M/E/TX

Reg. No:

Final Assessment Test – May 2024

VIT Vellore Institute of Technology

Course: **BMAT102L - Differential Equations and Transforms**

Class NBR(s): **1527 / 1529 / 1530 / 1533 / 1534 / 1539 / 1548 / 1549 / 1583 / 1584 / 1585 / 1588 / 1589 / 4953 / 4956**

Slot: **C1+TC1+TCC1**

Time: **Three Hours**

Max. Marks: **100**

- **KEEPING MOBILE PHONE/ELECTRONIC DEVICES EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Find the general solution of $x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+3y=x^4\log_e x\ (x>0)$ using method of undetermined coefficients. **[10]**

2. An LCR circuit connected in series has $R=10$ ohms, $C=10^{-2}$ farad, $L=\frac{1}{2}$ henry and an applied voltage $E=12$ volts. Assuming no initial current and no initial charge at $t=0$ when the voltage is first applied, find the subsequent current in the system at any time t without finding charge. **[10]**

3. Solve the Lagrange's equation $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$ **[10]**

4. Solve $\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial t}=u$ with $u(x,0)=6e^{-3x}$ using the method of separation of variables. **[10]**

5. (a) Find the Laplace transform of a periodic function $f(t)=\begin{cases}1 & if\ 0<t<1\\ t & if\ 1<t<2\end{cases}$ with period 2. **[5]**

   (b) Evaluate $\int_0^\infty e^{-2t}\,t\,u(t-2)\,dt$ using Laplace transform. **[5]**

6. Find the inverse Laplace transform of $\frac{1}{s(s^2+2s+2)}$ using convolution theorem. **[10]**

7. Obtain the solution of $u_x+u_t=0$ with $u(0,t)=0(t>0)$ and $u(x,0)=\cos x+\sin x(x>0)$ using Laplace transform. **[10]**

8. Obtain the half range cosine series for $f(x)=x(2-x)$ in the interval $(0,\ 2)$ and hence find the value of $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots$ **[10]**

9. Obtain the Fourier series for $f(x)=x^3-\pi^2x$ in the interval $(-\pi,\ \pi)$ and hence find the sum of the series $\sum_{n=1}^{\infty}\frac{1}{n^6}$ using Parseval's identity. **[10]**

10. Find the Fourier transform of $f(x) = e^{-a|x|}$ $(a > 0)$ and hence evaluate $\int_0^\infty \frac{\cos px}{a^2 + p^2} dp$. **[10]**

11. (a) Find $Z\left\{\cos\left(\frac{n\pi}{2}\right) + \sin\left(\frac{n\pi}{2}\right)\right\}$. **[5]**

(b) $Z^{-1}\left\{\frac{2z^2 + 3z}{z^2 - 2z - 8}\right\}$ using partial fractions method. **[5]**

12. Obtain the solution of $u_{n+2} - 5u_{n+1} + 6u_n = 5^n$ with $u_0 = 0,\ u_1 = 0$ using Z –Transform. **[10]**

⇔⇔⇔ **M/E/TX**⇔⇔⇔