



VIT

Vellore Institute of Technology

REG. NO.:

SLOT: A1+TA1+TAA1

SCHOOL OF ADVANCED SCIENCES
DEPARTMENT OF MATHEMATICS
FALL SEMESTER - 2024-2025
Continuous Assessment Test - II

Programme Name & Branch : B. Tech
 Course Code & Course Name : BMAT201L & Complex Variables and Linear Algebra
 Class Number(s) : Common Slot QP
 Faculty Name(s) : Common Slot QP
 Date of Examination : 13. 10. 2024
 Exam Duration : 90 Minutes

Maximum Marks : 50

General Instruction(s):

• Answer ALL Questions.

• **Open Book Test:** Only one text book and hand written materials are allowed.

- Evaluate $\int_C z e^{\pi z} \left(\frac{1}{z^4 - 16} + 1 \right) dz$, where C is the ellipse $9x^2 + y^2 = 9$, using Cauchy's residue theorem. (10 marks)
- Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4 \cos \theta} d\theta$. (10 marks)
- Solve the systems of equations $x_1 + x_2 + x_3 - x_4 = -2$; $2x_1 - x_2 + x_3 + x_4 = 0$; $3x_1 + 2x_2 - x_3 - x_4 = 1$; $x_1 + x_2 + 3x_3 - 3x_4 = -8$ by Gauss-Jordan elimination method. (10 marks)
- Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and hence find its inverse. Also, find the eigen values of A , A^T , and A^{-1} . (10 marks)
- (a) Verify the following are subspaces of $\mathbb{R}^3(\mathbb{R})$ or not? (10 marks)
 - $W = \{(x, y, z) \in \mathbb{R}^3 : xyz = 0\}$,
 - $W = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 - z^2 = 0\}$.
 (b) Determine the following sets are linearly independent or not.
 - $\{x, 2x - x^2, 3x + 2x^2\} \subseteq \mathcal{P}_3(\mathbb{R})$, where $\mathcal{P}_3(\mathbb{R})$ is a polynomial space.
 - $\left\{ \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ -3 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ -1 & 7 \end{bmatrix} \right\} \subseteq \mathcal{M}_{2 \times 2}(\mathbb{R})$, where $\mathcal{M}_{2 \times 2}(\mathbb{R})$ is a matrix space.