

Greedy Technique

DR. P. GAYATHRI

PROFESSOR

SCOPE

VIT VELLORE

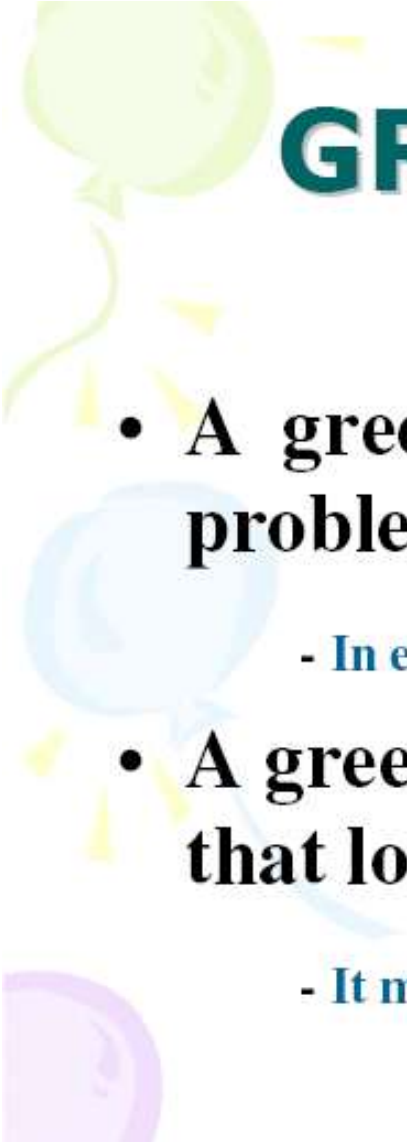
Greedy Approach

Best suited for optimization problems – Have some condition to satisfy and objective function.

- Maximization problem – demands maximum result
- Minimization problem – demands minimum result

Feasible solution - Any solution that satisfies the given condition is called feasible solution.

Optimal solution – A feasible solution that meets the objective function is called optimal solution



GREEDY TECHNIQUE

- A greedy algorithm builds a solution to a problem in steps.
 - In each iteration it adds a part of the solution.
- A greedy algorithm always makes the choice that looks best at the moment.
 - It moves from local optimal solution to global optimal solution.



BASIC EXAMPLES

- Knapsack Problem
- Minimum Spanning Tree Problem
 - Prim's Algorithm
 - Kruskal's Algorithm
- Shortest Path Problem
 - Dijkstra's Algorithm
 - Floyd Warshall's Algorithm
- Tree Vertex Splitting Problem
- Coin Changing Problem

Knapsack problem

Variations

Fractional Knapsack - Fractions are allowed. This applies to items such as:

- Bread - taking half

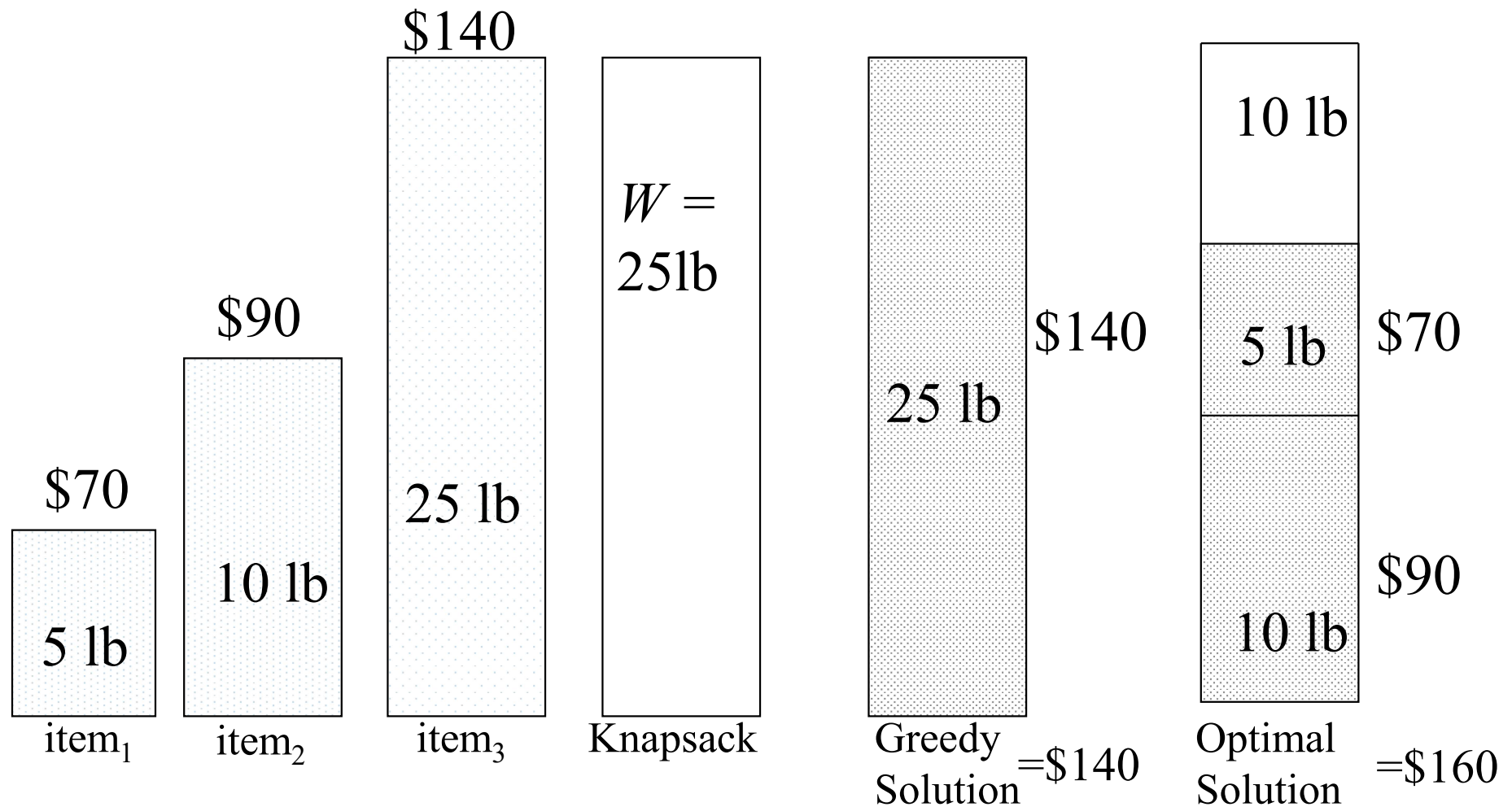
0/1 - No fractions.

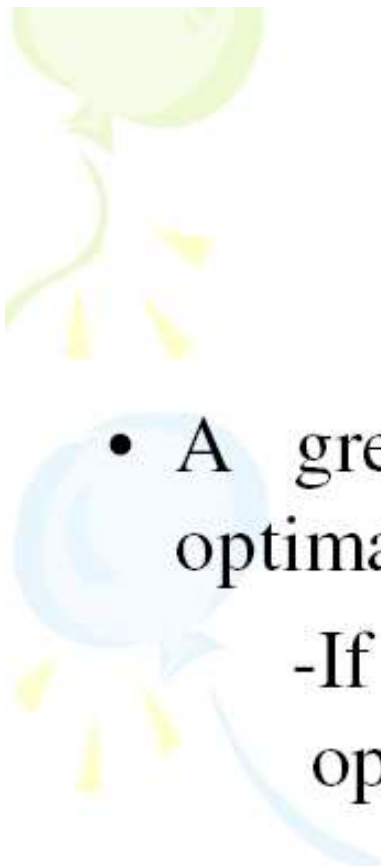
- 0/1 (1 brown pant, 1 green shirt...)

Greedy for 0/1 knapsack problem

Selection criteria : *Maximum beneficial item*

$$S = \{ (item_1, 5, \$70), (item_2, 10, \$90), (item_3, 25, \$140) \}$$





OBSERVATION

- A greedy algorithm may or may not be optimal.
 - If it is not optimal, it is sometimes close to optimal.

Fractional knapsack

KNAPSACK PROBLEM

- Input:

Number of objects available = n

Capacity of the knapsack = m

Weight of the object $i = w_i$

Profit for the object $i = p_i$

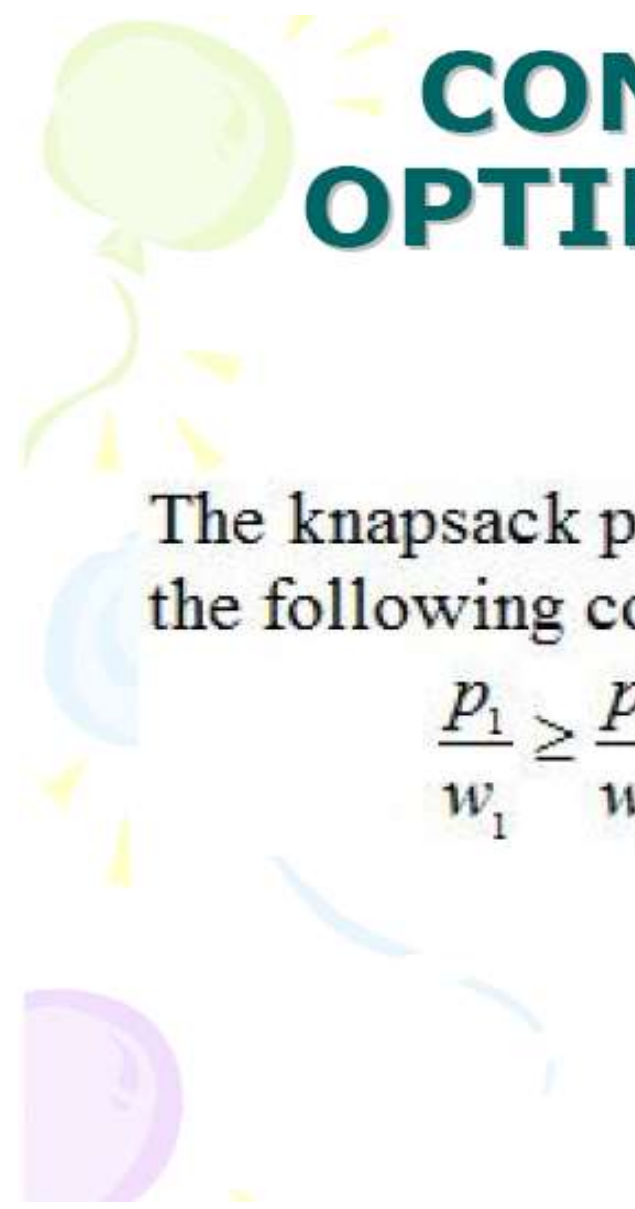
- Output: A full knapsack that maximizes the profit

- Problem Formulation

Maximize $Z = \sum_{i=1}^n x_i p_i$

Subject to $\sum_{i=1}^n w_i x_i \leq m$

with $0 \leq x_i \leq 1$ for $i = 1, 2, \dots, n$



CONDITION FOR OPTIMAL SOLUTION

The knapsack problem yields an optimal solution if the following conditions hold.

$$\frac{p_1}{w_1} \geq \frac{p_2}{w_2} \geq \frac{p_3}{w_3} \geq \dots \geq \frac{p_n}{w_n}$$



ALGORITHM

Greedy Knapsack (m, n, p_i, w_i)

For $i = 1$ to n

Set $x[i] = 0.0$

$U = m$

For $i = 1$ to n

If ($w[i] > U$)
then Break

Else

$x[i] = 1.0$

$U = U - w[i]$

If ($i \leq n$)

$x[i] = U/w[i]$

Example

Let $m=20$, $n=3$, $(p_1, p_2, p_3) = (25, 24, 15)$, $(w_1, w_2, w_3) = (18, 15, 10)$

$$\text{Obj1} = 25/18 = 1.39$$

$$\text{Obj2} = 24/15 = 1.6$$

$$\text{Obj3} = 15/10 = 1.5$$

Changed sequence: $(p_1, p_2, p_3) = (24, 15, 25)$, $(w_1, w_2, w_3) = (15, 10, 18)$

$$i=1 \ x[1] = 0$$

$$i=2 \ x[2] = 0$$

$$i=3 \ x[3] = 0$$

$$U = 20$$

i=1 $15 > 20 - F$ $x[1] = 1.0$ $U = 20 - 15 = 5$

i=2 $10 > 5 - T$ break

If $(2 \leq 3) - T$ $X[2] = 5/10 = 1/2 = 0.5$

Therefore, Result = (1,0.5,0)

Profit = (24, 7.5, 0) = 31.5

Weight = (15, 5,0) = 20