Continuous Assessment Test (CAT - 1), February 2024

Programme	:	**B.Tech.**	Semester	:	**Winter 2023-2024**
Course Title	:	**Engineering Physics**	Course Code	:	**BPHY101L**
School	:	**School of Advanced Sciences**	Slot	:	**D1+TD1**
Duration	:	**90 mins**	Max. Marks	:	**50**
Class No	:				

Part – A (5 ×10 = 50)

Answer ALL Questions

Sl. No	Questions	Max Marks	CO	BL
1	Derive the classical wave equation for a one-dimensional traveling wave excited on a string under constant tension T with a clear and well labelled diagram.	**10**	**CO1**	**L2**
2	Obtain the wave function for a stationary wave produced on a string of length L, fixed at both the ends (nodes), and by applying suitable boundary conditions derive the expression for frequency of first three harmonics and sketch the wave patterns.	**10**	**CO1**	**L3**
3	**a)** Calculate the Reflection (R) and Transmission (T) coefficients of a wave travelling on two strings joined together with a massless knot under same tension but of different linear densities (μ_1 and μ_2), when speed of wave in first string (v_1) is double that on the second string, using related equations. Also draw the incident, reflected and transmitted waves. **b)** A long string of linear density 10 g/cm is joined to another long string of linear density 40 g/cm and the combination is held under constant tension. A transverse sinusoidal wave of amplitude 30 cm and wavelength 250 cm is launched along the lighter string. Calculate the wavelength and amplitude of the wave when it is travelling along the heavier string.	**5** **5**	**CO1**	**L4**
4	Show that light is an electromagnetic wave that travels with speed $\boldsymbol{c}$ (3×10^8 m/s) in free space. Further, starting with a traveling $\boldsymbol{E}$-wave ($\boldsymbol{E_y} = \boldsymbol{E_o}\, sin(kz - \omega t)$), obtain the expression for the corresponding $\boldsymbol{B}$-*wave*.	**10**	**CO1**	**L2**
5	**a)** Explain the physical significance of divergence of a vector ($\boldsymbol{\nabla} . \boldsymbol{A}$) and curl of a vector ($\boldsymbol{\nabla} \times \boldsymbol{A}$). Further, comment on their appropriate use in Maxwell's equations. **b)** An electromagnetic wave traveling with velocity of light has an electric field $\boldsymbol{E_z} = 100\, sin\,(80y + \omega t)$. Find its wavelength ($\lambda$), angular frequency ($\omega$), direction of propagation and expression of magnetic field, $\boldsymbol{B}$.	**5** **5**	**CO1**	**L4**

Winter 2023-34, Engineering Physics, Slot D1, CAT-1 Key

QP setter : V. Suresh

1. Standard derivation of the classical wave equation for 1D traveling wave produced on a string under uniform tension T, linear mass density μ ....

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$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \; ; \quad v = \text{Speed the wave}$$

$$v = \sqrt{\frac{T}{\mu}}$$

2.

Equation of stationary wave

$$y = A\sin(kx-\omega t) + A\sin(kx+\omega t) = A\sin kx\cos\omega t \quad \text{— (1)}$$

Boundary condition $y(0) = y(L) = 0 \Rightarrow \sin kL = 0$

$$k_nL = n\pi \quad \text{— (2)}$$

$$k_n = \frac{2\pi}{\lambda_n} \; ; \; \frac{2\pi L}{\lambda_n} = n\pi$$

$$\boxed{\lambda_n = \frac{2L}{n}} \quad \text{— (3)}$$

$\lambda_1 = 2L$ $(n=1)$

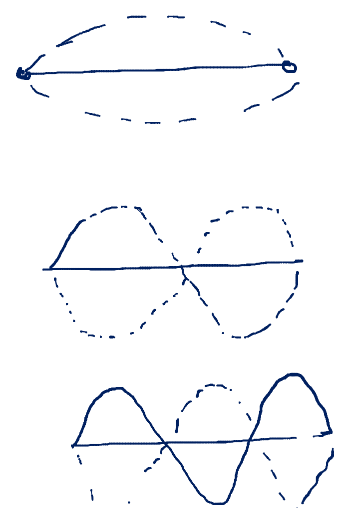
$\lambda_2 = L$ $(n=2)$

$\lambda_3 = 2L/3$ $(n=3)$

$n=1$

$n=2$

$n=3$



$$\boxed{f_n = \frac{V}{\lambda_n} = n\cdot\frac{V}{2L}} \quad \text{— (4)}$$

$f_1 = \frac{V}{2L}$

$f_2 = \frac{V}{L}$

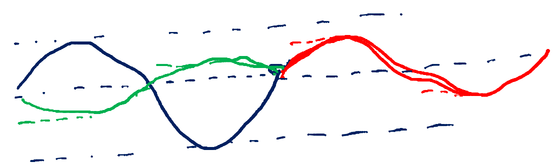
$f_3 = \frac{3V}{2L}$

3a.

$$R = \frac{V_2 - V_1}{V_1 + V_2} \; ; \; T = \frac{2V_2}{V_1 + V_2}$$

(i) $V_1 = \sqrt{T/\mu_1}$; $V_2 = \sqrt{\frac{T}{\mu_2}}$;

$V_1 = 2V_2 \Rightarrow R = -1/3$; $T = 2/3$



- Reflected wave ($1/3^{rd}$ of Ai)
- Transmitted wave ($2/3^{rd}$ of Ai)
- Incident wave (Ai)

Ai is the amplitude of incident wave.

3b.

$\mu_1 = 10 g/cm$ $\qquad$ $\mu_2 = 40 g/cm$

$A_i = 30$ cm $\qquad$ $A_T = ?$

$\lambda_1 = 250$ cm $\qquad$ $\lambda_2 = ?$

$$V_1 = \sqrt{\frac{T}{\mu_1}} \; ; \; V_2 = \sqrt{\frac{T}{\mu_2}} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{\mu_2}{\mu_1}}$$

$$V = \lambda f \Rightarrow \frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{\mu_2}{\mu_1}}$$

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{\mu_1}{\mu_2}} \Rightarrow \lambda_2 = \frac{250}{2} = \underline{\underline{125 \text{ cm}}} \; ; \qquad \frac{V_2}{V_1} = \frac{\lambda_2}{\lambda_1} = \frac{1}{2} \text{ or } V_1 = 2V_2$$

$$T = \frac{A_T}{A_i} = \frac{2V_2}{V_1+V_2} = \frac{2V_2}{3V_2} \Rightarrow A_T = A_i \cdot \frac{2}{3} = \underline{\underline{20 \text{cm}}}$$

$$\boxed{\begin{aligned} \lambda_2 &= 125 \text{ cm} \\ A_T &= 20 \text{cm} \end{aligned}}$$

4. Maxwell's III equation,

$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$, Applying $\vec{\nabla}\times$ on both sides

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{-\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2 E = \frac{-\partial}{\partial t}\left(\mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

↳ Maxwell's contribution.

In Vacuum

$$+\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E = \frac{1}{v^2} \frac{\partial^2 E}{\partial t^2}$$

$V = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = C$. So light is an EM wave

$E_x = E_0 \sin(kz - \omega t)$; $E_y = E_z = 0$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \frac{-\partial \vec{B}}{\partial t}$$

This results in the fact that if we have $E_x = E_0 \sin(kz - \omega t)$

$B_y = B_0 \sin(kz - \omega t)$

$\vec{E} \perp \vec{B}$ and $(\vec{E} \times \vec{B})$ is in the direction of propagat[ion]

5a. Physical significance of Div and Curl and their usage in Maxwell's equations.

5b.

$E_z = 100 \sin(80y + wt)$ or $\vec{E} = 100 \sin(80y + wt)\,\hat{k}$

It's a traveling E wave, with E-field oscillations along Z-axis and propagation along -y-axis.

$k = 80 = \frac{2\pi}{\lambda} \Rightarrow \lambda = \pi/40$;

$\frac{w}{k} = c \Rightarrow w = ck = 240 \times 10^8$;

$\vec{B} = \frac{100}{c} \sin(80y + wt)(-\hat{i})$