



BEEE203L

CIRCUIT THEORY

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MODULE – IV

Analysis of Transient Response of Circuits

Contents

- Review of Laplace transformation;
- Laplace transform of network
- Time domain solution for RL,RC, RLC networks for DC excitation
- Time domain solution for RL,RC, RLC networks for AC excitation
- Transient Behaviour of circuit elements under switching conditions and their representations
- Evaluation of initial and final conditions in RL,RC,RLC circuits with DC excitation
- Evaluation of initial and final conditions in RL,RC,RLC circuits with AC excitation

Review of LT

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- ▶ Laplace transform converts transforms the *differential equations* into *algebraic equation*
- ▶ Laplace transformation is used to transform the circuit from the time domain to the frequency domain, obtain the solution and apply the inverse Laplace transform to the result to transform it back to the time domain
- ▶ The Laplace transform is significant for a number of reasons
 - ▶ It can be applied to a wider variety of inputs
 - ▶ It provides an easy way to solve circuit problems involving initial conditions
 - ▶ It is capable of providing us, in one single operation, the total response of the circuit comprising both the natural and forced responses

Definition of the LT

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For a given function $f(t)$ the laplace transform is denoted by $F(s)$ or $\mathcal{L}(f(t))$

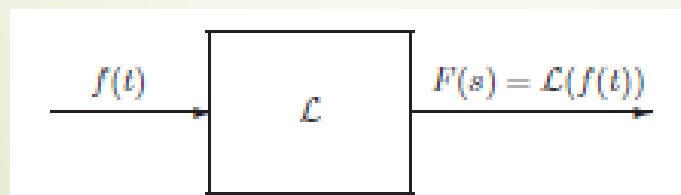
$$\mathcal{L}(f(t)) = F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt$$

where s is a complex variable given by

$$s = \sigma + j\omega$$

Equation (1) is called as *one-sided* Laplace transform

- The lower unit 0^- indicate a time just before $t = 0$
- The equation (1) illustrates the concept of transformation $f(t)$ to $F(s)$ i.e time-domain to s-domain (frequency)



► Laplace transform of unit step

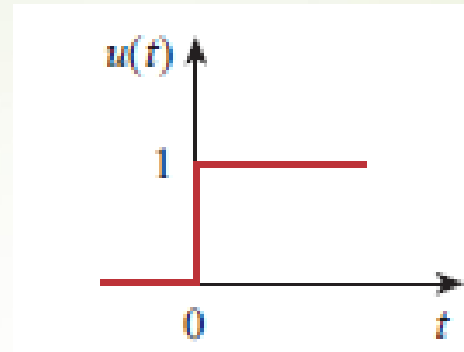


Figure: unit step function

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\mathcal{L}[u(t)] = \int_{0^-}^{\infty} 1e^{-st} dt = -\frac{1}{s}e^{-st} \Big|_0^{\infty}$$

$$= -\frac{1}{s}(0) + \frac{1}{s}(1) = \frac{1}{s}$$

LT pairs

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$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$

$f(t)$	$F(s)$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t + \theta)$	$\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$
$\cos(\omega t + \theta)$	$\frac{s \cos \theta - \omega \sin \theta}{s^2 + \omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
$\frac{df}{dt}$	$sF(s) - f(0^-)$
$\int_0^t f(t) dt$	$\frac{1}{s} F(s)$

Properties of LT

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Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency shift	$e^{-at}f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2 f}{dt^2}$	$s^2 F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3 f}{dt^3}$	$s^3 F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \dots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(t) dt$	$\frac{1}{s} F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds} F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s) ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1 - e^{-sT}}$
Initial value	$f(0)$	$\lim_{s \rightarrow \infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s \rightarrow 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

Inverse LT

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- The frequency domain expressions are transformed back to time domain by applying inverse Laplace transform

$$\mathcal{L}^{-1}(F(s)) = f(t) = \frac{1}{2\pi j} \int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s) e^{st} ds$$

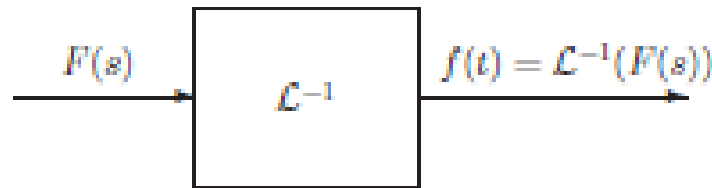


Figure: Representation of the inverse Laplace transform

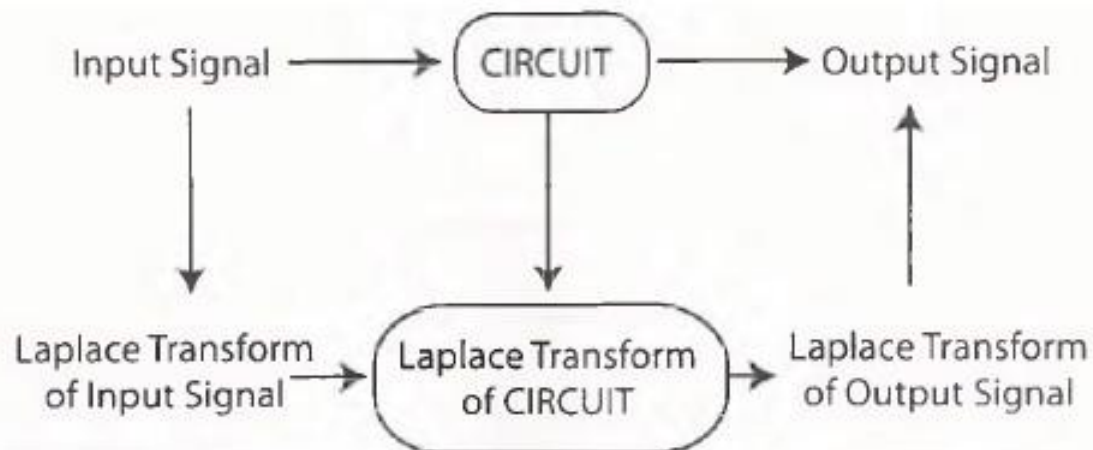
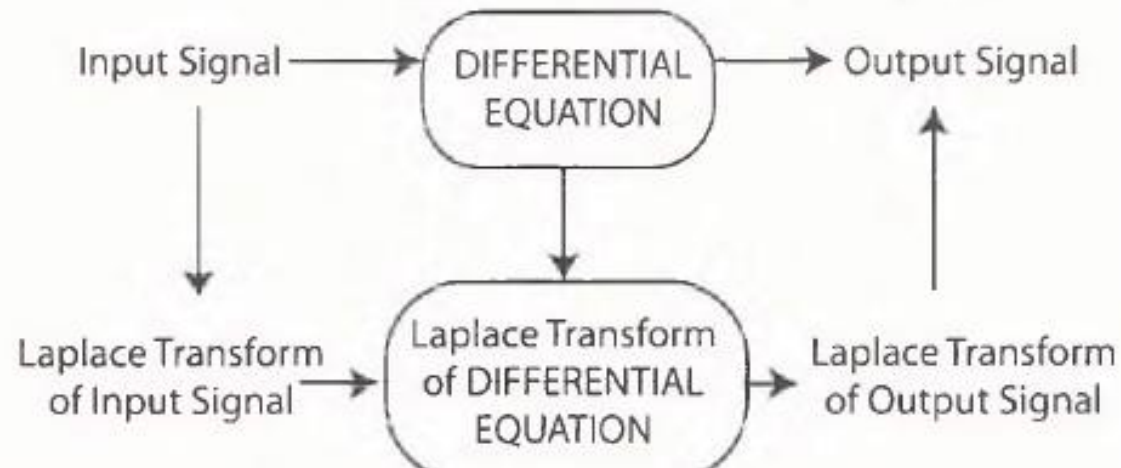
Applications of LT

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- Laplace transform is one of the most powerful mathematical tools for analysis, synthesis, and design
- Steps in applying the Laplace Transform:
 - Transform the circuit from the time domain to the s-domain
 - Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any circuit analysis technique
 - Take the inverse transform of the solution and thus obtain the solution in the time domain

Solution to DE

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Application of LT to Electrical systems

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For a resistor, the voltage-current relationship in the time domain is

$$v(t) = Ri(t)$$

Taking the Laplace transform, we get

$$V(s) = RI(s)$$

For an inductor,

$$v(t) = L \frac{di(t)}{dt}$$

Taking the Laplace transform of both sides gives

$$V(s) = L[sI(s) - i(0^-)] = sLI(s) - Li(0^-)$$

or

$$I(s) = \frac{1}{sL}V(s) + \frac{i(0^-)}{s}$$

For a capacitor,

$$i(t) = C \frac{dv(t)}{dt}$$

which transforms into the s-domain as

$$I(s) = C[sV(s) - v(0^-)] = CsV(s) - Cv(0^-)$$

or

$$V(s) = \frac{I(s)}{sC} + \frac{v(0^-)}{s}$$

Findings

- ▶ We should observe from the above equations that the initial conditions are part of the transformation.
- ▶ This is one advantage of using the Laplace transform in circuit analysis
- ▶ Another advantage is that a complete response transient and steady state of a network can be obtained
- ▶ If we assume zero initial conditions for the inductor and the capacitor, the above equations reduce to:

Resistor: $V(s) = RI(s)$

Inductor: $V(s) = sLI(s)$

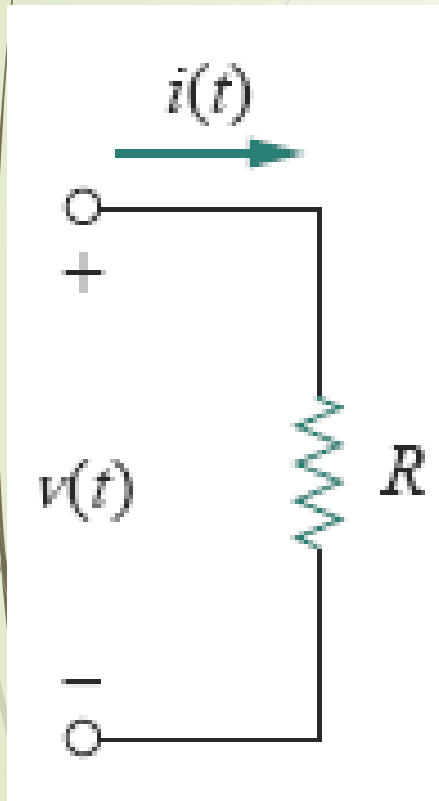
Capacitor: $V(s) = \frac{1}{sC}I(s)$

With zero initial conditions

Resistive circuit

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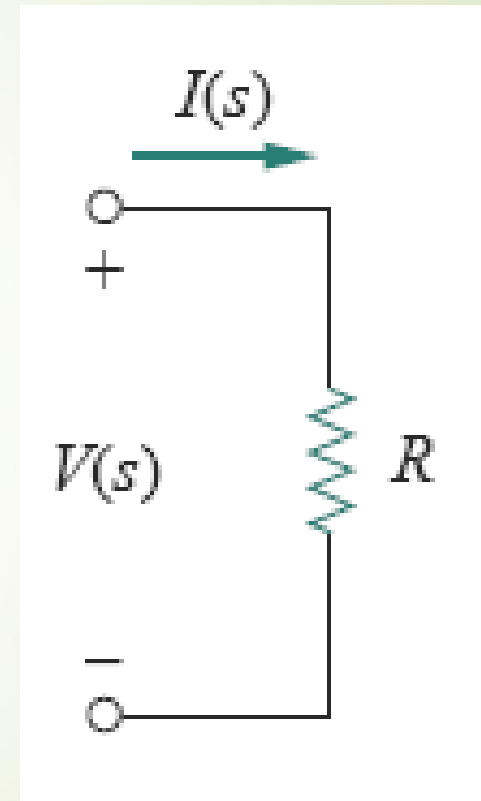
in time domain



impedance

$$\frac{V(s)}{I(s)} = Z(s) = R$$

in frequency domain

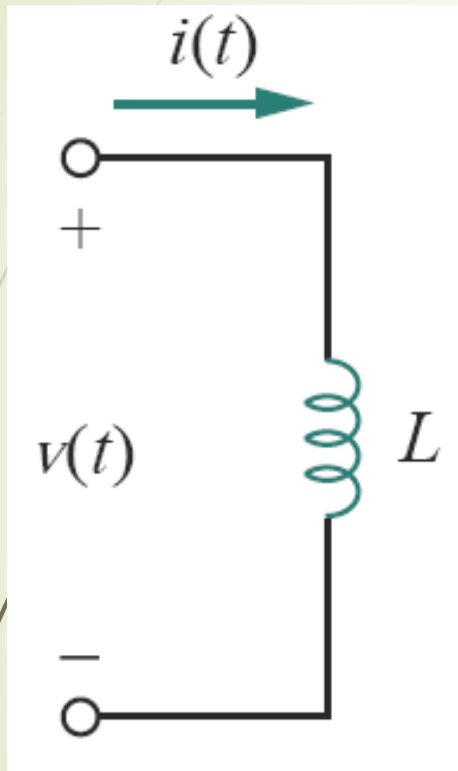


$$v(t) = Ri(t) \Rightarrow L[v(t)] = L[Ri(t)] \Rightarrow V(s) = RI(s)$$

Inductive circuit

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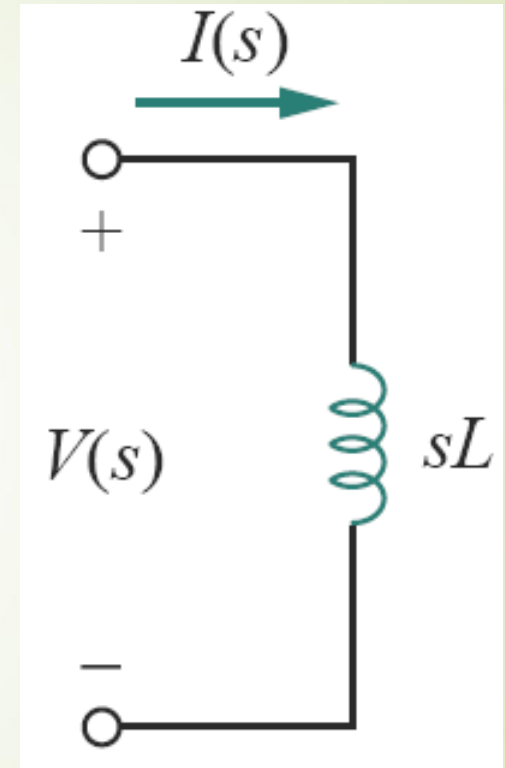
in time domain



impedance

$$\frac{V(s)}{I(s)} = Z(s) = sL$$

in frequency domain

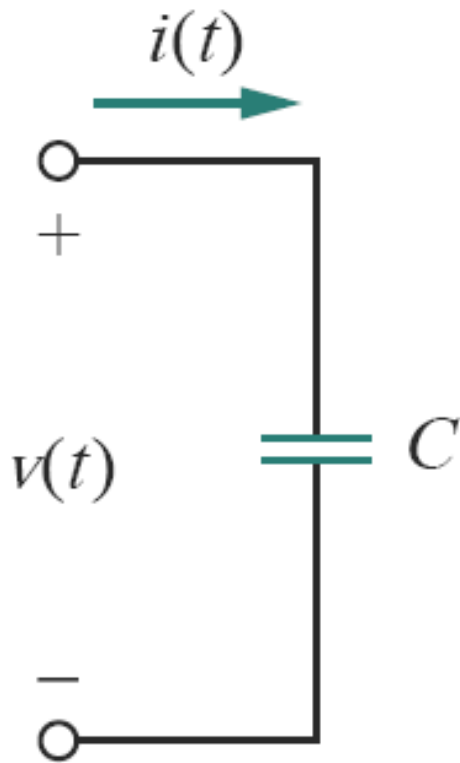


$$v(t) = L \frac{di(t)}{dt} \quad \Rightarrow \quad \mathcal{L}[v(t)] = \mathcal{L}\left[L \frac{di(t)}{dt}\right] \quad \Rightarrow \quad V(s) = sLI(s)$$

Capacitive circuit

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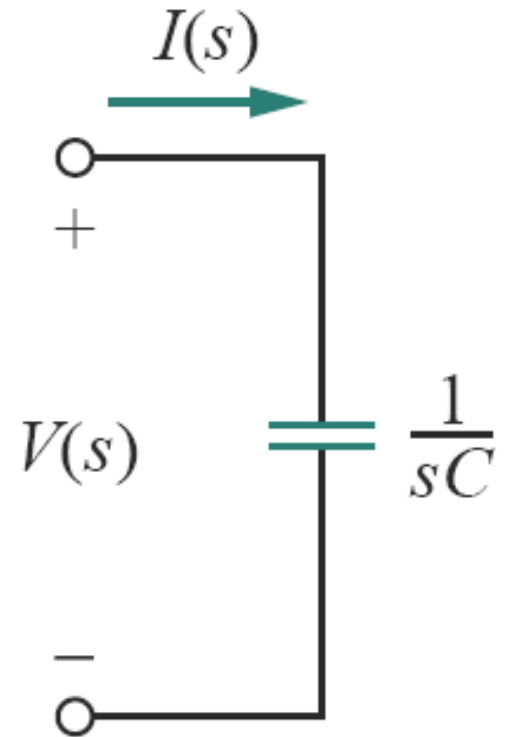
in time domain



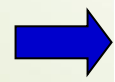
impedance

$$\frac{V(s)}{I(s)} = Z(s) = \frac{1}{sC}$$

in frequency domain



$$i(t) = C \frac{dv(t)}{dt}$$



$$\mathcal{L}[i(t)] = \mathcal{L}\left[C \frac{dv(t)}{dt}\right]$$



$$I(s) = sCV(s)$$

$$V(s) = \frac{1}{sC} I(s)$$

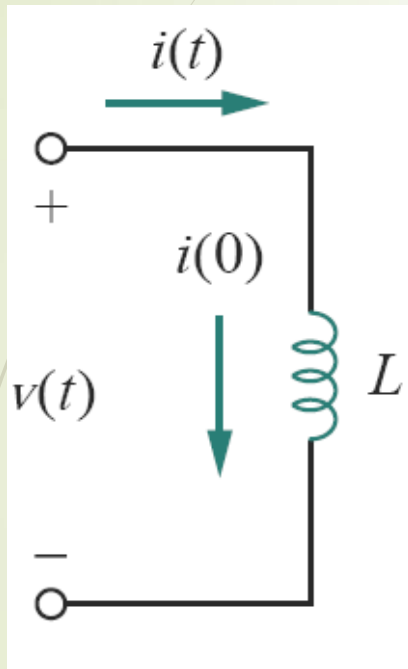
With Initial conditions

Inductive circuit with initial current

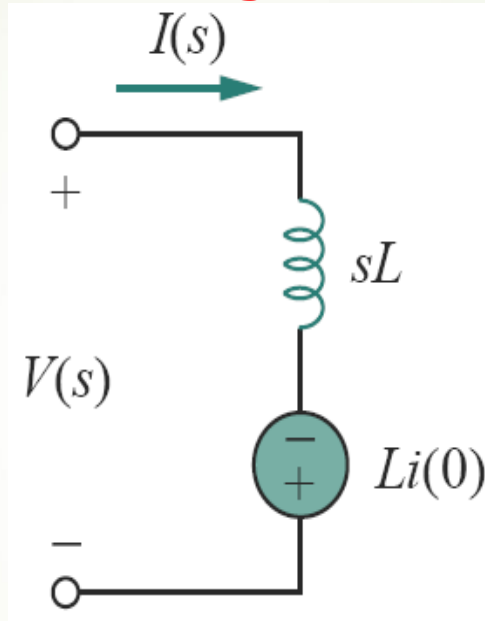
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in frequency domain

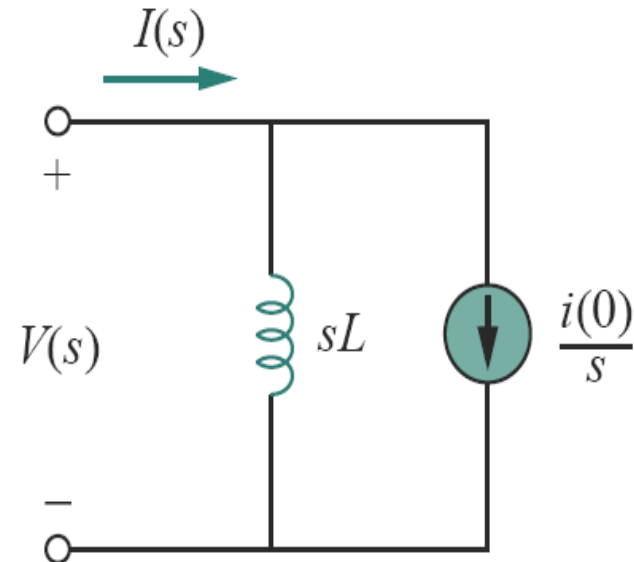
in time domain



With voltage source



With current source



$$\mathcal{L}[v(t)] = \mathcal{L}\left[L \frac{di(t)}{dt}\right]$$

$$V(s) = sLI(s) - Li(0^-)$$

or

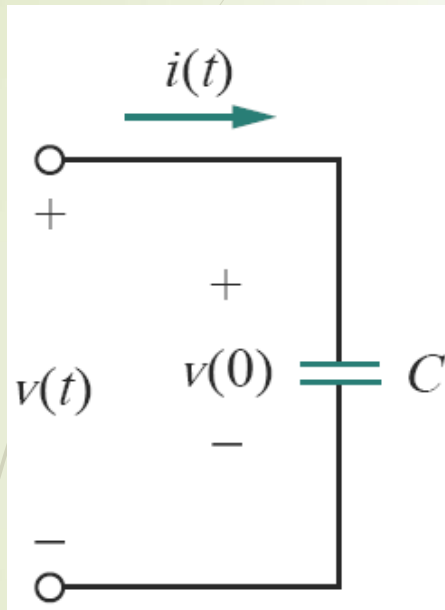
$$I(s) = \frac{1}{sL} V(s) + \frac{i(0^-)}{s}$$

Capacitive circuit with initial current

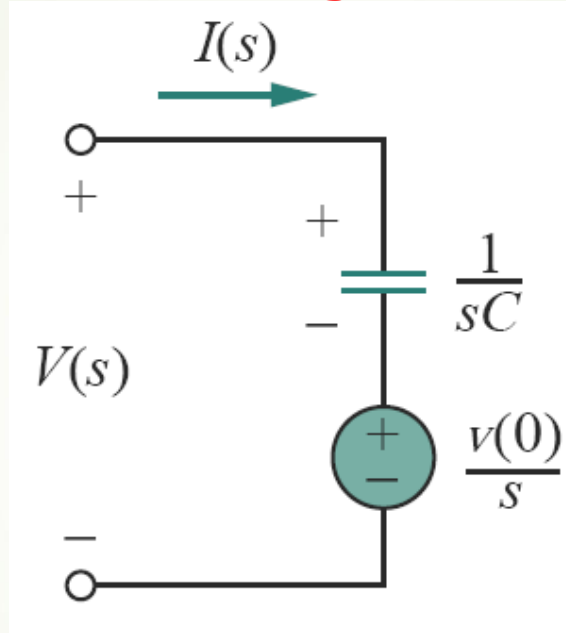
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in frequency domain

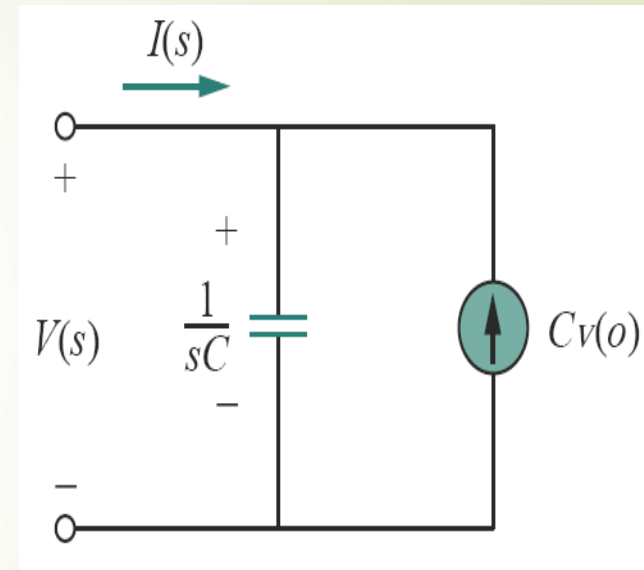
in time domain



With voltage source



With current source



$$\mathcal{L}[i(t)] = \mathcal{L}\left[C \frac{dv(t)}{dt}\right]$$

$$I(s) = sC V(s) - C v(0^-)$$

or

$$V(s) = \frac{1}{sC} I(s) + \frac{v(0^-)}{s}$$