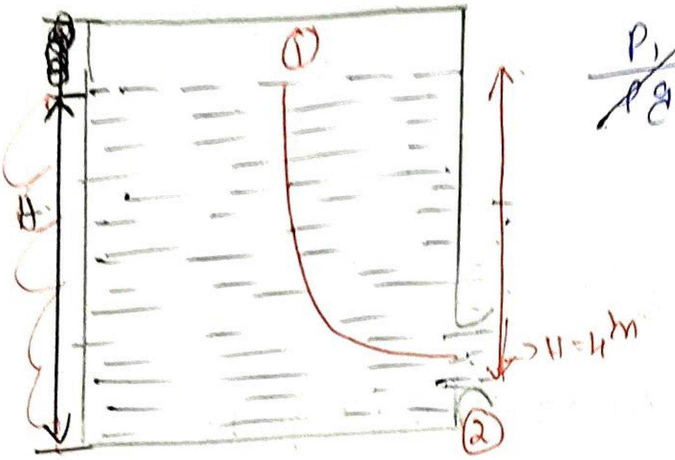
$$23.335 - 0.0254 = \frac{P_2}{1000 \times 9.81}.$$

$P_2 = 23.3 \times 9810 N/m^2$

$= \frac{228573\ N/m^2}{10^4}$

$\Rightarrow 22.857\ N/cm^2$ #

Determine the velocity and mass flow rate of efflux from the circular hole (0.1m dia) at the bottom of the water tank (at this instant). The tank is open to the atmosphere. and H=4M.



$$\cancel{\frac{P_1}{\rho g}}^{\to 0} + \cancel{\frac{V_1^2}{2g}}^{\to 0} + Z_1 = \cancel{\frac{P_2}{\rho g}}^{\to 0} + \frac{V_2^2}{2g} + \cancel{Z_2}^{\to 0}$$

$P_1 = P_2 = P_{atm}$.

$V_1 << V_2$. (or) $V_1 \approx 0$.

$Z_1 = H$

$Z_2 = 0$

$$H = \frac{V_2^2}{2g}$$

$V_2 = \sqrt{2gH}$

velocity @section 2.

$= \sqrt{2 \times 9.81 \times 4}$

$= \sqrt{78.48}$

$= 8.85 m/s.$

$\dot{m} = \rho A V$

mass flow rate.

→diameter.

$= 1000 \times \frac{\pi}{4} (0.1)^2 (8.85)$

$= 69.5 (kg/s)$

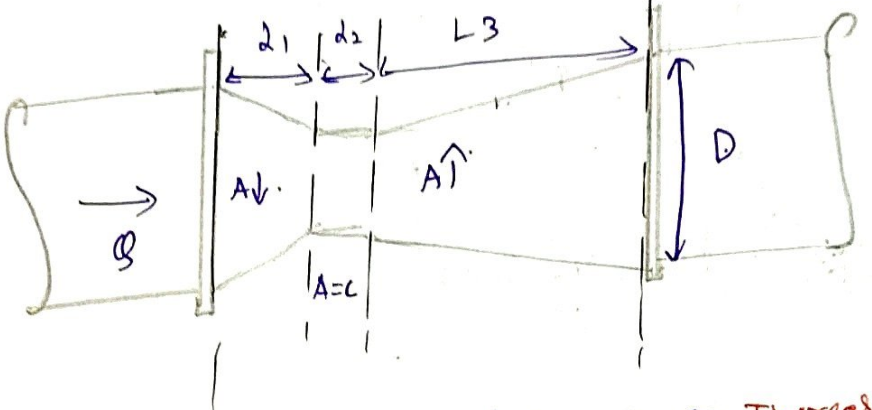
Application

1. Venturimeter.
2. orifice meter.
3. Pitot table.

→ flow rate in pipe.

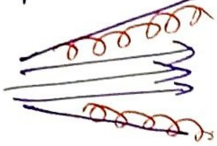
X. IM

Venturimeter



$L_3 > L_1$

↳ • To avoid loss of Pressure in boundary we are Increasing the ~~Po~~ Length of (d_3).



region①

⇒ A↓ velocity (v)↑

$\rho AV = C$:- Contuity equation.

$A_1 \downarrow V_1 \uparrow = C = A_2 V_2$.

bernolyes equation.

$$\frac{P}{\rho g} + \frac{v^2}{2g} = C$$

$$\downarrow \quad \uparrow \quad = \uparrow\downarrow$$

$$\boxed{A\downarrow V\uparrow P\downarrow}$$ #.

region②

⇒ A↑ velocity (v)↓.

$\rho AV = C$

$A_3 \uparrow V_3 \downarrow = C$

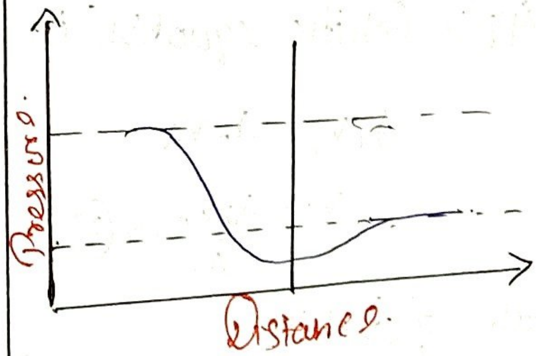
by bernolyes equation

$$\frac{P}{\rho g} + \frac{V^2}{2g} = C.$$

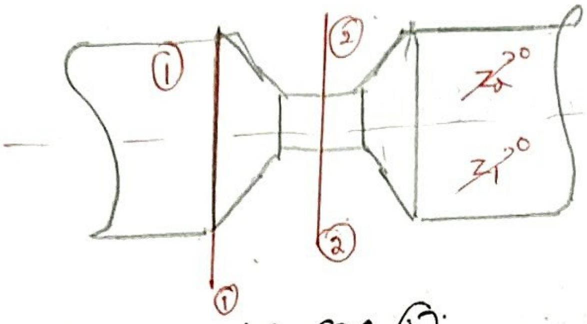
$$\uparrow \quad \downarrow \quad = \uparrow\downarrow.$$

A↑V↓P↑.

To plot Graph



To measure flow rate through venturimeter.



by apply bernolies eq. b/w ① & ②

$$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} = C}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1}^{0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2}^{0}$$

need value

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} \rightarrow ①$$

$$\boxed{h = \frac{P_1 - P_2}{\rho g}} \qquad h = \frac{V_2^2 - V_1^2}{2g} \rightarrow ②$$

Apply conitic equation in section ① & ②

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2 V_2}{A_1} \rightarrow ③$$

sub eq ③ in ②

$$h = \frac{V_2^2 - \left(\frac{A_2 V_2}{A_1}\right)^2}{2g}$$

$$2gh = V_2^2 \cdot \left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

$$2gh = V_2^2 \cdot \left[\frac{A_1^2 - A_2^2}{A_1^2}\right]$$

$$V_2 = \frac{A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} \Rightarrow Q = A_1 V_1 = A_2 V_2 \Rightarrow \boxed{Q_{thoy} = A_2 V_2 = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$$

Discharge.

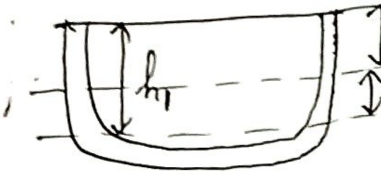
$$\uparrow$$

$$\boxed{\frac{Q_{actual}}{Q_{th}} = cd}$$

① An oil in Specific gravity 0.8 is flowing through a venturimeter having inlet diameter 20cm and throat diameter 10cm the oil. a Mercury difference manometer shows a reading of 25cm. Calculate. The discharge of oil through the horizontal venturimeter. take $C_d = 0.98$

Given:-

$$Q_{th} = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$



$x = 25cm \Rightarrow \frac{25}{100} = 0.25$

They given heavy

$$h = \frac{P_1 - P_2}{\rho g} = \underbrace{h_1 - h_2}_{x} \left[\frac{\overset{\text{mercury (heavy)}}{S_h}}{\underset{\text{oil (light)}}{S_l}} - 1\right]$$

- 20cm ⇒ 0.20m
- 10cm ⇒ 0.10m
- 25cm ⇒ 0.25

$$h = 0.25 \left[\frac{13.6}{0.8} - 1\right]$$

$$\boxed{h = 4m \text{ of oil}}$$

$d_1 = 20cm$

- $a_1 = \pi/4 \times D_1 \Rightarrow \pi/4 \times 20^2 = 314.16cm^2 \Rightarrow \pi/4 \times (0.20)^2 = 0.0314\ m^2$
- $a_2 = \pi/4 \times D_2 \Rightarrow \pi/4 \times 10^2 \Rightarrow 78.54\ cm^2 \Rightarrow \pi/4 \times (0.10)^2 = 0.00785m^2$

$\rightarrow (0.314)(0.00785) = 0.000247m^4$

$$Q = \underset{0.98}{C_d} \cdot \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

$\sqrt{2 \times 9.81 \times 4.0} \Rightarrow \sqrt{78.48} \Rightarrow 8.86$

$= ~~0.98\times~~ (0.0314^2 - 0.00785^2) = (0.000986 - 0.0000617)$
$= 0.000924m^4$

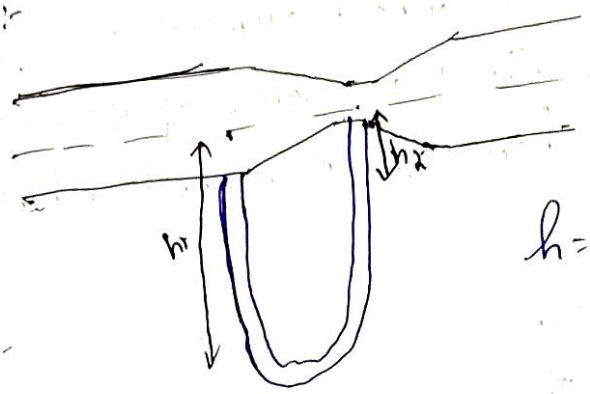
$\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \Rightarrow \frac{0.000247}{\sqrt{0.000924}} \Rightarrow \frac{0.000247}{0.0304} = 0.00813$

$Q = 0.98 \times 0.00813 \times 8.86$

$Q = 0.98 \times 0.0720 \Rightarrow \boxed{Q = 0.0706 m^3/s}$

(or) 71 liters/sec

Case -I :-



$$\boxed{x = h_1 - h_2.}$$

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

S_h → specific gravity (heavy)

S_o → fluid passing to pipe

Case -II :- (Invert manometer)

$$h = x\left[1 - \frac{S_1}{S_o}\right]$$

S_1 → lighter.

S_o → fluid passing to pipe.

Case -III :- vertical (higher Pressure).

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2.$$

$$\left[\frac{P_1 - P_2}{\rho g} + Z_1 - Z_2\right] = \frac{V_2^2 - V_1^2}{2g}$$

$$h = \left[\frac{P_1}{\rho g} + Z_1\right] - \left[\frac{P_2}{\rho g} + Z_2\right] = x\left[\frac{S_h}{S_o} - 1\right]$$

S_h → heavier.

Case - IV

$$h = \left[\frac{P_1}{\rho g} + Z_1\right] - \left[\frac{P_2}{\rho g} + Z_2\right] = x\left[\cancel{\frac{S_h}{S_o} - 1}\right]$$

$$\left[1 - \frac{S_1}{S_0}\right] \rightarrow \text{lighter.}$$

Problem ①

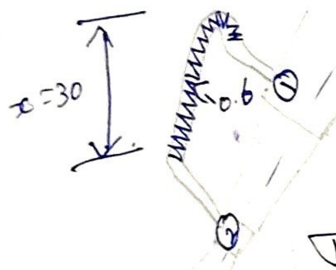
Find the discharge of water flowing through a pipe 30cm. diameter placed in an venturi meter is inserted, 15 cm. The difference of Pressure between the main and through is measured by a liquid of Sr 0.6 in an inverted U-tube which gives a reading of 30cm. The loss of head between the main and throat is 0.2 times the kinetic head of Pipe.

Sol:- In question they not mension Pressure value, So we want to consider hight @ formula of that

$$\frac{P_1 - P_2}{\rho g} + Z_1 - Z_2 = h = x\left[1 - \frac{Sl}{So}\right]$$

$$h = 0.3\left[1 - \frac{0.6}{1}\right]$$

$$\boxed{h = 0.12m}$$



The loss are there so, apply the bernolyes in section ① & ② @ modify according to loss in head.

$$\underbrace{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1}_{\text{Total energy section ①}} = \underbrace{\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2}_{\text{total energy section ②}}$$ → No loss formula.

$$(TE)_1 - (TE)_2 = 0.2 \times \frac{V_1^2}{2g}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + 0.2\frac{V_1^2}{2g}$$ → with loss

$$\underbrace{\frac{P_1-P_2}{\rho g}+Z_1-Z_2}_{\downarrow} = \frac{V_2^2-V_1^2}{2g}+\frac{0.2\,V_1^2}{2g}$$

$$0.12m = \frac{V_2^1-V_2^2}{2g}+0.2\frac{V_1^2}{2g} \rightarrow ①$$

by using Contitic equation:-

$$\boxed{A_1V_1 = A_2V_2}$$

$$\frac{\pi}{4}\times(0.3)^2\times V_1 = \frac{\pi}{4}(0.15)^2\times V_2.$$

$$V_2 = 4V_1 \rightarrow ② \Rightarrow V_1 = \frac{V_2}{4} \Rightarrow ②$$

by sut ② in ①

$$0.12+\frac{V_2^2}{2g}\left[\frac{0.8}{16}-1\right]=0$$

$$\frac{V_2^2}{2g}[0.05-1] = 0.12.0$$

$$V_2 = \sqrt{\frac{2.481\times 12}{0.95}}$$

$$\boxed{V_2 = 157.4cm.} \Rightarrow a_2V_2.$$

$\Rightarrow 176.7\times157.4$

$\Rightarrow 27800cm^3/s$

$\Rightarrow 27.8\,lit/s$ ≠

(or)

$$0.12 = \frac{V_2^2-\frac{1}{16}V_2^2}{2g}+\frac{0.2V_2^2}{2g} \Rightarrow \frac{(1-0.0625+0.2)V_2^2}{2g}$$

$1-0.0625+0.2 = 1.375$

$$0.12 = \frac{1.1375V_2^2}{2g} \Rightarrow V_2^2 = \frac{0.12\times2\times9.81}{1.1375}$$

$$V_2^2 = \frac{2.3544}{1.13575} = 2.0716$$

$$\boxed{V_2 = 1.439m/s}$$

$h_1 = 0.2\times\frac{V_1^2}{2g}$

$$0.2 = \frac{V_2^2-\frac{1}{16}V_2^2}{2g}+0.2\times\frac{\frac{1}{16}V_2^2}{2g}$$

$$0.2 = \frac{V_2^2}{2g}\left(1-\frac{1}{16}+0.2\times\frac{1}{16}\right) = \frac{V_2^2}{2g}(1-0.0625+0.0125) \quad 0.95$$

$$0.12 = \frac{0.95V_2^2}{2g} \Rightarrow V_2 = \frac{0.12\times2g}{0.95} \Rightarrow V_2^2 = \frac{2.3544}{0.95} = \sqrt{2.4762}.$$

$$Q = A_2V_2 = 0.01767\times1.574 = 0.02782\,m^3/s \Rightarrow \boxed{27.8\,L/s} \quad \boxed{V_2 = 1.574m/s}$$

$$A_2 \Rightarrow \frac{\pi}{4}\times(0.15)^2 = 0.01767m^2 \Rightarrow Q = A_2V_2 = 0.01767\times1.439 = 0.02545\,m^3/s \Rightarrow 25.45\,L/s$$

Pitot tobe

Difference between.

- Piezometer :- to find Static Pressure ⇒ when a fluid in rest condition.
- Pitot tobe :- to find Stagenation Pressure.

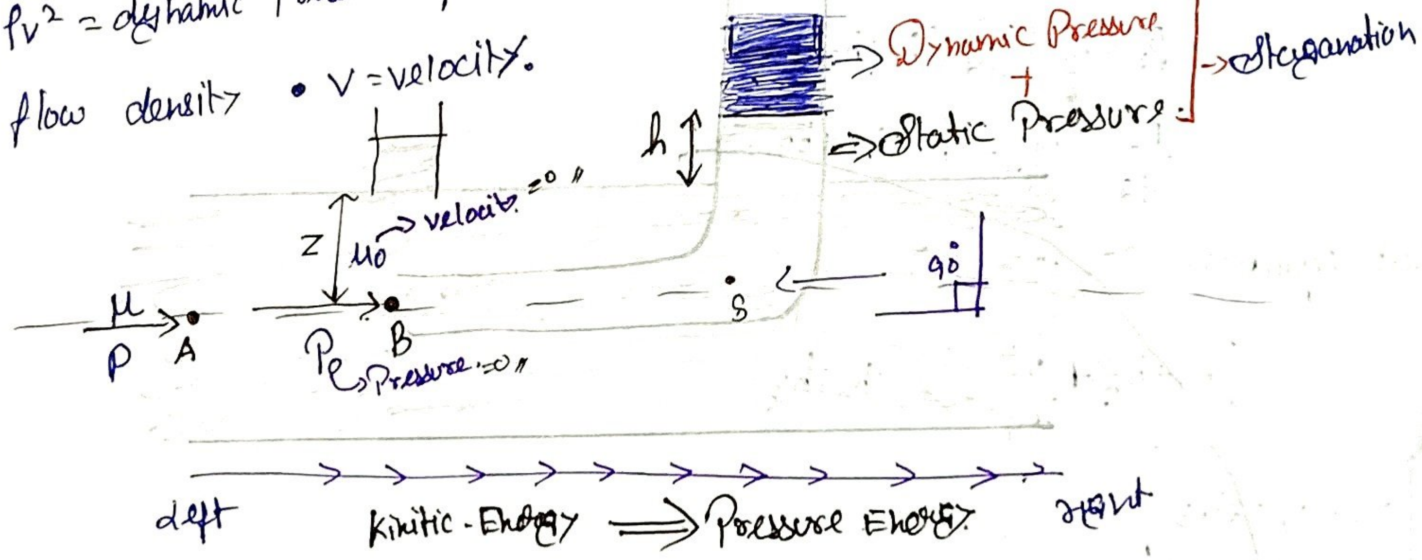
Stagenation Pressure :- The Pressure a fluid has when it is brought
to rest isentropically (without energy loss due to friction/heat).
It is the Sum of Static Pressure and dynamic pressure.
↓
moving

$$P_{stagenation} = P_{static} + \frac{1}{2}\rho v^2$$

P_{static} = Static Pressure.

$\frac{1}{2}\rho v^2$ = dynamic Pressure (from velocity)

= flow density • V = velocity.



ing point A to B on bernolyes eq^n

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_A}^{\to 0} = \frac{P_2}{\rho g} + \underbrace{\frac{V_2^2}{2g}}_{\text{velocity}=0} + \cancel{Z_B}^{\to 0}$$

same line

$$\frac{P}{\rho g} + \frac{u^2}{2g} = \frac{P_0}{\rho g} + 0$$

⇓

$$\frac{\rho U^2}{2} = P_2 = P_{stagnant}$$

Dynamic Press at 2 point

$$\frac{U^2}{2g} = \frac{P_0}{2g} - \frac{P}{\rho g}$$

$$P = \rho g z$$

$$P_0 = \rho g (z+h)$$

$$\frac{U^2}{2g} = (\cancel{z}+h) - \cancel{(z)}^{\to 0}$$

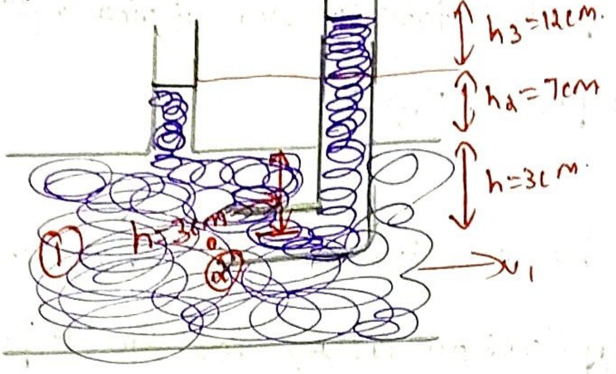
$$\frac{U^2}{2g} = h$$

$\boxed{U_{thy} = \sqrt{2gh}}$ ⇒ velocity of Centre of tube.

$V_{real} = C_U = C\sqrt{2gh}$ (or) $C_{cu}\sqrt{2gh}$.

Grate Problem:-

A Piezameter and a Pitot tobe are tapped into a. horizontal water pipe, as Shown is figure, to measure Static and Stagnation (Static + dynamic) Pressure for the.



① -> P_1

② -> P_2.

by apply bernolyies eq ① & ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1}^{\to 0} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}}^{\to 0} + \cancel{Z_2}^{\to 0} \Rightarrow \textcircled{a}$$

by using hydrostatic law:-

$$P = \rho g h.$$

$$P_1 = \rho g (h_1 + h_2) \Rightarrow \frac{P_1}{\rho g} = h_1 + h_2 \Rightarrow \textcircled{1}$$

$$P_2 = P_0 = \rho g (h_1 + h_2 + h_3)$$

$$\frac{P_2}{\rho g} = \frac{P_0}{\rho g} = h_1 + h_2 + h_3 \Rightarrow \textcircled{2}$$

Put equation ① & ② on ⓐ

$$(\cancel{h_1}^0 + \cancel{h_2}^0) + \frac{V_1^2}{2g} = \cancel{h_1}^0 + \cancel{h_2}^0 + h_3$$

$$\boxed{V_1 = \sqrt{2gh_3}}$$

$$V_1 = \sqrt{2 \times 9.81 \times 0.12}.$$

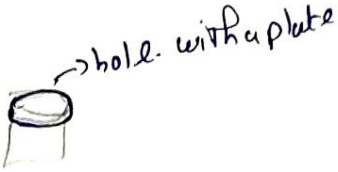
$$V_1 = \sqrt{2 \times 9.81\, m/s^2 \times 0.12 m}$$

$$V_1 = \sqrt{2 \times 9.81 \times 0.12\, (m^{\cancel{2}}/s^{\cancel{2}})}$$

$$\boxed{V_1 = 1.5342\, m/s}$$

Orifice meter

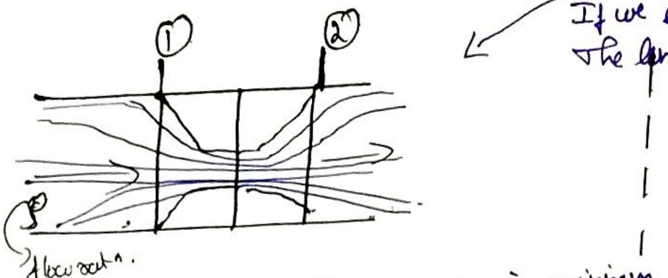
⇒ to calculate flow rate through pipe.

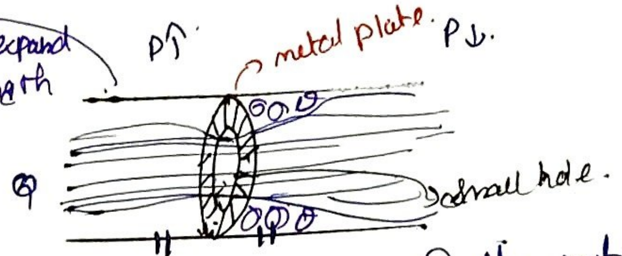


⇒ orifice it like a tank opening.

⇒ a flow measuring instrument that uses a sharp-edged hole in a plate to measure the discharge (flow rate) in pipe.

⇒ to understanding deeply we want to compare ventury meater & orifice meater.





⇒ we know where the flow rate is minimum	⇒ we dont know where the flow rate is minimum.
⇒ $C_d = 1$	⇒ $C_d = 0.6$ to 0.8
⇒ less loss	⇒ more loss.
⇒ Second section is long so we can get recovered. more loss in pressure.	⇒ Due to turbulence more loss
⇒ Expensive, bulk, req more space.	⇒ cheap easy to install.
⇒ accurate, low energy loss, but Costly & large.	⇒ Simple, cheap but more energy loss

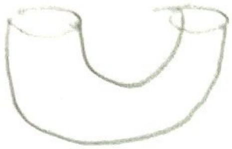
- $$Q_{th} = \frac{\overset{\text{Pipe section}}{a_1}\,\overset{\text{A in min}}{a_2}\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

- $$Q_{th} = \frac{\overset{\text{pipe section}}{a_1}\,\overset{\text{small hole}}{a_0}\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

Co-efficient of Contraction = $\boxed{C_c = \frac{A_2}{A_0}}$ ⇒ flow rate is minimum.

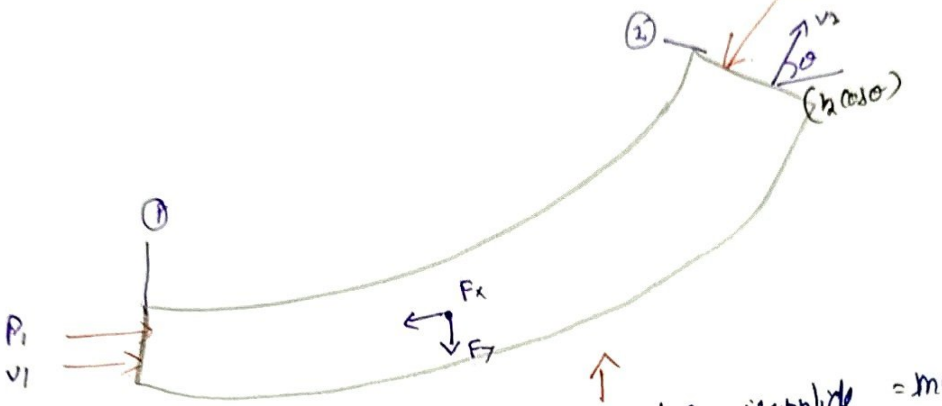
If a_0 not given in question use this formula

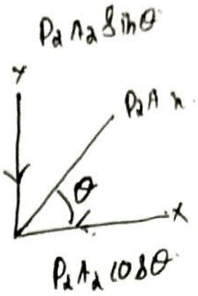
Mattonal Balance {⇒10M}#



⇒ what is total force acting on bending pipe

⇒ it is unsteady ⇒ NO friction loss





$$\Sigma F = \frac{m \cdot a}{t}$$

- F_{net} = total (net) force acting on object.
- m = mass of the object.
- a ⇒ acceleration Producer.

- If more force ↑ is applide = more acceleration ↑.
- If mass is bigger ↑ = acceleration will ↓ smaller

ex:- $a = F/m = 20/10 = 2 m/s^2$ #

$$\Sigma F = \overset{\text{mass flow rate}}{\dot{m}} \cdot (\Delta V)$$

- ΔV = changing in velocity $(V_2 - V_1)$
- m = mass flow rate.
- ΣF = net Force.

$$\Sigma F = \dot{m} \underbrace{(V_2 - V_1)}$$ → constant ⇒ impelx momentum equation.

In x-direction:-

$$P_1A_1 - F_x - P_2A_2\cos\theta = m\,(\overrightarrow{V_2\cos\theta - V_1})$$

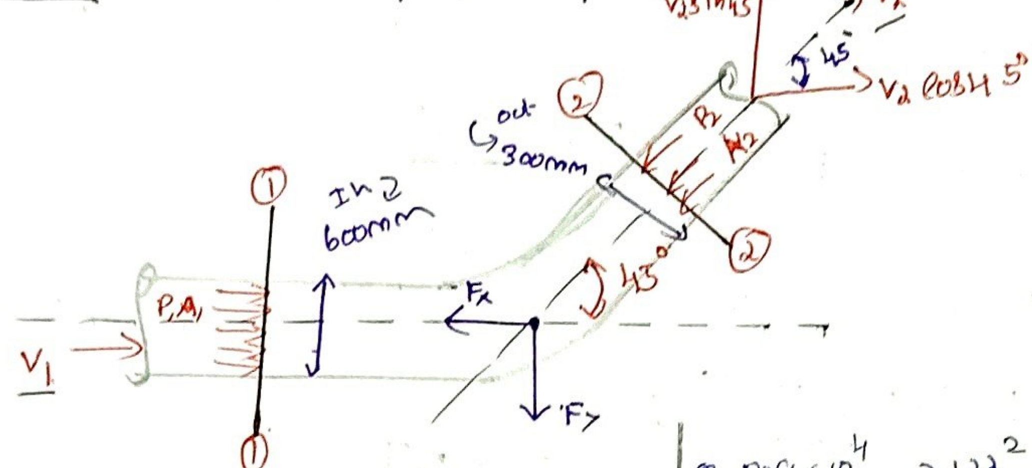
In y-direction:-

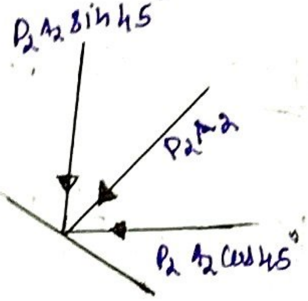
$$- P_2A_2\sin\theta - F_y = m\,(V_2\sin\theta - \overset{\text{no y value}}{0})$$

by:- $F_R = \sqrt{F_x^2 + F_y^2}$ #

Sum ①

A 45° reducing bend is connected in a pipe line, the diameter at the inlet and the outlet of the bend 600mm and 300mm respecti… Find the magnitude and direction of the fource exerted by water on the bend if the intensity of Pressure at the inlet to bend is 8.829 N/cm² and rate of flow of water is 600 lit/s





Resultant

- $F_R = \sqrt{F_x^2 + F_y^2}$

X-direction:-

$$\Sigma F = \dot{m}(V_{out} - V_{in})$$

$$P_1A_1 - P_2A_2 \cos 45° - F_x = \underset{(\rho Q)}{\dot{m}}(V_2 \cos 45 - V_1) \rightarrow ①$$

Y-direction:-

$$-P_2V_2 \sin\theta - F_y = \underset{(\rho Q)}{\dot{m}}(V_2 \sin 45 - 0) \rightarrow ②$$

$P_1 = 8.829 \, N/cm^2 \Rightarrow 8.829 \times 10^4 \, N/m.$

$P_2 = ?$; $Q = 0.6 \, m^3/sec \Leftarrow 600 \, lit/s$

$$\boxed{Q = A_1V_1 = A_2V_2}$$

- $A = \frac{\pi}{4} \times 0.6^2 = 0.2827$

- $V_1 = \frac{Q}{A_1} \Rightarrow \frac{0.6}{0.2827} = 2.122 \, m/s$

- $A = \frac{\pi}{4} \times 0.3^2 = 0.07068$

- $V_2 = \frac{Q}{A_2} = \frac{0.6}{0.07068} = 8.488 \, m/s$

by using berholyes equation.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{2.122^2}{2 \times 9.81} = \frac{P_2}{9810} + \frac{8.488}{2 \times 9.81}$$

$$9 + 0.2295 = \frac{P_2}{9810} + 3.672.$$

$$\frac{P_2}{9810} = 9.2295 - 3.672.$$
$$= 5.5575 \, m \text{ of water.}$$

$P_2 = 5.5575 \times 9810 \, N/m^2$

$\boxed{P_2 = 5.45 \times 10^4 \, N/m^2}$ #

$\boxed{m = \rho AV = \rho Q}$

$F_x = \rho Q[V_1 - V_2\cos\theta] + P_1A_1 - P_2A_2\cos\theta.$

$= 1000 \times 0.6 [2.122 - 8.488 \cos 45] + 8.829 \times 10^4 \times 0.2827 - 5.45 \times 10^4 \times 0.07068 \times \cos 45$

$\Rightarrow -2327.9 + 24959.6 - 2720.3 \Rightarrow (24959.6 - 5048.2)$

$\Rightarrow \underline{19911.4 \, N}$ #

$F_y = \rho Q[-V_2\sin\theta] - P_2A_2\sin\theta.$

$\Rightarrow 1000 \times 0.6[-8.488 \sin 45°[-5.45 \times 10^4 \times 0.07068 \times \sin 45°]$

$\Rightarrow -3601.1 - 2721.1 \Rightarrow \underline{-6322.2 \, N}$ #

∴ Resultent force. $F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(19911.4)^2 + (-6322.2)^2}$

$\Rightarrow 20890.9 \, N$ #

∴ The angle made by resultant force.

$\tan\theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.2} = 0.3175$ #

$\theta = \tan^{-1}(0.3175)$

$= 17°36'$ #

Millennium Problem

NAvier -Stokes Equation

by General term Newton -2 law applicable only (Solid Partical)

$$F = m \times a$$

- Navier -Stokes are the Scientist who modify this equation for (Liquid's)

$$\text{kg (Newton-2)} = m = \frac{m}{V} = \rho \rightarrow ① \#$$ -: mass [m]

Fource [F] :-

$$F = \overset{\text{gravity}}{F_g} + \underset{\text{Pressure}}{F_p} + \overset{\text{viscosity}}{F_V} + \underset{\text{Turbulant}}{F_T} + \overset{\text{compressiblity.}}{F_c} \Rightarrow \text{General equation of motion.}$$

- Let we want to consider incompressible flow :- [omit F_c].

$$F = F_g + F_p + F_v + F_T \Rightarrow \text{Reynold's Equation.}$$

- [omit F_T]

$$F = F_g + F_p + F_V \Rightarrow \text{Navier-Stokes Equation.} \rightarrow ②$$

⇒ consider Ideal fluid [omit F_V]

$$F = F_g + F_p \Rightarrow \text{Euler's Equation.}$$

by differential form :-

$$\frac{dP}{\rho} + g\,dz + v\,dv = 0$$

by Integration

$$\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = 0$$

$$\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{constant.}$$

$$\frac{P}{\rho g} + z + \frac{v^2}{2g} = \text{constant} \quad \text{Bernoulli's}$$

$$\boxed{\frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + \underset{\text{losses}}{h_2}} \quad \text{Eq}$$

acceleration :- (Direction) → ③

$$\underbrace{\left(-\frac{\partial v}{\partial t} + v + \nabla v\right)}_{\text{analogs [mass} \times \text{acceleration]}}$$

The formula of Navier-Stokes equation.

$$\boxed{\underbrace{P}_{①}\underbrace{\left(-\frac{\partial v}{\partial T} + v + \nabla v\right)}_{③} = \underbrace{-\nabla P + \nabla T + f}_{②}}$$

① ③ ② → Equation's.