Z/K/TX

Reg. No: [redacted]

Final Assessment Test – November 2024

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 2508 / 2509 / 2510 / 2511 / 2512 / 2513 / 2514 / 2515 / 2517 / 2518 / 2519 / 2520 / 2521 / 2522 / 2523 / 2525

Slot: A2+TA2+TAA2

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer **ALL** Questions

(10 X 10 = 100 Marks)

1.a) Show that the function $f(z)=\begin{cases}\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, & z\neq 0\\ 0, & z=0\end{cases}$ [10]

satisfies the Cauchy-Riemann equations at $z=0$. Is the function analytic at $z=0$?

OR

1.b) Prove that $u=x^2-y^2-2xy-2x+3y$ is harmonic. Find a function v such that $f(z)=u+iv$ is analytic. Also express $f(z)$ in terms of z. [10]

2. Find the image of the triangular region bounded the lines $x=1, y=1$ and $x+y=1$ under the transformation $w=z^2$. [10]

3. Find the bilinear transformation which maps the points $\infty, i, 0$ in the z-plane into $-1,-i,1$ in the w-plane. Also, find the fixed points of the transformation $w=\dfrac{z-1}{z+1}$. [10]

4. Obtain the Laurent series of the function $f(z)=\dfrac{7z-2}{z(z+1)(z-2)}$ about $z=-1$. [10]

5. Evaluate $\displaystyle\int_0^\infty \frac{x\sin mx}{x^2+a^2}dx\,(a,m>0)$ using contour integration. [10]

6. Find the dimension of the null space of $A=\begin{bmatrix}1&1&0&0&1\\0&0&1&-2&0\\4&2&0&0&3\\1&1&1&-2&1\\2&2&0&0&2\\1&1&2&-4&1\end{bmatrix}$. [10]

7.a) Show that the linear transformation $T:R^3(R)\to R^3(R)$ defined by $T(x,y,z)=(x+y+z,y+z,z)$ is invertible and hence find T^{-1}. [10]

OR

7.b) Let $T:R^3(R)\to R^2(R)$ be a linear transformation defined by $T(x,y,z)=(3x+2y-4z, x-5y+3z)$. Find the matrix of T relative to the bases $B_1=\{(1,1,1),(1,1,0),(1,0,0)\}$ and $B_2=\{(1,3),(1,5)\}$. [10]

8. Given that $B=\{(1,2,0),(8,1,-6),(0,0,1)\}$ is a basis of $R^3(R)$. Obtain an orthogonal and orthonormal bases of $R^3(R)$ using Gram-Schmidt orthogonalization process. [10]

9. If $A=\begin{bmatrix}1&2&-2\\2&5&-4\\3&7&-5\end{bmatrix}$, verify the Cayley-Hamilton theorem and evaluate $A^8-A^7+5A^6-A^5+A^4-A^3+6A^2+A-2I$. [10]

10. Apply the Gauss-elimination method to solve the following system of equations $2x+y+z=4; x+y-2z=3; -x-2y+z=1$. Also, find the eigen values of the coefficient matrix. [10]