MODULE - II

AC Circuits

Contents

POWER

∴ **In dc circuits,**

$$\text{Power, } P = VI$$

$$P = \frac{V^2}{R}$$

$$P = I^2 R$$

In AC circuits,

Instantaneous Power: $p(t)$

For AC circuits, the voltage and current are

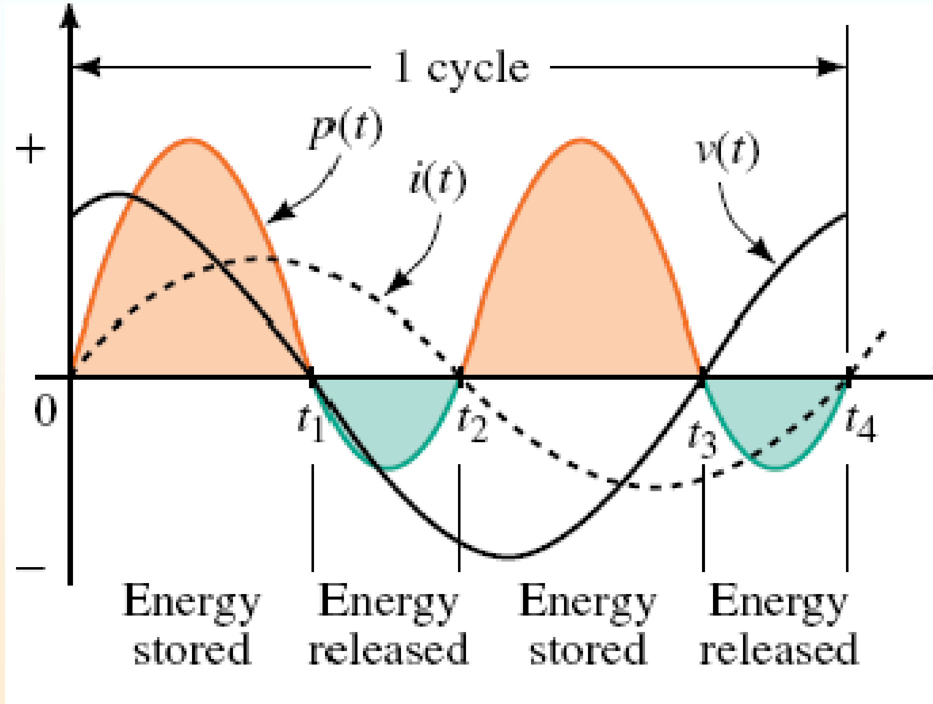
$$v(t) = V_M \cos(\omega t+\theta_v)$$

$$i(t) = I_M \cos(\omega t+\theta_i)$$

The *instantaneous power* is simply their product

$$p(t) = v(t)\, i(t) = V_M I_M \cos(\omega t+\theta_v) \cos(\omega t+\theta_i)$$

$$= \tfrac{1}{2} V_M I_M [\underbrace{\cos(\theta_v - \theta_i)}_{\text{Constant Term}} + \underbrace{\cos(2\omega t+\theta_v +\theta_i)}_{\text{Wave of Twice Original Frequency}}]$$



Average Power (P)

- Calculate average power (integrate power over one cycle and divide by period)

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\ dt = \frac{1}{T}\int_{t_0}^{t_0+T} \left[V_M \cos(\omega t + \theta_v)\right] \left[I_M \cos(\omega t + \theta_i)\right] dt$$

$$= \frac{1}{2} V_M\ I_M \cos(\theta_v - \theta_i)$$

- Recall that passive sign convention says:

P > 0, power is being absorbed

P < 0, power is being supplied

Complex Power (**S**)

- Complex power is expressed in units of volt-amperes like apparent power
- Complex power has no physical significance; it is a purely mathematical concept

Definition of *complex power*, S

$$\mathbf{S} = \mathbf{V}_{\mathrm{rms}} \mathbf{I}_{rms}^{*} = \mathrm{V}_{\mathrm{rms}} \angle \theta_v \ \mathrm{I}_{\mathrm{rms}} \angle -\theta_i$$

$$= \mathrm{V}_{\mathrm{rms}} \mathrm{I}_{\mathrm{rms}} \angle \theta_v - \theta_i$$

$$= \mathrm{V}_{\mathrm{rms}} \mathrm{I}_{\mathrm{rms}} \cos(\theta_v - \theta_i) + j \mathrm{V}_{\mathrm{rms}} \mathrm{I}_{\mathrm{rms}} \sin(\theta_v - \theta_i)$$

$$= P + jQ$$

$$\mathbf{S} = P + jQ = I_{rms}^2 \,\mathrm{Re}(\mathbf{Z}) + j I_{rms}^2 \,\mathrm{Im}(\mathbf{Z}) = I_{rms}^2 \mathbf{Z}$$

Real Power (**P**)

- *P* is the *real* or *average power*
- Alternate expressions for the *real* or *average power* (*P*)

$$P = \mathrm{Re}(\mathbf{S}) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$= |\mathbf{S}| \cos(\theta_Z) = (I_{rms} |\mathbf{Z}|) I_{rms} \left[\frac{\mathrm{Re}(\mathbf{Z})}{|\mathbf{Z}|} \right]$$

$$= I_{rms}^2 \,\mathrm{Re}(\mathbf{Z})$$

Reactive Power (Q)

- Q is the *reactive* or *quadrature power*, which indicates temporary energy storage rather than any real power loss in the element; and Q is measured in units of volt-amperes reactive, or var
- Alternate expressions for the *reactive* or *quadrature power* (Q)

$$Q = \mathrm{Im}(\mathbf{S}) = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$= |\mathbf{S}| \sin(\theta_Z) = (I_{rms}|\mathbf{Z}|) I_{rms} \left[\frac{\mathrm{Im}(\mathbf{Z})}{|\mathbf{Z}|} \right]$$

$$= I_{rms}^2 \, \mathrm{Im}(\mathbf{Z})$$

In circuits excited by ac sources, the voltage and current are sinusoidal quantities which varies with time. **When voltage and current are time varying quantities, the power is also a time varying quantity**.

For time varying quantities, the power is defined as average over a period of time. Since the average value of sinusoidal voltage and current are zero, we can take, the rms value of voltage and current.

We know that, the rms values of voltage and current are complex and so the **power is also complex**. "The complex power is denoted by S and it is defined as the product of rms voltage and the conjugate of rms current".

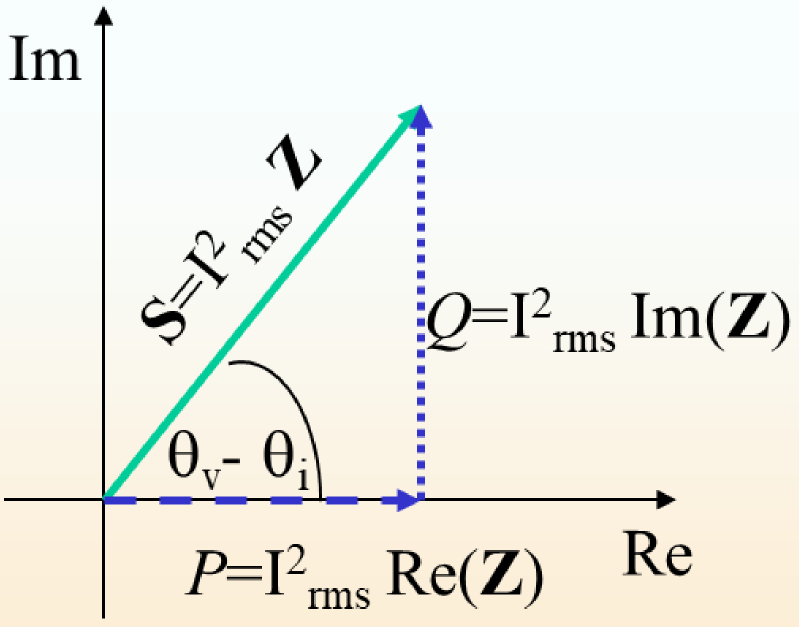
Various powers in AC circuits	**Unit**
1. Instantaneous power (*p*)	**- VA**
2. Complex power (S = P ± j Q)	**- VA**
3. Apparent Power [abs(S)]	**- VA**
4. True or Active or Real or Average power (P)	**- Watts**
5. Reactive power (Q)	**- VAR**

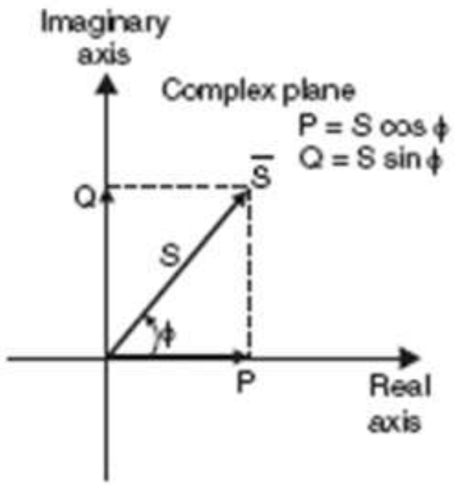
Power Triangle

- The *power triangle* relates *pf angle* to *P* and *Q*

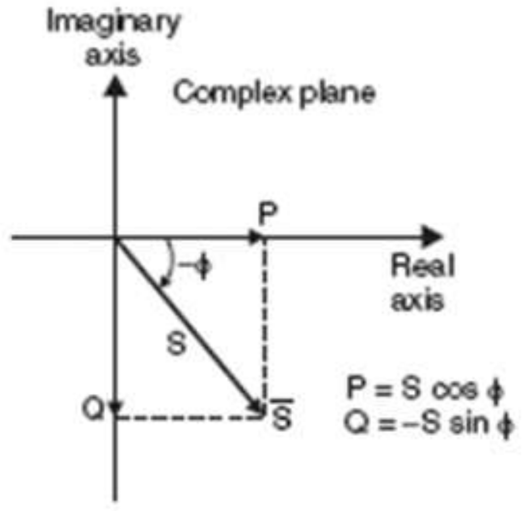
$$\tan(\theta_v - \theta_i) = \frac{Q}{P} = \frac{\textit{reactive/quadrature power}}{\textit{real/average power}}$$

  - the phasor current that is in phase with the phasor voltage produces the *real* (*average*) power
  - the phasor current that is out of phase with the phasor voltage produces the *reactive* (*quadrature*) power

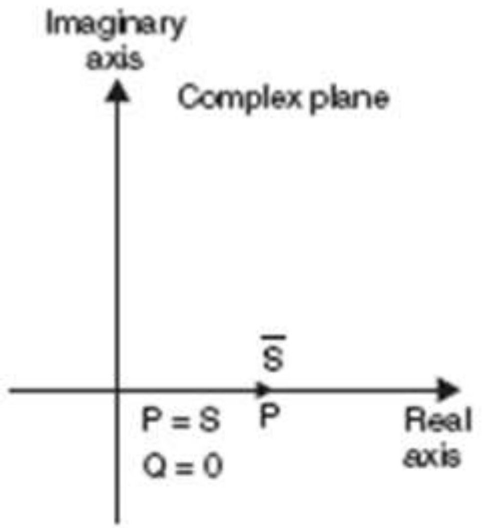




Vector of $\bar{S}$ when ϕ is positive.



Vector of $\bar{S}$ when ϕ is negative.



Vector of $\bar{S}$ when ϕ is zero.

With reference to Fig., we can write,

$$\bar{S} = |\bar{S}|\cos\phi + j|\bar{S}|\sin\phi$$

Let, $\bar{S} = P + jQ$

$$\therefore P = |\bar{S}|\cos\phi$$

$$Q = |\bar{S}|\sin\phi$$

We know that, $|\bar{S}| = S = VI$

$$\therefore P = VI\cos\phi \text{ in } W$$

$$Q = VI\sin\phi \text{ in } VAR$$

In Fig., the triangle formed by P, Q and S is also called **power triangle**.

The real part of S is called active power or simply power. The imaginary part of S is called reactive power. The power is denoted by P and expressed in the units of watts, *W*. The reactive power is denoted by Q and expressed in the units of volt-ampere-reactive, *VAR*.

In ac circuits, the phase angle φ may be positive, zero or negative. (Remember that φ is phase difference between V and I)

When φ is positive,
- the current lags voltage.
- the circuit is inductive.
- the active power is positive.
- the reactive power is positive.

When φ is zero,
- the current is in-phase with voltage.
- the circuit is resistive.
- the active power is positive.
- the reactive power is zero.

When φ is negative,
- the current leads the voltage.
- the circuit is capacitive.
- the active power is positive.
- the reactive power is negative.

The ratio of active power and apparent power is defined as power factor". The power factor is a measure of active power in the apparent power.

$$P = S \cos \varphi$$

$$\therefore \text{ Power factor } = \frac{\text{Active power}}{\text{Apparent power}} = \frac{P}{S}$$

Complex power in terms of V and I

$$\bar{S} = \bar{V}\bar{I}^*$$

$$\bar{V} = V_{rms}\angle\theta_v$$

$$\bar{I} = I_{rms}\angle\theta_i$$

Complex power in terms of Z

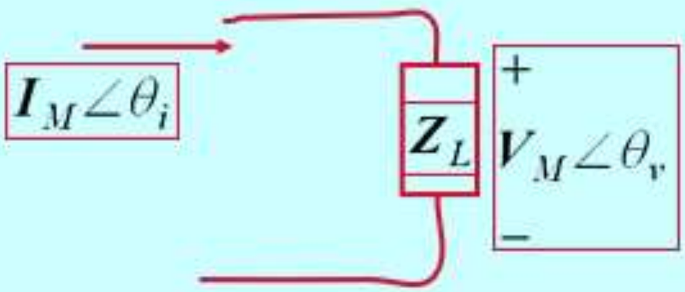
$$\bar{S} = \frac{|\bar{V}|^2}{Z^*} \qquad \bar{S} = |\bar{I}|^2 Z$$

Complex power should be calculated using RMS values of V and I

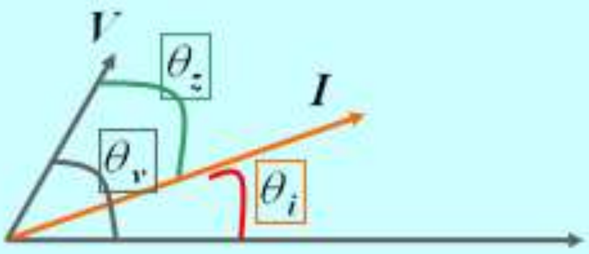
$$\bar{S} = \bar{V}\bar{I}^* = \bar{V}\frac{\bar{V}^*}{Z^*} = \frac{|\bar{V}|^2}{Z^*}$$

Power Factor

- Consider A Complex Current Thru a Complex Impedance Load



- The Current and Load-Voltage Phasors (Vectors) Can Be Plotted on the Complex Plane



- By Ohm & Euler

$$\mathbf{V} = \mathbf{ZI} \Rightarrow \angle V = \angle Z + \angle I$$

$$\theta_v = \theta_z + \theta_i$$

$$or \quad \theta_z = \theta_v - \theta_i$$

Power Factor (pf)

- Deviation of *power factor* ($0 \le pf \le 1$)

$$pf = \frac{average\ power}{apparent\ power} = \frac{P}{V_{rms}\ I_{rms}}$$

$$= \frac{V_{rms}\ I_{rms}\ \cos(\theta_v - \theta_i)}{V_{rms}\ I_{rms}} = \cos(\theta_v - \theta_i) = \cos(\theta_{Z_L})$$

- A low power factor requires more rms current for the same load power which results in greater utility transmission losses in the power lines, therefore utilities penalize customers with a low *pf*

Power Factor Angle (θ_{Z_L})

- *power factor angle* is $\theta_v - \theta_i = \theta_{Z_L}$ (the phase angle of the load impedance)
- *power factor* (*pf*) special cases
  - purely resistive load: $\theta_{Z_L} = 0°$ $\Rightarrow$ $pf=1$
  - purely reactive load: $\theta_{Z_L} = \pm 90°$ $\Rightarrow$ $pf=0$

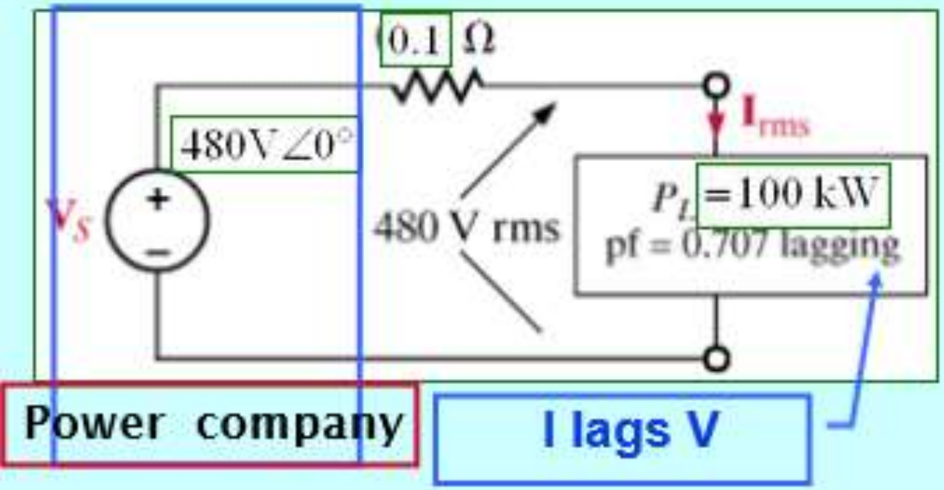
Power Factor Angle	I/V Lag/Lead	Load Equivalent
$-90° < \theta_{\mathbf{Z}_L} < 0°$	Leading	Equivalent RC
$0° < \theta_{\mathbf{Z}_L} < 90°$	Lagging	Equivalent RL

pf – Why do We Care?

- Consider this case
  - V_{rms} = 460 V
  - I_{rms} = 200A
  - pf = 1.5%
- Then
  - $P_{apparent}$ = 92kVA
  - P_{actual} =1.4 kW
- ∴ This Load requires The Same Power as a Hair Dryer
- However, Despite the low power levels, The WIRES and CIRCUIT BREAKERS that feed this small Load must be Sized for 200A!
  - The Wires would be nearly an INCH in Diameter

Example ➡ Power Factor

- The Local Power Company Services this Large Industrial Load



- Find I_{rms} by Pwr Factor

$$P = V_{rms} \times I_{rms} \times pf$$

$$I_{rms} = P / (pf \times V_{rms})$$

- Then the I^2R Loses in the 100 mΩ line

$$P_{losses} = I_{rms}^2 R_{line} = \frac{P^2 R_{line}}{V_{rms}^2} \times \frac{1}{pf^2}$$

$$P_{losses}(pf = 0.707) = \frac{10^{10} \times 0.1}{480^2} \times \frac{1}{0.707^2} (W)$$

$$= 4.34 kW \times 2$$

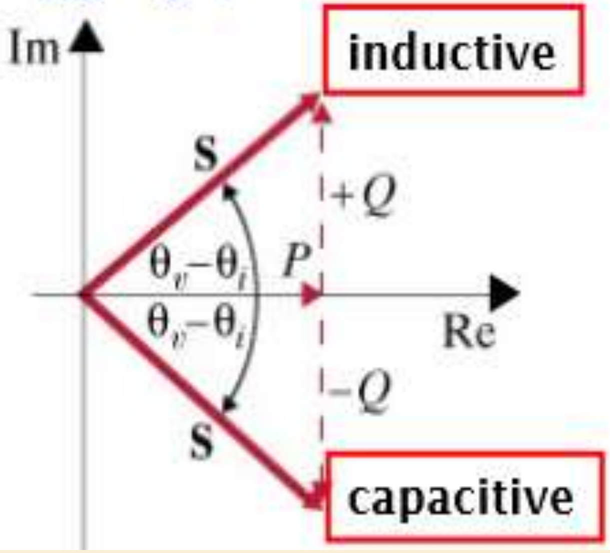
- Improving the pf to 94%

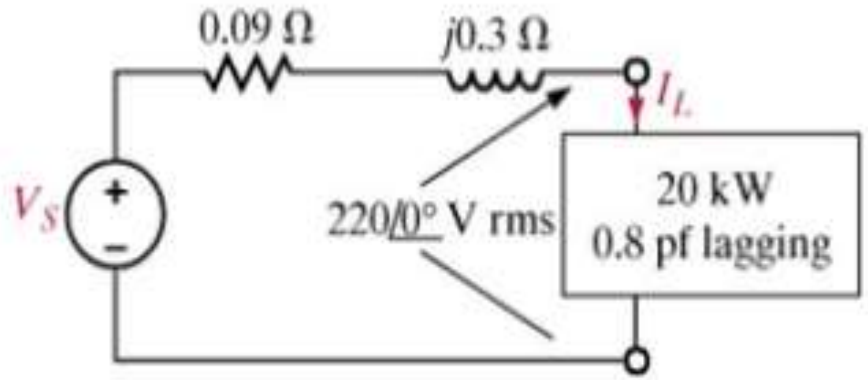
$$P_{losses}(pf = 0.94) = \frac{10^{10} \times 0.1}{480^2} \times \frac{1}{0.94^2} (W)$$

$$= 4.34 kW \times 1.13$$

$$\boldsymbol{P_{saved}} = 0.87 \times 4.34 \boldsymbol{kW} = 3.77 \boldsymbol{kW}$$

- For the Circuit At Right
  - $\mathbf{Z}_{line}$ =0.09 Ω + j0.3 Ω
  - P_{load} = 20 kW
  - $\mathbf{V}_{load}$ = 220∠0°
  - pf = 80%, lagging
  - f = 60 Hz → ω = 377s^{-1}
- Lagging pf → Inductive





- From the Actual Power

$$P = \mathrm{Re}\{\mathbf{S}\} = |\mathbf{S}|\cos(\theta_v - \theta_i) = S \times pf$$

- Thus

$$\therefore \quad S_L = \frac{P}{pf} = \frac{20kW}{0.8} = 25kVA$$

- And Q from Pwr Triangle

$$Q^2 = S_L^{\,2} - P^2 \Rightarrow Q = 15\,\mathrm{kVAR}$$

Example - Complex Power cont

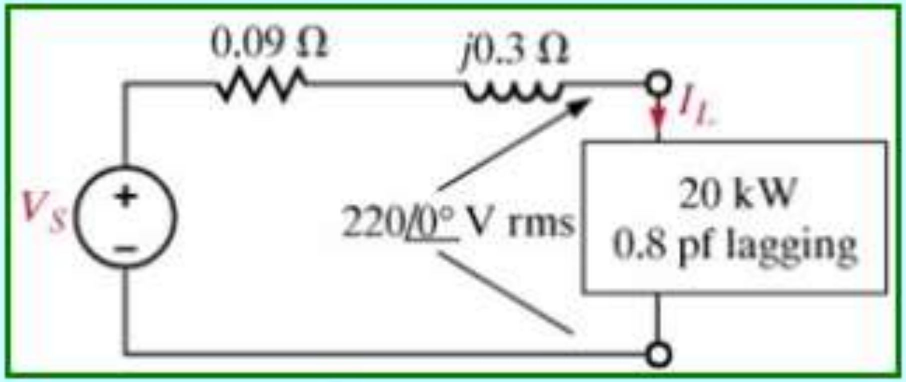
- Then $\mathbf{S}_L$

$$\mathbf{S}_L = (20 + j15)kVA = 25kVA\angle 36.87°$$

- Recall the **S** Mathematical Definition

$$\mathbf{S}_L = \mathbf{V}_L \mathbf{I}_L^*$$

- Note also that [**U***]* = **U**
- In the S Definition, Isolating the Load Current and then Conjugating Both Sides



$$\Rightarrow \mathbf{I}_L = \left[\frac{\mathbf{S}_L}{\mathbf{V}_L}\right]^* = \left[\frac{25kVA\angle 36.87°}{220V\angle 0°}\right]^*$$

$$\mathbf{I}_L = 113.64\angle -36.86°(A)$$

- Alternatively

$$\mathbf{I}_L = \left[\frac{20{,}000 + j15{,}000}{220}\right]^*$$

$$\mathbf{I}_L = 90.91 - j68.18(A)$$

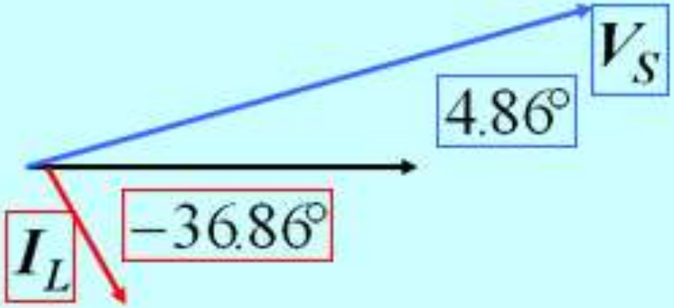
Example - Complex Pwr cont.2

- Now Determine $\mathbf{V}_S$

$$\mathbf{V}_S = \Delta\mathbf{V}_{line} + \Delta\mathbf{V}_L$$

$$\mathbf{V}_S = (0.09 + j0.3)\mathbf{I}_L + 220\angle 0°$$

$$\mathbf{V}_S = (0.09 + j0.3)(90.91 - j68.18) + 220(V)$$



- Then $\mathbf{V}_S$

$$\mathbf{V}_S = 248.63 + j21.14$$

$$\mathbf{V}_S = 249.53V_{rms}\angle 4.86°$$

- To find the Src Power Factor, Draw the **I** & **V** Phasor Diagram

- Then The Phase Angle

$$\theta_v - \theta_i = 4.86° - (-36.86°) = 41.72°$$

$\therefore$ V Leads I $\Rightarrow$ Inductive Load

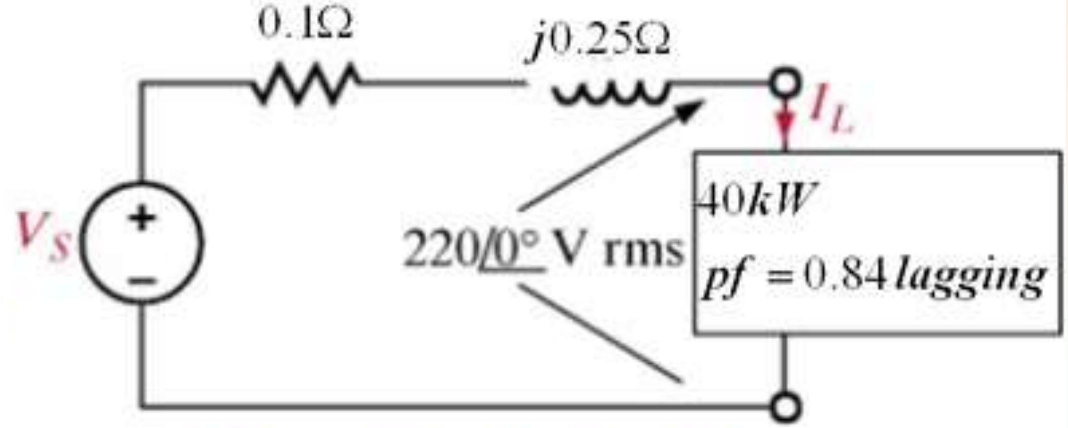
and also

$$pf = \cos(\theta_v - \theta_i) = \cos 41.72°$$

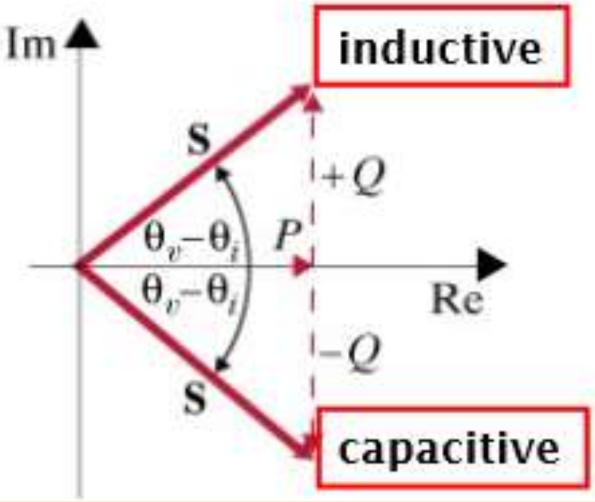
$$\therefore \quad pf = 0.7464$$

Example - Complex Power kVAR

- For the Circuit At Right, Determine
  - Real And Reactive Power losses in the Ln
  - Real And Reactive Power at the Source



- Lagging pf → Inductive



- From the Actual Power

$$P = \mathrm{Re}\{\mathbf{S}\} = |\mathbf{S}|\cos(\theta_v - \theta_i) = S \times pf$$

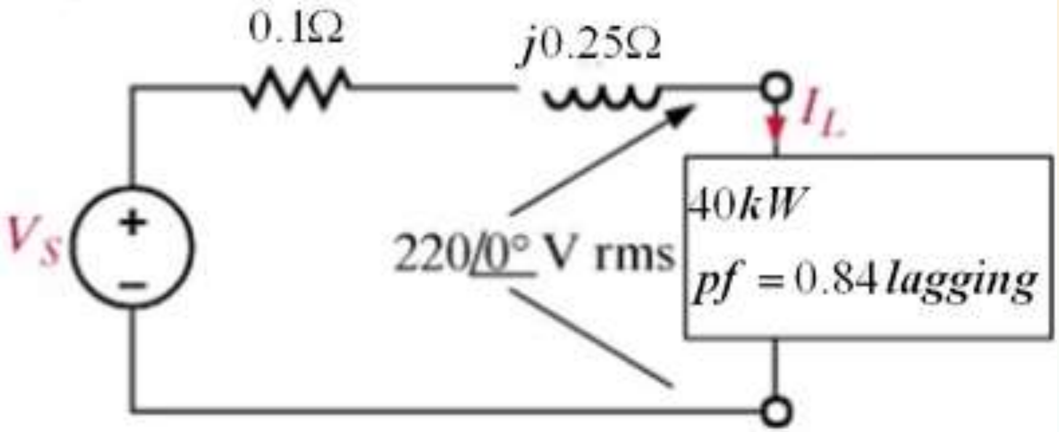
- Thus

$$\therefore \quad S_L = \frac{P}{pf} = \frac{40kW}{0.84} = 47.62kVA$$

- And by S Definition

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* \Rightarrow I_L = \frac{S_L}{V_L} = 216.45(A)_{rms}$$

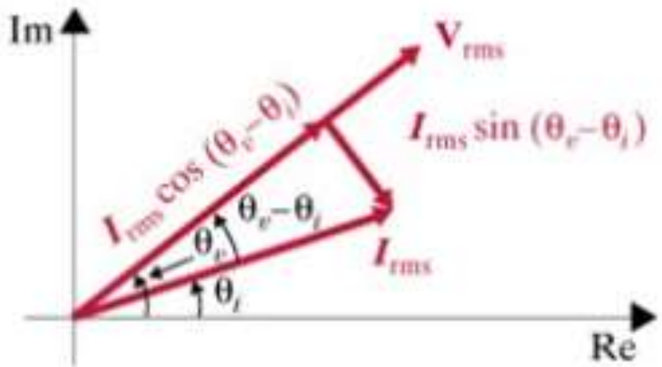
Example - Complex kVAR cont.



- Also from the S Relation

$$|\mathbf{Q}_L| = \sqrt{|\mathbf{S}_L|^2 - P^2} = 25{,}839(VAR)$$

- Now the Power Factor Angle



- pf = cos($\theta_v - \theta_i$); hence

$$\theta_v - \theta_i = \operatorname{acos}(0.84) = 32.86°$$

- Then for Line Loses

$$\mathbf{S}_{line} = \Delta\mathbf{V}_{line}\mathbf{I}^*_{line} = (\mathbf{Z}_{line}\mathbf{I}_L)\mathbf{I}^*_L$$
$$= \mathbf{Z}_{line} I_L^2$$

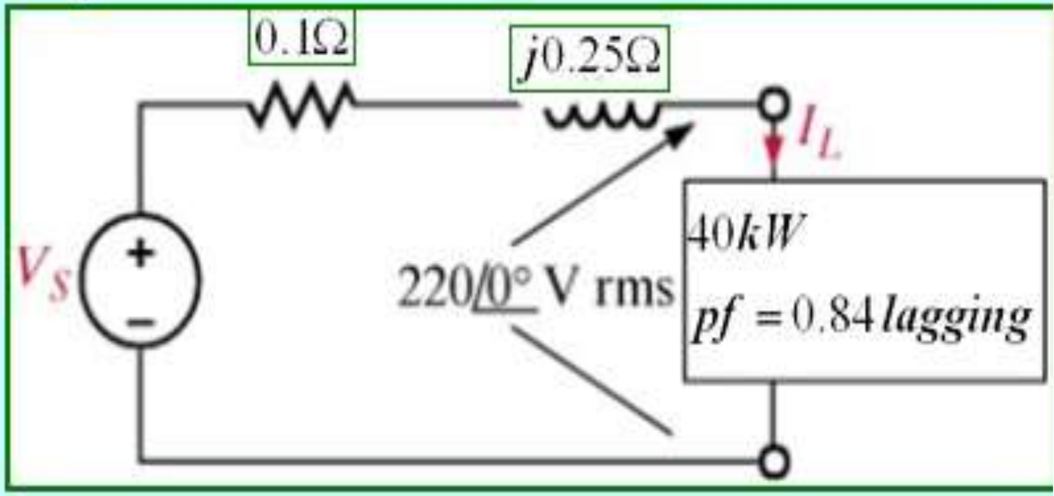
- Quantitatively

$$\mathbf{S}_{line} = (0.1 + j0.25)(216.45)^2$$
$$\therefore \quad \mathbf{S}_{line} = (4685 + j11713)VA$$

Example - Complex kVAR cont.2

- Find Power Supplied by Conservation of Complex Power

$$\mathbf{S}_{Supplied} = \mathbf{S}_{line} + \mathbf{S}_{Load}$$



- In this Case

$$\mathbf{S}_{Sup} = (4.685 + j11.713) + (40 + j25.839)$$
$$= (4.685 + 40) + j(11.713 + 25.839)$$
$$= 44.685 + j37.552 \quad kVA$$
$$= 58.37\angle 40.04° \quad kVA$$

- Then to Summarize the Answer
  - P_{line} = 4.685 kW
  - Q_{line} = 11.713 kVAR
  - P_S = 44.685 kW
  - Q_S = 37.552 kVAR