Spearman's Rank Correlation Coefficient:-

This measure is useful when quantitive measure for certain factor cannot be fixed but the individual in the group can be arranged, by obtaining for each individual a number indicating rank in the group.

(such as evaluation of leadership or Judgement of beauty)

It is defined as

$$R = 1 - \frac{6 \Sigma D^2}{N^3 - N}$$

D refers to the difference of ranks between paired item in two series

The value of this ~~cof~~ coefficient interpreted in the same way as Karl Pearson's correlation coefficient, ranges b/w +1 and -1. When r_s is +1, there is complete agreement in the order of the ranks and ranks are in same direction. When $r_s = -1$, there is complete disagreement in the order of the rank and they are in opposite direction.

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Ex

R_1	R_2	$D = R_1 - R_2$	D^2
1	1	0	0
2	2	0	0
3	3	0	0
			$\Sigma D^2 = 0$

R_1	R_2	$D = R_1 - R_2$	D^2
1	3	-2	4
2	2	0	0
3	1	2	4
			$\Sigma D^2 = 8$

$$R = 1 - \frac{6 \Sigma D^2}{N^3 - N}$$

$$R = 1 - \frac{6 \times 0}{3^3 - 3} = 1$$

$$\boxed{R = 1}$$

$$R = 1 - 6\frac{\Sigma D^2}{N^3 - N}$$

$$R = 1 - \frac{6 \times 8}{3^3 - 3}$$

$$R = 1 - 2$$

$$\boxed{R = -1}$$

The spearman rank correlation coefficient is nothing but Karl pearson's coeff. b/w the ranks. Hence it can be interpreted in the same ~~samm~~ manner as Pearsonian correlation coefficient.

In Rank correlation, we may have three types of problems:-

(1) when ranks are given

(2) when ranks are not given

(3) when ranks are equal.

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Q:- Seven method of imparting business education were ranked by MBA students of two univ as follows:-

Method of Teaching:	I	II	III	IV	V	VI	VII
Rank by Students of "Univ A":	2	1	5	3	4	7	6
Rank by students of "Univ B":	1	3	2	4	7	5	6

calculate rank correlation coeff. and comment on its value.

Solution:- Let $R_A \rightarrow$ rank by univ A student's
$R_B \rightarrow$ rank by univ B students

Method of Teaching	R_A	R_B	$D = R_A - R_B$	D^2
I	2	1	1	1
II	1	3	-2	4
III	5	2	3	9
IV	3	4	-1	1
V	4	7	-3	9
VI	7	5	2	4
VII	6	6	0	0
				$\Sigma D^2 = 28$

$$R = 1 - \frac{6 \Sigma D^2}{N^3 - N}$$

$$R = 1 - \frac{6 \times 28}{7^3 - 7} = 1 - 0.5 = 0.5$$

$$\boxed{R = 0.5}$$

there is moderate degree of +ve correlation b/w the rank given by students of univ A and univ B.

Q: Two Judges in a beauty competition rank the 12 entries as follows

X	1	2	3	4	5	6	7	8	9	10	11	12
Y	12	9	6	10	3	5	4	7	8	2	11	1

what degree of disagreement b/w the judgement of two judges

Ans -.454

Q: Ten competitors in a beauty contest are ranked by three Judges in the following order:-

Ist Judge:	1	6	5	10	3	2	4	9	7	8
IInd Judge:	3	5	8	4	7	10	2	1	6	9
IIIrd Judge:	6	4	9	8	1	2	3	10	5	7

Use rank correlation coeff. to determine which pair of Judges has nearest approach to common taste in beauty.

R_1	R_2	R_3	$D_1^2 = R_1 - R_2$	$D_2^2 = (R_2 - R)^2$	$D_3^2 = (R_1 - R_3)^2$
1	3	6	4	9	25
6	5	4	1	1	4
5	8	9	9	1	16
10	4	8	36	16	4
3	7	1	16	36	4
2	10	2	64	64	0
4	2	3	4	81	1
9	1	10	64	81	1
7	6	5	1	1	4
8	9	7	1	4	1
			$\Sigma D_1^2 = 200$	$\Sigma D_2^2 = 214$	$\Sigma D_3^2 = 60$

$\Sigma D_1^2 = 200$, $\Sigma D_2^2 = 214$ $\Sigma D_3^2 = 60$

$$R_{(I \text{ and } II)} = 1 - \frac{6 \Sigma D^2}{N^3 - N} = \frac{1 - 6 \times 200}{10^3 - 10} = -0.212$$

$$R_{(II\ \&\ III)} = 1 - \frac{6 \Sigma D_2^2}{N^3 - N}$$

$$R_{(II\text{ and }III)} = -.297$$

Now $$R_{(I\ \&\ III)} = 1 - \frac{6 \Sigma D^2}{N^3 - N}$$

$$= 1 - \frac{6 \times 60}{10^3 - 10}$$

$$R_{(I\ \&\ III)} = .636$$

coeff. of correlation is max. in the judgement of I^{st} and III^{rd} judges, so they have nearest approach to common taste in beauty.

(2) When ranks are equal :-
when actual data is given not the ranks then rank can be assigned by taking either highest value as 1 or lowest values as 1.

Q: ~~Calculate~~ Ten salesmen registered the following figures of sales performance, their sales experience is given in table in years.

Salesman :	1	2	3	4	5	6	7	8	9	10
Sales exp (yrs):	5	3	2	7	6	8	4	13	1	9
Sales (Rs thousand):	83	48	76	93	116	89	86	173	84	132

Using the spearman's rank correlation coeff, find out whether the sales exp and sales performance are correlated.

Soln:- First we rank the data:-

Salesman	Exp X	R_x	Sales Y	R_y	$D^2 = (R_x - R_y)^2$
1	5	5	83	3	4
2	3	3	48	1	4
3	2	2	76	2	0
4	7	7	93	7	0
5	6	6	116	8	4
6	8	8	89	6	4
7	4	4	86	5	1
8	13	10	173	10	0
9	1	1	84	4	9
10	9	9	132	9	0

$$R = 1 - \frac{6 \Sigma D^2}{N^3 - N}$$

$\boxed{R = .842}$ yes they are correlated.

(3) When Ranks are equal or repeated:-

In some cases we need to rank two or more individual or entries having equal value. In such cases each individual will be assigned an average rank.

If two individual are ranked equal at 5^{th} plance, then they each given as rank

$$\frac{5+6}{2} = 5.5$$

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If three individual ranked equal at 5^{th} place then each given as rank

$$\frac{5+6+7}{3} = \frac{18}{3} = 6$$

In this case, an adjustment in the formula for calculating the rank coefficient of correlation is made.

this adjustment $= \frac{1}{12}(m^3 - m)$ to ΣD^2
where m = no of items with common rank.

$$R = 1 - \frac{6(\Sigma D^2 + C)}{N^3 - N},$$

where c is adjustment given by

$$C = \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \text{-------}$$

m_1 = no. of time first observation is repeated
m_2 = no. of time second observation is repeated.

ie

$$R = 1 - \frac{6\left(\Sigma D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + \cdots\right)}{N^3 - N}$$

Q:- Calculate rank coeff. of correlation:-

X:	140	142	140	160	150	155	160	157	140	170
Y:	43	45	42	50	45	52	57	48	49	53

x	Y	R_x	R_y	$D = R_x - R_y$	D^2
$\boxed{140}$	43	9	9	0	0
142	$\boxed{45}$	7	7.5	-.5	.25
$\boxed{140}$	42	9	10	-1	1
160	50	2.5	4	-1.5	2.25
150	$\boxed{45}$	6	7.5	-1.5	2.25
155	52	5	3	2	4
160	57	2.5	1	1.5	2.25
157	48	4	6	-2	4
$\boxed{140}$	49	9	5	4	16
170	53	1	2	-1	1

$\Sigma D^2 = 33$

Giving highest value as rank 1.

in x :- 160 is ranked as $\frac{2+3}{2} = 2.5$

in 140 " " " $\frac{8+9+10}{3} = \frac{27}{3} = 9$

Ily in Y, 45 is ranked as $\frac{7+8}{2} = 7.5$

$m_1 = 2, \quad m_2 = 3 \quad m_3 = 2$

Adjustment

$$C = \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \frac{1}{12}(m_3^3 - m_3)$$

$$C = \frac{1}{12}(8-2) + \frac{1}{12}(27-3) + \frac{1}{12}(8-2)$$

$$C = \frac{6}{12} + \frac{24}{12} + \frac{6}{12}$$

$$C = \frac{1}{2} + 2 + \frac{1}{2}$$

$$\boxed{C = 3}$$

ie $$R = 1 - \frac{6(\Sigma D^2 + C)}{N^3 - N}$$

$$R = 1 - \frac{6 \times (33+3)}{10^3 - 10}$$

$$R = 1 - \frac{6 \times 36}{1000 - 10}$$

$$R = 1 - \frac{216}{990}$$

$$\boxed{R = 0.7818}$$