MODE

Mode is that value in a set of observation which occurs with the greatest frequency.

Ex 3, 5, 8, 5, 4, 5, 9, 3
Mode of the series is 5.

Note:- when there are two or more values having the same max. frequency, one cannot say which is the modal value and hence mode is said to be ill-defined. Such a series is called bimodal or multimodal.

Ex:-

Size of item:-	110	120	130	140
No of time it occurs:-	2	3	3	2

since 120 & 130 have same frequency ie mode is ill defined in this case.

Calculation of Mode:- For Discrete frequency distribution

Ex:-

x:	1	2	3	4	5	6	7	8
f:	4	9	16	25	22	15	7	3

Here max. frequency is 25 and value of x corresponding to that is 4. So
mode = 4

but in anyone of the following cases, mode can be calculated by method of grouping:-

(1) if max. frequency is repeated.
(2) max. frequency occurs in the very begining or at the end of the distribution.
(3) If there are other irregularity in the distribution.
4) If the difference b/w the max frequency

and the frequency preceding it or succeeding it is very small.

method of grouping :-

Procedure :-

(a) write the actual frequencies
(b) add the frequencies in two's.
(c) add again in two's leaving out the first frequency.
(d) add in three's.
(e) add in three's leaving out the first frequency.
(f) add in three's leaving out the first two frequencies.

Q find the mode

size :	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
freq :	48	52	56	60	63	57	55	50	52	41	57	63	52	48	40

Soln :-

Size	Frequency (f) (1)	(2)	(3)	(4)	(5)	(6)
5	48	100				
6	52		108			
7	56	116		156		
8	60		123		168	
9	63	120		180		179
10	57		112		175	
11	55	105		157		162
12	50		102			
13	52	93		161	143	
14	41		98			150
15	57	120		140		
16	63		115		173	163
17	52	100				
18	48		88			
19	40					

classmate
Date
Page

Analysis Table

column NO. (I)	Max. frequency (II)	combination of values of x giving max freq. (III)
(1)	63	9, 16
(2)	120	9, 10, 15, 16
(3)	123	8, 9
(4)	180	8, 9, 10
(5)	175	9, 10, 11
(6)	179	7, 8, 9

From column (III), we find that the value 9 is repeated max no. of time (6 times) therefor **mode = 9**

Calculation of Mode in continuous frequency distribution:-

Class having max. frequency is called modal class

say l_1 = lower limit
h = length of class
f_1 is freq. of modal class
f_0 is freq of class preceding the modal class
f_2 is freq of class succeeding the modal class.

$$\text{Mode} = l_1 + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2}$$

Q Find the mode of the following distribution

Class:	0-10	10-20	20-30	30-40	40-50	50-60
freq:	5	8	7	12	28	20

60-70	70-80
10	10

Soln:- max. freq. is 28
Modal class 40-50
$f_1 = 28$, $f_0 = 12$, $f_2 = 20$
$h = 10$, $l_1 = 40$

$$\text{Mode} = 40 + \frac{10 \times (28-12)}{2\times 28 - 12 - 20}$$

$$\text{Mode} = 40 + 6.666$$

$$\boxed{\text{Mode} = 46.67}$$

Q The median and mode of the following wage distribution are known to be Rs 33.50 and Rs. 34 respectively. Find values of f_3, f_4, f_5.

Wages	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
freq	4	16	f_3	f_4	f_5	6	4	230

Wages	frequency f	cf
0-10	4	4
10-20	16	20
20-30	f_3	$20+f_3$
30-40	f_4	$20+f_3+f_4$
40-50	f_5	$20+f_3+f_4+f_5$
50-60	6	$26+f_3+f_4+f_5$
60-70	4	$30+f_3+f_4+f_5$
	$\Sigma f = 230$	

given that $\boxed{f_3 + f_4 + f_5 = 200}$ — (1)

median is 33.50, which lies in class 30-40.

ie using $\text{median} = l_1 + \frac{h}{f}\left(\frac{N}{2} - C\right)$

$h = 10$, $f = f_4$ $N = 230$, $C = 20 + f_3$

$$33.50 = 30 + \frac{10}{f_4}\left(115 - 20 - f_3\right)$$

$3.50 f_4 = 10(95 - f_3)$

$.350 f_4 = 95 - f_3$

$.350 f_4 + f_3 = 95$ ——— (2)

Now Mode = 34

ie Modal class is 30-40

$$\text{Mode} = l + \frac{h(f_4 - f_3)}{2f_4 - f_3 - f_5}$$

$$34 = 30 + \frac{10(f_4 - f_3)}{2f_4 - f_3 - f_5}$$

$4(2f_4 - f_3 - f_5) = f_4 - f_3$

$.8 f_4 - .4 f_3 - .4 f_5 = f_4 - f_3$

$-.2 f_4 + .6 f_3 - .4 f_5 = 0$ ——— (3)

Using $f_3 + f_4 + f_5 = 200$

$f_3 + f_5 = 200 - f_4$

ie

$$4 = \frac{10(f_4 - f_3)}{2f_4 - 200 + f_4}$$

$4(3f_4 - 200) = 10(f_4 - f_3)$

$2f_4 + 10 f_3 = 800$

$f_4 + 5 f_3 = 400$ ——— (3)

from (3) & (2)

$f_4 = 100$, $f_3 = 60$ and $f_5 = 40$.

8 calculate the mode

marks	No. of students
above 0	80
above 10	77
above 20	72
above 30	65
above 40	55
above 50	43
above 60	28
above 70	16
above 80	10
above 90	8
above 100	0

Soln Commulation frequency is given, first we will convert it into simple frequency distribution

Marks	freq
0-10	3
10-20	5
20-30	7
30-40	10
40-50	$12 = f_0$
50-60	$15 = f_1$
60-70	$12 = f_2$
70-80	6
80-90	2
90-100	8

Max freq. is 15 ie Modal class is 50-60.

here $f_1 = 15$ $f_0 = 12$ $f_2 = 12$

$l = 50$

$$\text{mode} = l + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2}$$

$$= 50 + \frac{10(3)}{2 \times 15 - 12 - 12} = 55.$$

Q The following are the marks obtained by student in a class test. Find the modal marks

Marks	Students
32-35	10
36-39	37
40-43	65
44-47	80
48-51	51
52-55	35
56-59	18
60-63	4

Hint:- Modal class 43.5 - 47.5
Ans 44.864

Q From the following data, determine modal weight.

weight	100-110	110-120	120-130	130-140	140-150	150-160	160-170	170-180
Marks	4	6	20	32	33	17	8	2

Soln as max. freq. is 33, but difference b/w max frequency and preceding freq. is very less ie we can use grouping method to find mode

weight	Marks (I)	(II)	(III)	(IV)	(V)	(VI)
100-110	4	10				
110-120	6		26			
120-130	20	52		30		
130-140	32				58	
140-150	33	50	65			85
150-160	17			82		
160-170	8	10	25		58	
170-180	2					27

column no	max freq	combinative of class giving max freq.
(1)	33	140-150
(2)	52	120-130, 130-140
(3)	65	130-140, 140-150
(4)	82	130-140, 140-150, 150-160
(5)	58	110-120, 120-130, 130-140, 140-150, 150-160, 160-170
(6)	85	120-130, 130-140, 140-150

we have

Total no.

110-120	120-130	130-140	140-150	150-160	160-170
1	3	5	5	1	1

This is bi-modal distribution. ie mode can be found by applying

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

Median = 139.69

Mean = 139.51

Mode = 3 Median − 2 Mean = 140.05 Ans.

Q Compute Mode for the following data

2, 5, 3, 2, 1, 4, 6, 3, 7.

Soln 2 and 3 have freq. 2 so it is bimodal data. Here we will find mode, using [Mode = 3 Median − 2 Mean]

$\bar{x} = 3.6667$

Median = 3

ie Mode = 3×3 − 2×3.6667

Mode = 1.6666

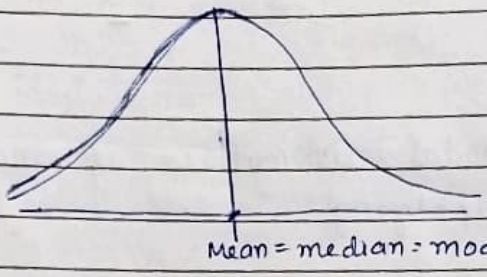
Karl Pearson gave relationship b/w mean, median and mode

$$\text{Mean} - \text{Median} = \frac{1}{3}(\text{Mean} - \text{Mode})$$

or $$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

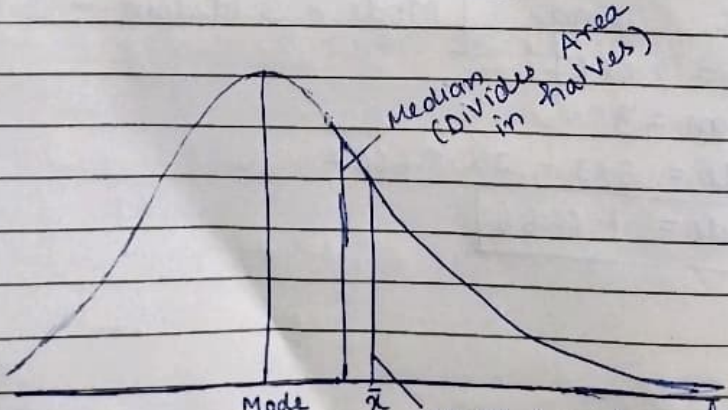
Graphical representation:-

A distribution in which mean, median and mode coincide ie (mean = mode = median) is known as symmetrical distribution.



When the values of mean, median and mode are not equal, the distribution is known as asymmetrical or skewed. In such distributions a very important relationship exists among mean, median and mode.

Distance b/w mean and median is about one third the distance between the mean and mode.

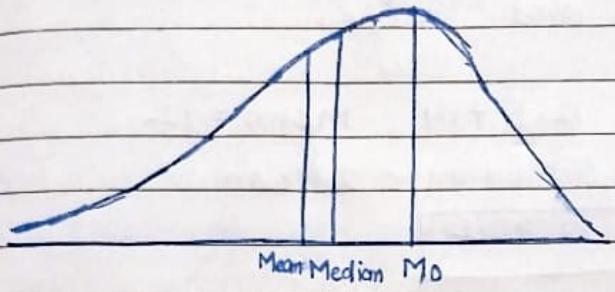


Positively Skewed

Mode < Median < Mean

Negatively Skewed

Mean < Median < Mode



Q:- In a Asymmetrical distribution A.M = 15, Mode = 12. Find the value of Median.

Soln

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

$$3\,\text{Median} = 45 - 3$$

$$\boxed{\text{Median} = 14}$$

Q:- Compute Mode for following data

Size :	0-4	4-8	8-12	12-16	16-20
freq :	10	20	30	35	35.

Soln:- Highest freq. is 35 and it corresponds to two bottom class intervals. ie given distribution is bimodal. So we will find mode using relationship b/w Median, mean and mode.

Here Median = 12.5714, Mean = 12

Mode = 3 Median − 2 Mean

Mode = 13.7142

Geometric Mean

For a set of n observations, Geometric mean is defined as

$$G.M = (x_1 x_2 x_3 \cdots x_n)^{1/n} = \text{Antilog}\left(\frac{\sum_{i=1}^{n} \log x_i}{n}\right)$$

ie n^{th} root of their product.

For discrete series

$$G.M = \text{Antilog}\left(\frac{\Sigma f \log x}{n}\right)$$

for cont series

$$G.M = \text{Antilog}\left(\frac{\Sigma f \log m}{n}\right)$$

m is mid value of class

Harmonic Mean:-

Harmonic mean of n no. of observations is the reciprocal of the arithmetic mean of reciprocal of the given observations.

ie

$$H.M = \frac{1}{\frac{1}{n}\left(\frac{1}{x_1} + \frac{1}{x_2} + -- + \frac{1}{x_n}\right)}$$

$$H.M = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

← for individual observations

For discrete series

$$H.M = \frac{n}{\sum_{i=1}^{n} \left(f_i \cdot \frac{1}{x_i}\right)}$$

for continuous series:-

$$H.M = \frac{n}{\sum_{i=1}^{n} \left(f_i \frac{n}{m_i}\right)}$$

m_i is mid point of class.

Positional Measures

Besides median, there are other measures which divides a series into equal parts. Important measures among these are

(1) quartiles (2) deciles (3) percentiles

As just one point divides a series into two parts

Quartiles are those values of variate which divides the series (total frequency) into four equal parts. As only 3 points will divide the series into 4 equal parts. Therefore, there are three quartiles Q_1, Q_2, and Q_3.

Deciles :- The nine points which divides the series in to 10 equal parts are called deciles. They are denoted by D.

Percentiles :- The 99 points, which divides the series into hundred equal parts are called percentiles. denoted by P.

Deciles formula

D_x = Size of $\frac{x}{10}(N+1)^{th}$ item

Percentile / formula

Computation of Quartiles:-

First Quartile:-

For individual observations and Discrete series:-

Q_1 = size of $\frac{N+1}{4}^{th}$ item

For continuous series

Q_1 = size of $\frac{N}{4}^{th}$ item i.e $\boxed{Q_1 = l + \frac{h}{f}\left(\frac{N}{4} - c\right)}$

Second Quartile:-

Q_2 coincides with median.

Third Quartile:-

For individual observations and Discrete series:

Q_3 = size of $\frac{3(N+1)^{th}}{4}$ item

For continuous series:-

Q_3 = size of $\frac{3N^{th}}{4}$ item

i.e $\boxed{Q_3 = l + \frac{h}{4}\left(\frac{3N}{4} - c\right)}$

The first Quartile Q_1 is the point which has 25% observations before it and 75% observations after it. Sly third Quartile Q_3 has 75% observation before & 25% observation after it.

Q Calculate Q_1 & Q_3 for the following data:-

Roll No:	1	2	3	4	5	6	7
Marks:	20	28	40	12	30	15	50

Soln arranging the marks in increasing order

12 15 20 28 30 40 50

Q_1 = Size of $\frac{N+1}{4}^{th}$ item

$Q_1 = \frac{7+1}{4}^{th}$ item's size

$= 2^{nd}$ item size

$\boxed{Q_1 = 15}$

Q_3 = Size of $\frac{3(N+1)}{4}^{th}$ item

Q_3 = Size of $\frac{3(7+1)}{4}^{th}$ item

$Q_3 = 6^{th}$ item size

$\boxed{Q_3 = 40}$

Q Compute Q_1 and Q_3 for following data:

Class :	10-20	20-30	30-40	40-50	50-60	60-70	70-80
f :	12	19	5	10	9	6	6

Solution :-

Marks	frequency	c.f
10-20	12	12
20-30	19	31
30-40	5	36
40-50	10	46
50-60	9	55
60-70	6	61
70-80	6.	67

$N = 67$

Q_1 = size of $\frac{N}{4}^{th}$ item

= size of $\frac{67}{4}^{th}$ item

= Size of 16.75^{th} item

ie Q_1 lies in 20-30 interval.

$$Q_1 = l + \left(\frac{N}{4} - c\right)\frac{h}{f}$$

$f = 19 \quad c = 12 \quad N = 67, \; l = 20$

$$Q = 20 + \left(\frac{67}{4} - 12\right) \times \frac{10}{19} = 20 + 2.5 = 22.5.$$

$$\boxed{Q_1 = 22.5}$$

Q_3 = Size of $\frac{3N}{4}^{th}$ item

Q_3 = Size of $\frac{3}{4} \times 67^{th}$ item

Q_3 = Size of 50.25^{th} item

ie Q_3 lies in 50-60 class

$l = 50, \quad f = 9 \quad c = 46$

$$Q_3 = l + \left(\frac{3N}{4} - c\right)\frac{h}{f}$$

$$Q_3 = 50 + \left(50.25 - 46\right) \times \frac{10}{9}$$

$$Q_3 = 50 + 4.072$$

$$\boxed{Q_3 = 54.072}$$

Formula for Deciles:-

D_r = Size of $r\frac{(N+1)}{10}^{th}$ item (for individual and discrete series)

Ex D_4 = Size of $\frac{4}{10}(N+1)^{th}$ item (" ")

& D_r = Size of $\frac{r}{10}N^{th}$ item (for cont. series)

Ex: D_6 = Size of $\frac{6}{10}N^{th}$ item (" ")

Formula for Percentiles:-

Similarly.

P_r = Size of $r\frac{(N+1)}{100}^{th}$ item (for individual and discrete)

Ex P_{60} = Size of $\frac{60\times(N+1)}{100}^{th}$ item (" ")

& P_r = Size of $\frac{r\,N}{100}^{th}$ item (for continuous series)

Ex P_{40} = Size of $\frac{40\,N}{100}^{th}$ item (" ")

In last question:-

P_{60} = Size of $\frac{60\,N}{100}^{th}$ item

$P_{60} = 40.2^{th}$ item,

$P_{60} = l + \frac{h}{f}\left(\frac{60N}{100} - c\right)$

$P_{60} = 40 + \frac{10}{10}(40.2 - 36) = 44.2$

D_4 = Size of $\frac{4N}{10}^{th}$ item

D_4 = Size of 26.8^{th} item

ie D_4 lies in 20-30 class

$$D_4 = l + \frac{h}{f}\left(\frac{4N}{10} - c\right)$$

$$D_4 = 20 + \frac{10}{19}(26.8 - 12)$$

$$\boxed{D_4 = 27.79}$$ Ans.